Homework assignment 4

Problem 1: Eady edge waves

Consider flow over topography in the case of Eady edge waves: suppose the quasi-geostrophic potential vorticity is constant, $\beta = 0$, and S [the stratification parameter in pressure coordinates] is constant. The meridional potential temperature gradient is constant, so the vertical shear is also constant. Let the topography be at height

$$z = h_0 \cos(kx)$$

Think carefully about the boundary conditions to apply at $p = p_0$ and find the structure of the forced waves. Discuss in comparison with the case solved in class.

Problem 2: Quasi-geostrophic Rossby waves

Consider the quasi-geostrophic equation for potential vorticity in pressure coordinates on the β-plane. Suppose that S, the stratification parameter, is constant. Find the free Rossby wave dispersion relationship for this system substituting solutions of the form $\psi \sim \exp(\mathrm{i}kx + \mathrm{i}ly + \mathrm{i}mp - \mathrm{i}\omega t)$. These waves are the three dimensional generalization of the two dimensional Rossby waves discussed in class.

Now consider the dispersion relationship for waves between rigid boundaries at $p = p_0$ and $p = p_t$. In this case you might use solutions of the form $\phi(x, y, p, t) \sim \exp(\mathrm{i}kx + \mathrm{i}ly - \mathrm{i}\omega t)\cos(m\pi(p - p_t)/(p_0 - p_t))$. Why? What's the disperion relationship in this case?

Problem 3: Two–layer waves

The two–layer model is commonly used for atmospheric and oceanic studies. It has an upper layer of mean depth H_1 and density $\rho_0 - \Delta\rho$ and a lower layer of mean depth H_2 and density ρ_0. The equations for the QG streamfunctions in each of the layers are

$$\frac{d}{dt}q_1 + J(\psi_1, q_1) = 0 \tag{1}$$
$$\frac{d}{dt}q_2 + J(\psi_2, q_2) = 0 \tag{2}$$
$$q_1 = \nabla^2\psi_1 - F_1(\psi_1 - \psi_2) + \beta y \tag{3}$$
$$q_2 = \nabla^2\psi_2 - F_2(\psi_2 - \psi_1) + \beta y \tag{4}$$

with $F_1 = f_0^2/g'H_1$, $F_2 = f_0^2/g'H_2$ and g' the "reduced gravity" being $g\Delta\rho/\rho$.

1) Suppose you have potential vorticities with only upper layer anomalies

$$q_1 = \beta y + A\cos(kx) \qquad q_2 = \beta y \tag{5}$$

Invert to find the flows. How big are the lower layer flows compared to the upper layer flows? What direction are they?

2) Find the plane Rossby wave solutions. For a given k and ℓ, what are the frequencies and vertical structures (ratio of the amplitudes $|\psi_2|/|\psi_1|$)? Compare these to the Rossby waves in the continuously stratified you studied in problem 2.