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Norwegian Deepwater Program: Damping of Vortex-Induced Vibrations

Kyrre Vikestad, MARINTEK, Carl M. Larsen, NTNU, and J. Kim Vandiver, MIT

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Abstract

Vortex-induced vibration (VIV) of a long riser in sheared current is often considered as an energy balance problem: Excitation forces in the power-in region add an equal amount of energy to the system as is dissipated by damping forces outside this region and structural damping. A riser may have different excitation and damping regions depending on the actual oscillation frequency, cross-section properties and local flow velocity. A damping model must hence be able to handle higher and lower flow velocities than the excitation velocity range. In this paper the fluid damping models proposed by Venugopal [1] are compared with the experiments conducted by Gopalkrishnan [2] and Vikestad [3]. The results show that the models are conservative at high and low reduced velocities.

Introduction

VIV is dependent on many factors such as the Reynolds number, flow velocity and turbulence of the incident current, surface roughness, cross-section shape, inclination, motion of the structure, etc. No solutions or models exist which can account for all these factors.

The solution has therefore been to simplify the interaction problem as much as possible hopefully without loosing the information important for the prediction of the resulting VIV and fatigue life of the riser. One such simplification is to assume that each cross-section along the riser is oscillating harmonically in the cross-flow direction only and that the incident flow is constant in time. Then the fluid forces can be split into two parts; the inertia part in phase with the acceleration and the excitation/damping part in phase with the velocity of the cross-section. If the force in phase with velocity has the same sign as the velocity it is excitation, if it has opposite sign it is damping.

In this paper we compare the damping model presented by Venugopal [1] to other experiments. The model predicts the damping force for oscillations in still water, at velocities lower than the excitation velocity range, and at higher. The Venugopal model was based on published empirical damping data from sub-critical flow experiments. In this paper the Venugopal model is further tested against the experimental data gathered by Vikestad [3]. The model was found to be slightly conservative in that it tends to underestimate the damping, but not overly so.

The special feature of Vikestad's experiments is that they reveal the damping on a freely vibrating spring-mounted cylinder. The damping is evaluated at a frequency which is different than the local vortex shedding frequency. This emulates the situation when two frequencies are competing on a long riser subjected to sheared current. Response from one excitation region is damped out in the excitation region of the other frequency.

Damping of VIV

Dynamic equilibrium

The dynamic equilibrium equation for a single degree of freedom system with mass m, structural damping c_{str}, stiffness k, and an excitation force F_v is given as:

$$m\ddot{x} + c_{str}\dot{x} + kx = F_v \qquad (1)$$

If F_v is a single harmonic force, $F_v = F_0 \sin(\omega t+\phi)$, and ϕ is the phase between the force and the motion, $x = x_0 \sin(\omega t)$, Eq. (1) can be written as

$$(m + am)\ddot{x} + (c_{str} + c_f)\dot{x} + kx = 0 \qquad (2)$$

where am is the added mass:

$$am = \frac{F_0 \cos(\phi)}{\omega^2 x_0} \quad [\mathrm{kg}] \qquad (3)$$

and c_f is the excitation force:

$$c_f = -\frac{F_0 \sin(\phi)}{\omega x_0} \quad \left[\frac{\mathrm{N}}{\mathrm{m/s}}\right] \qquad (4)$$

If c_f is negative it is exciting the system (putting energy into the system). For a continuous system, such as a riser, the coefficient, c_f, has units of force per unit length per unit velocity.

Venugopal's damping model

Venugopal [1] proposed three damping expressions, one each for still water, low reduced velocity, and high reduced velocity. The reduced velocity is given as $U_R = 2\pi U/(\omega D)$ where U is the incident velocity, D is the diameter, and ω is the frequency of cross-flow oscillation. High and low reduced velocities refer to higher and lower than the reduced velocities corresponding to power in. The following expressions correspond to the damping coefficient c_f, for a continuous system, and have dimensions [(N/m)/(m/s)]). The models are based on several experiments, with Gopalkrishnan's [2] as one of the more important.

