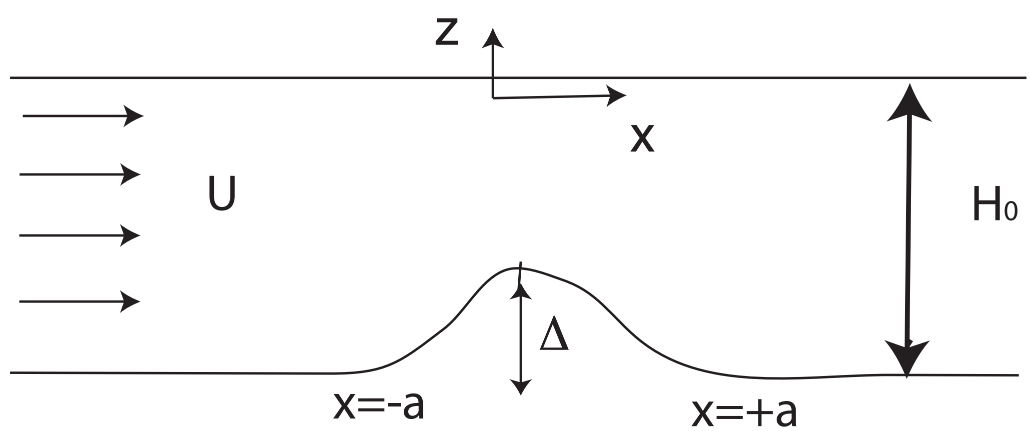
Homework assignment 2

Problem 1

A constant density ocean flows over a submarine ridge as shown in the sketch. the upstream flow is uniform with velocity U. Assume that the flow is steady, and that the ridge width is large compared to the fluid depth, so that the shallow water equations apply. Assume that the velocity field, like the topography, is independent of y. Assume that the Coriolis parameter is constant.



1. Solve for the velocity and the surface elevation, assuming that the change in surface elevation is small compared to Δ. Then state the condition under which this assumption is true. What is the change in surface elevation across the ridge? How does the flow change when $\beta \neq 0$?

Hint: You may wish to start by proving that, in steady flow, the potential vorticity and the Bernoulli functional,

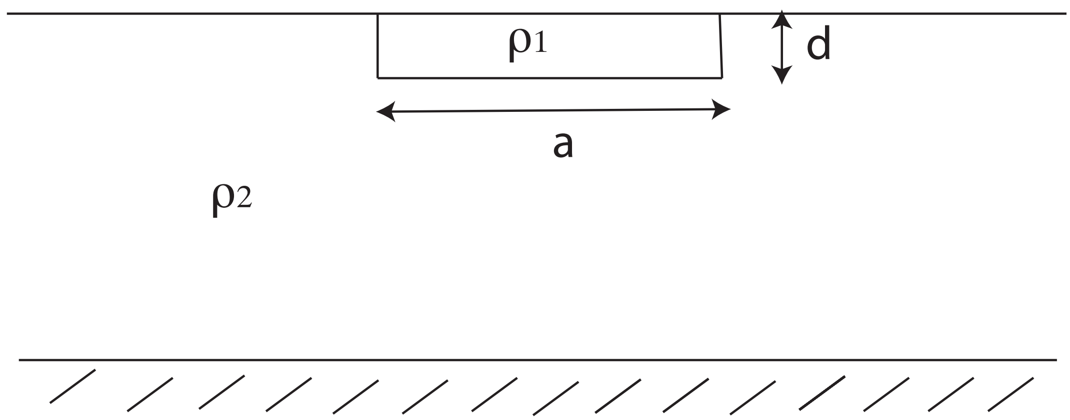
$$\frac{1}{2}u^2 + \frac{1}{2}v^2 + g\eta, \tag{1}$$

are constant along streamlines.

2. Repeat your claculations using quasi-geostrophic theory and compare the two asnwers.

Problem 2

A ribbon of fluid of width a, depth d, and constant density ρ_1, lies atop a homogeneous fluid of density $\rho_2 > \rho_1$. The fractional density difference $(\rho_2 - \rho_1)/\rho_1$ is small. The reference frame is in rotation at a constant angular velocity $f_0/2$, and $d \ll H_0$ (see sketch). Initially both fluids are at rest and the free surface is flat. After a sufficient time, the system evolves to a steady state.



Using shallow-water dynamics, calculate the shape of the interface and free surface, and the velocity in both layers, in the final state. Illustrate your solution with a qualitatively accurate sketch. What would be the finale state be like if $f_0 = 0$?

Hints: 1) Use the conservation of potential vorticity in both layers. 2) Assume that the free surface displacement is small compared to the interface displacement and that $|v_2| \ll |v_1|$, but justify these assumptions a posteriori. 3) Be alert to symmetries.