The damping force coefficient on a cylinder section with diameter D, oscillating with cross-flow amplitude of x_0, frequency ω, in a fluid with density ρ, viscosity ν, and incident velocity U (if current) is given as:

1. Damping in still water:

$$c_{sw} = \frac{\omega\pi\rho D^2}{2}\left[\frac{2\sqrt{2}}{\sqrt{Re_\omega}} + k_{sw}\left(\frac{x_0}{D}\right)^2\right] \qquad (5)$$

where $Re_\omega = \omega D^2/\nu$. The first part corresponds to the skin friction according to Stoke's law. The second part is the pressure-dominated force. The factor k_{sw} is a value found from curve fitting to be 0.25.

2. Low reduced velocity damping:

$$c_1 = c_{sw} + \rho DUC_{vl} \qquad (6)$$

The damping is increasing linearly with respect to the incident flow velocity. The coefficient C_{vl} was found to be 0.18 based on measurements.

3. High reduced velocity damping:

$$c_2 = \rho\frac{U^2}{\omega}C_{vh} \qquad (7)$$

This coefficient is independent of the amplitude ratio. The coefficient C_{vh} was found to be 0.2 based on measurements. Note that the damping in Eq. (7) is independent on the diameter. However, Vandiver [4] shows that by introducing the local vortex shedding frequency

$$\omega_s = 2\pi St\frac{U}{D} \qquad (8)$$

the high reduced velocity damping coefficient may be expressed as

$$c_2 = \frac{1}{2\pi St}\frac{\rho DC_{vh}U\omega_s}{\omega} \qquad (9)$$

where ω is the vibration frequency.

Damping expressed with non-dimensional coefficients.

The damping force model, $F_{Damp} = -c_f\dot{x}$, may be expressed in a non-dimensional way by transforming it into an equivalent lift force (with negative lift coefficient),

$$F_L(t) = \frac{1}{2}\frac{\rho DU^2C_L}{\omega x_0}\dot{x} \qquad (10)$$

By requiring the same energy loss per cycle using F_L we get the following relation between the dimensional force coefficient and the non-dimensional lift coefficient:

$$c_f = -\frac{1}{2}\frac{\rho DU^2C_L}{\omega x_0} \qquad (11)$$

If the reduced velocity is low, and especially in still water, the cylinder oscillating in the cross-flow direction will mainly go back and forth in its own wake, as it is for still water oscillations. In that case the damping force may be expressed using a drag coefficient and Morison's equation. The force is at every location along the riser supposed to be in phase with the velocity of the cylinder, and expressed as:

$$F_D(t) = -\frac{1}{2}\rho DC_D|\dot{x}|\dot{x} \qquad (12)$$

The value of C_D is generally found from experiments where the calculated dissipated energy per oscillation cycle, $T\overline{F_D\dot{x}}$, (period T times the average dissipated energy) is equal to the measured average energy dissipation per cycle. Here we will use negative sign for energy lost from the system, and positive if the force puts energy into the system. Dissipated energy per cycle is (using $x = x_0 \sin(\omega t)$)

$$T\overline{F_D\dot{x}} = \int_0^T F_D\dot{x}\,dt = \int_0^T -\frac{1}{2}\rho DC_D|\dot{x}|\dot{x}^2 dt = -\frac{4}{3}\rho D\omega^2 x_0^3 C_D \qquad (13)$$

From Eq. (13) we may find the drag coefficient that gives the same damping per cycle as was measured in experiments. By doing the same exercise with the force coefficient c_f, we find that the dissipated energy per cycle from the damping force $F_{Damp} = -c_f\dot{x}$, is

$$T\overline{F_{Damp}\dot{x}} = \int_0^T F_{Damp}\dot{x}\,dx = -\pi\omega x_0^2 c_f \qquad (14)$$

Equating the two expressions leads to

$$c_f = \frac{4}{3\pi}\rho D\omega x_0 C_D \qquad (15)$$

By using non-dimensional force coefficients and the reduced velocity, we may express the Venugopal damping model in terms of the drag coefficient in Eq. (12) or as the lift coefficient in Eq. (10).

We will then arrive at the following results:

1. Damping in still water:
Expressed using drag coefficient $C_{D,\,sw}$

$$C_{D,sw} = \frac{3\sqrt{2}\pi^2}{4\sqrt{Re_\omega}} \frac{1}{\left(\frac{x_0}{D}\right)} + \frac{3k_{sw}\pi^2}{8}\left(\frac{x_0}{D}\right) \quad (16)$$

2. Low reduced velocity damping:
Expressed using drag coefficient $C_{D,lv}$

$$C_{D,lv} = \frac{3}{8}\frac{U_r}{\left(\frac{x_0}{D}\right)} C_{vl} + C_{D,sw} \quad (17)$$

Expressed as a lift coefficient $C_{L,lv}$

$$C_{L,lv} = \frac{-8\sqrt{2}\pi^3}{\sqrt{Re_\omega}} \frac{\left(\frac{x_0}{D}\right)}{U_R^2} - 4\pi^3 k_{sw} \frac{\left(\frac{x_0}{D}\right)^3}{U_R^2} - 4\pi C_{vl} \frac{\left(\frac{x_0}{D}\right)}{U_R} \quad (18)$$

3. High reduced velocity damping:
Expressed using drag coefficient $C_{D,hv}$

$$C_{D,hv} = \frac{3}{16\pi} \frac{U_R^2}{\left(\frac{x_0}{D}\right)} C_{vh} \quad (19)$$

Expressed as lift coefficient $C_{L,hv}$

$$C_{L,hv} = -\frac{2x_0}{D} C_{vh} \quad (20)$$

The damping model is visualized in Fig.1 (as a lift coefficient) and in Fig.3 (as a drag coefficient). It can be compared directly with the experimental results due to Gopalkrishnan [2], which is plotted as a lift coefficient in Fig.2 and as a drag coefficient in Fig.4.

It is important to keep in mind that the damping models are used outside the excitation bandwidth (U_R about 5 to 8). Gopalkrishnan's general lift coefficient data are shown also within the excitation range, and it is seen that here we will have damping for large amplitude ratios. This must be taken into account when defining lift forces within the excitation zone.

The different ways of expressing the damping does not change the important issue; the energy going out from the system per length of the riser per cycle is independent of how we chose to illustrate it. We also notice that the damping force coefficient is very dependent on the amplitude. This means that dynamic equilibrium must be found by iteration in order to find a stable solution that satisfies that the averaged power in equals averaged power out.

Present experiments

Vikestad [3] did in his doctoral work experiments with externally forced elastically mounted cylinder in fluid flow. The results were further analyzed in Vikestad [5]. The apparatus used is shown in Fig.5 and some key properties are given in Table 1. The external force was in the cross-flow direction, and was imposed by a harmonic motion of the support with a given frequency f_{ex}. The corresponding cylinder amplitude was x_{ex}. The response amplitude at the same frequency was therefore a balance between the excitation forces and the damping forces. The damping forces were calculated as a negative lift coefficient for the given flow velocity, oscillation frequency, and resulting response, corresponding to the measured dissipated energy.

In the experiments 23 different velocities were used ($U/(f_0D)$ varied from 2.8 to 13.1, f_0 was the still water natural frequency). They were combined with 12 different excitation frequencies (0.5 to 2.0 times f_0) and three different support-motion amplitudes (corresponding to static amplitudes of 0.13, 0.26, and 0.39 times the diameter of the cylinder).

The available combinations of response level at the external frequency, x_{ex}/D, and the corresponding reduced velocity, $U_R = 2\pi U/(\omega_{ex}D)$, are given in Fig.6. Data points for U_R between 4 and 9 were not used because that is the excitation region. For each of these points the lift force coefficient was calculated. The new information available in these experiments is measured hydrodynamic damping on a cylinder free to respond to local vortex shedding. Similar experiments have, to the best of our knowledge, not been performed before.

The results for low reduced velocities are given in Fig.7. For each lift coefficient found in the experiment, the corresponding predicted lift coefficient using Venugopal's model is given. The figure shows that the damping found in the experiments for all but one case is higher than the model. It may be tempting to define a new model, but the data basis in the present experiment is not sufficient. The amplitude ratio is to small, and also the reduced velocity range should cover lower values.

The results for high reduced velocities are given in Fig.8. Here the lift coefficients are plotted as a function of the amplitude ratio. The predicted lift coefficients using Venugopal's model are shown as the solid line. The figure shows that for the majority of cases, the Venugopal model is conservative.

The still water damping model as defined by Venugopal is shown in Eq. 5. It has been compared to measurements by Vikestad [6]. This comparison is shown in Figure 9 and shows the results from three decay tests of a very dense elastically mounted rigid cylinder. The damping force coefficient was calculated from the energy loss for each cycle, taking the known structural damping into account. The scatter at low A/D is due to the resolution of the position meter. At high A/D the model predicts lower than measured still water damping, and is therefore conservative, when used to predict response amplitude or fatigue damage rate.

Conclusions

From the present work we conclude that:

1. The damping force must be given as a function of the amplitude over diameter ratio and reduced velocity. This means that the VIV problem must be solved by iteration.
2. The damping data from Vikestad for damping in the VIV-region of other frequencies than the VIV frequency indicates more damping than estimated by Venugopal's model (based on a single frequency response).
3. Venugopal's model seems to be conservative, but not so much that a revision is required. The conservatism should still be there to account for uncertainties in the experimental data.

In the present investigation only sub-critical flow has been tested and compared. The general assumption (at least in the VIV-prediction programs) that the sub-critical flow results may be extrapolated into the critical flow regime has not undergone much testing. In lack of such tests, the damping models (and probably also the excitation models) should be used with care for high Re.

Acknowledgments

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Nomenclature

A = oscillation amplitude found in experiments [m]
am = added mass [kg]
c_1 = low reduced velocity damping force coefficient [N/(m/s)/m]
c_2 = high reduced velocity damping force coefficient [N/(m/s)/m]
C_D = drag force coefficient [-]
$C_{D,hv}$ = Venugopal's high U_R model expressed as drag force coefficient [-]
$C_{D,lv}$ = Venugopal's low U_R model expressed as drag force coefficient [-]
$C_{D,sw}$ = Venugopal's still water model expressed as drag force coefficient [-]
c_f = fluid damping force coefficient [N/(m/s)] (or [(N/m)/(m/s)])
C_L = lift force coefficient [-]
$C_{L,hv}$ = Venugopal's high U_R model expressed as a lift force coefficient [-]
$C_{L,lv}$ = Venugopal's low U_R model expressed as a lift force coefficient [-]
c_{str} = damping force coefficient due to structural damping [N/(m/s)] (or [N/(m/s)/m])
c_{sw} = damping force coefficient in still water [N/(m/s)/m]
c_{vh} = 0.2, experimentally found parameter [-]
c_{vl} = 0.18, experimentally found parameter [-]
D = cylinder diameter [m]
F_0 = amplitude of F_v [N]
F_D = fluid drag force in phase with the velocity of the structure squared [N/m]
F_{Damp} = damping force [N/m]
f_{ex} = external disturbance frequency [Hz]
F_L = harmonic fluid excitation force in phase with the velocity of the structure [N/m]
f_{osc} = f_{ex}, oscillation frequency due to external disturbance [Hz]
F_v = harmonic fluid force [N]
k = restoring force coefficient [N/m]
k_{sw} = 0.25, curve fitting parameter [-]
m = structural mass [kg]
Re_ω = $\omega D^2/\nu$ [-]
St = Strouhal number [-]
U = incident flow velocity [m/s]
U_R = reduced velocity, $2\pi U/(\omega D)$ [-]
T = period of one cycle [s]
t = time variable [s]
x_0 = displacement amplitude [m]
x = cross-flow displacement of the cross-section[m]
$\dot{x}$ = cross-flow velocity of the cross-section [m/s]
$\ddot{x}$ = cross-flow acceleration of the cross-section [m/s^2]
x_{ex} = response amplitude at frequency f_{ex} [m]
ν = kinematic viscosity of the fluid [-]
ρ = density of the fluid [kg/m^3]
ϕ = phase angle between force and motion [rad]
ω = circular oscillation frequency, $2\pi/T$ [rad/s]
ω_{ex} = $2\pi/f_{ex}$ [rad/s]
ω_s = local circular vortex shedding frequency [rad/s]

References

1. Venugopal, M.: *Damping and Response of a Flexible Cylinder in a Current*, Ph.D. thesis, Dept. of Ocean Eng., MIT, USA, 1996.
2. Gopalkrishnan, R.: *Vortex-Induced Forces on Oscillating Bluff Cylinders*, Sc. D. thesis, Dept. of Ocean Eng., MIT, and Dept. Applied Ocean Phys. and Eng., WHOI, USA, 1993.
3. Vikestad, K.: Multi-frequency response of a cylinder subjected to vortex shedding and support motions, Ph.D. thesis, Faculty of Marine Tech., NTNU, Norway, 1998.
4. Vandiver, J.K.: "Predicting Lock-in on Drilling Risers in Sheared Flows", *Proc. Flow-Induced Vibration 2000*, Lucerne, Switzerland, June 2000.
5. Vikestad, K.: *Hydrodynamic damping of non-VIV oscillations of a circular cylinder section subjected to constant current,* MARINTEK Report No. MT51 F99-381, November 1999.
6. Vikestad, K.: *Vortex-induced vibration (VIV) on an elastically mounted circular cylinder in constant and variable flow, MARINTEK* Report No. MT51 F99-213, June 1999.

Metric Conversion Factors

m × 3.2808 E+00 = ft
kg × 6.85 E-02 = slug

N × 2.248 E-01 = lb.

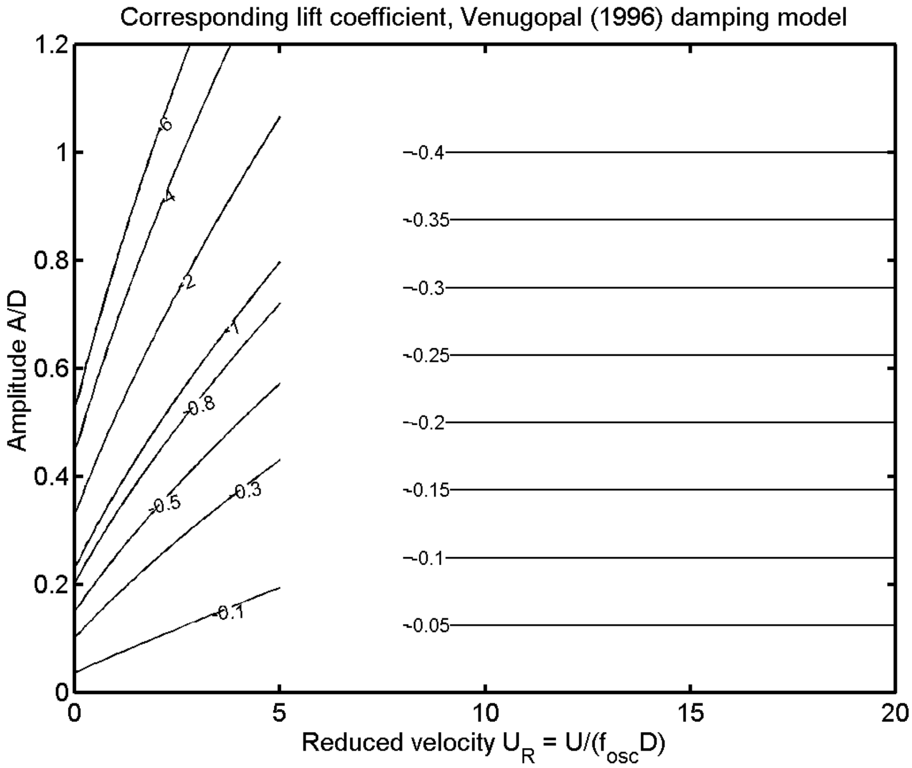

Fig. 1–Venugopal [1] model expressed as lift coefficient.

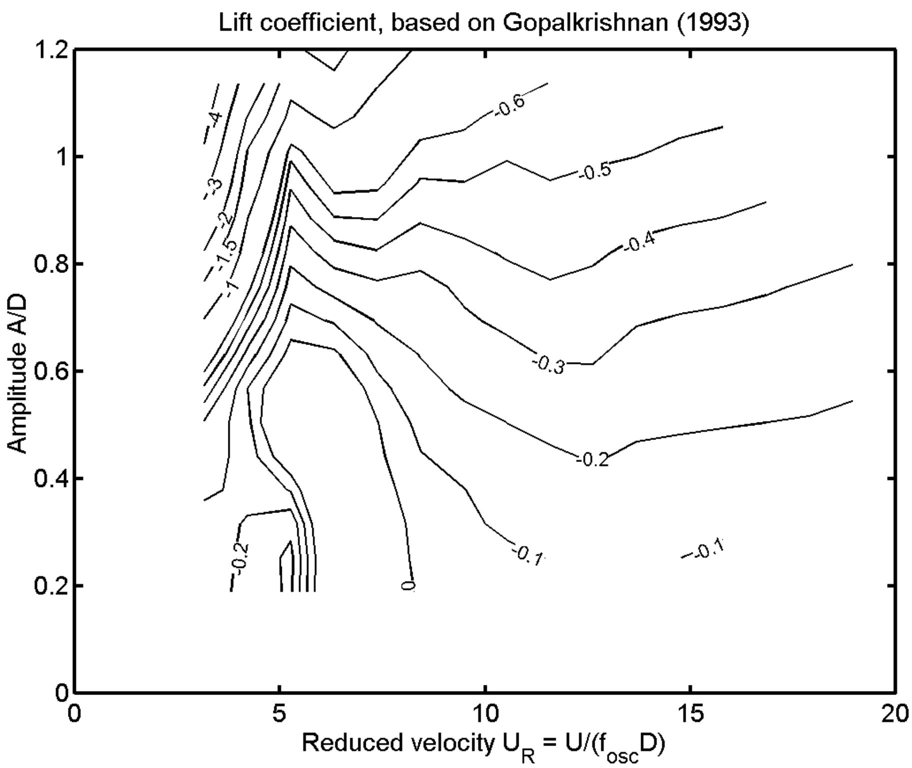

Fig. 2–Lift coefficients extracted from Gopalkrishnan [2].

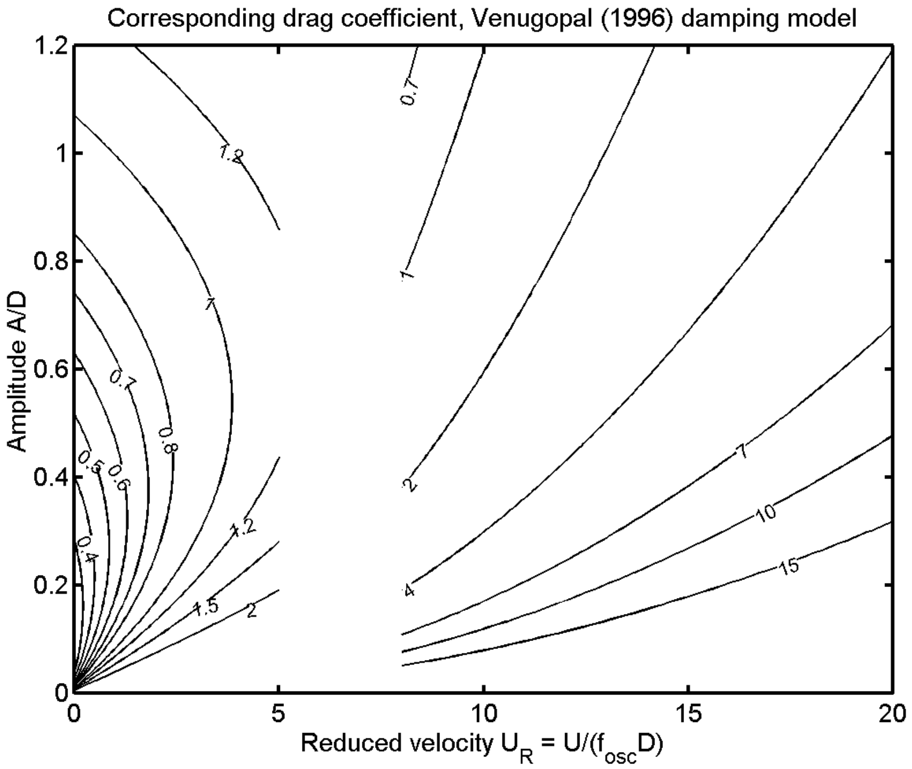

Fig. 3–Venugopal [1] model expressed as drag coefficient.

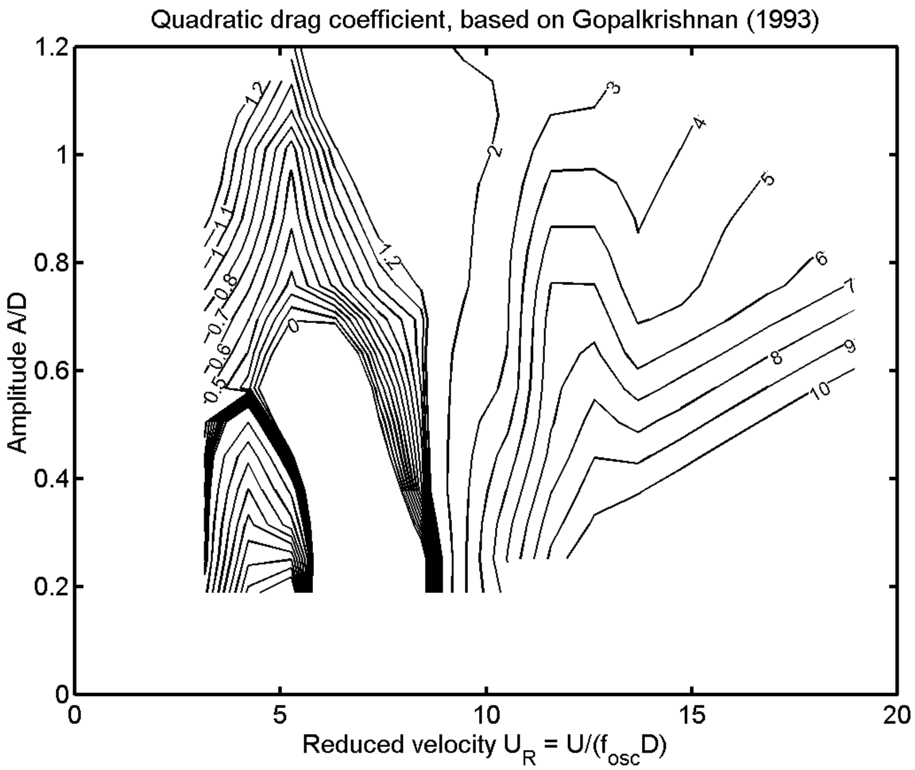

Fig. 4– Drag coefficients based on Gopalkrishnan [2].

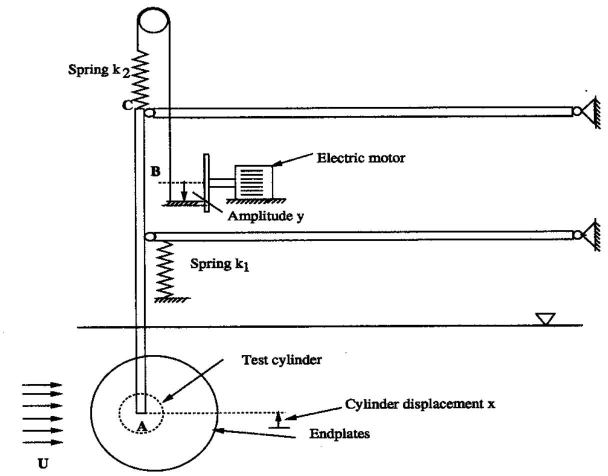

Fig. 5–The apparatus used by Vikestad [3].

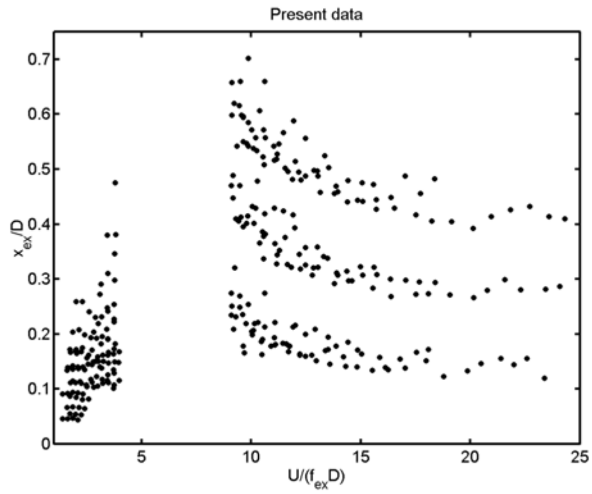

Fig. 6–Available data points

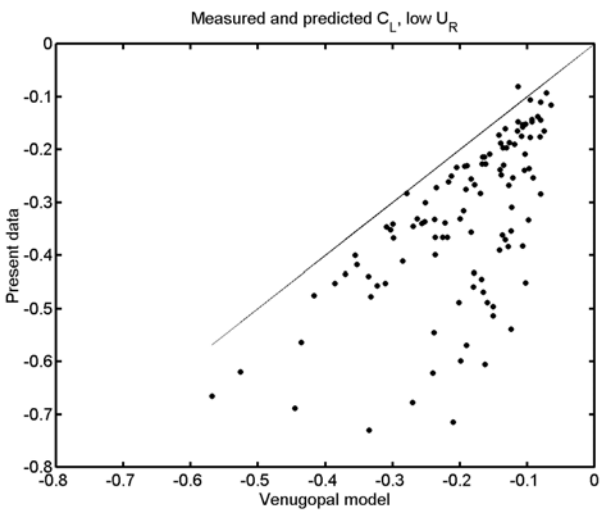

Fig. 7–Low reduced velocity damping results compared to model.

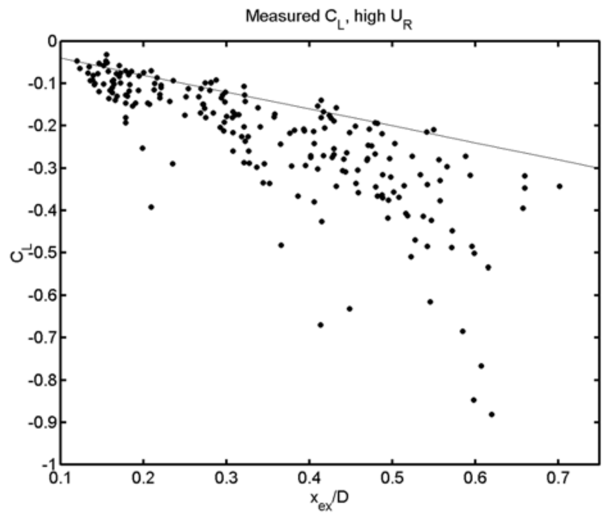

Fig. 8–High reduced velocity damping results compared to model.

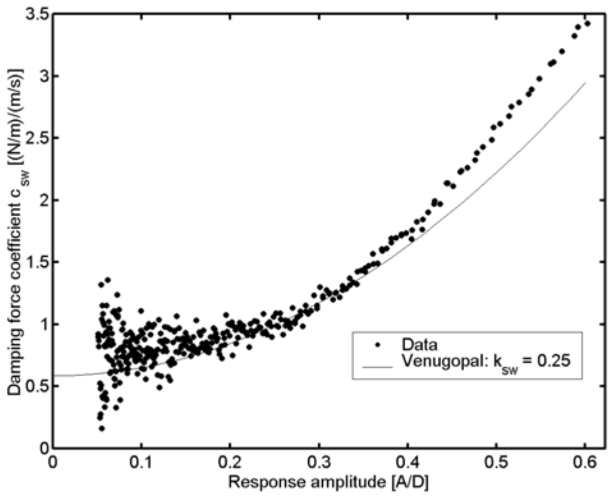

Fig. 9–Still water damping from decay tests (From Vikestad [6])

TABLE 1–Apparatus properties*

Cylinder length	L	2	m
Cylinder diameter	D	0.10	m
Total stiffness	k_{tot}	415	N/m
Natural frequency in still water	f_0	.497	Hz
Specific gravity (dry mass/displaced water)		1.664	[-]
Damping ratio (c/(2mω)) in air		.07-.1%	[-]
Damping ratio in water without cylinder		.7 – 1%	[-]

*The dense system used for the decay tests [6] had similar properties, except D=0.075m, k_{tot}=796 N/m, f_0=0.45 Hz, and specific gravity 9.24.