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A Procedure for Predicting the Fatigue Damage of Structural Members in Unsteady Winds

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Abstract

The practice of assuming full, steady state, vortex-induced-vibration response, to wind is excessively conservative [1]. This paper presents a practical method for estimating the reduction of the fatigue damage rate due to the natural unsteady fluctuations in wind speed. Two dimensionless parameters are shown to be particularly important. The first is the ratio of the lock-in bandwidth to the turbulence intensity level of the wind. The second is the ratio of the length of time the wind speed remains in the lock-in interval to the rise time of the member to steady state response levels. The paper also shows that one must account for the discrete nature of wind speed and direction scatter diagrams in estimating the probability of occurrence of the critical wind speed.

Introduction

A 1992 review [2] of state-of-the-art design calculations for wind-induced fatigue damage of structural members revealed substantial excess conservatism due to the assumption of steady state structural response to wind. Further work was recommended to establish the effect of unsteady wind speed variations, turbulence, and finite structural response rise time. These factors have now been investigated by means of wind tunnel tests, analysis of extensive North Sea wind records, and development of appropriate dynamic and probabilistic models.

The theoretical results, wind tunnel data, and statistical analysis of the wind data have been published in two recent papers and a Ph.D. thesis [3, 4 & 5]. This paper synthesizes these results in the form of a proposed, practical procedure for predicting the fatigue life of structural members excited by vortex shedding in wind.

In this paper the members are assumed to vibrate in the first mode in the cross-flow direction. The only types of members studied are restrained at both ends, with boundary conditions which may vary from pinned to fixed. Other structural configurations such as cantilevers could be addressed by the methods described here, but no emphasis has been given to such applications in this study.

Assumptions, Definitions and Objectives

Accepted practice in the industry today is to compute D_{ss}, the steady state fatigue damage rate that results from wind at the critical wind speed, V_c, which causes maximum vibration response. This damage rate is then multiplied by the fraction of the year that the wind is expected to blow at or near to the critical velocity for that member. In this paper three adjustment factors, γ_0, γ_1, and γ_{bin} are introduced, so as to obtain the adjusted damage rate, D_a, as shown below.

$$D_a = \gamma_0 * \gamma_1 * D_{ss} * (\gamma_{bin} * \Sigma P_{V,\theta}) \quad (1)$$

Where $\Sigma P_{V,\theta}$ is the summation of probabilities over the appropriate wind speed and direction bins of the wind scatter diagram. One such scatter diagram is presented in Table 1, which summarizes ten minute mean wind speed and direction data measured over a fifteen year period at Ekofisk in the North Sea [6]. This data will be used in an example calculation.

The first two factors, γ_0 and γ_1, account for the unsteady behavior of natural winds and the finite rise time required by the structural member to reach steady state, given the wind is at the critical wind speed. The third correction factor, γ_{bin}, accounts for the use of discrete wind speed and direction scatter diagrams rather than smooth wind speed probability density functions.

To understand the purpose of γ_0 and γ_1 it is helpful to define two time scales which are important in predicting the fatigue damage rate in unsteady winds. The first is the expected value of the "duration of visit" or uninterrupted length of time that the actual unsteady wind speed stays sufficiently close to the critical wind speed to result in vortex-induced-vibration with lock-in characteristics. The second is the "rise time" or length of time required for structural response to reach steady state.

In this paper the statistical characteristics of unsteady winds are of great importance and therefore must be defined carefully. By unsteady, we do not mean small length scale, high frequency turbulence. Turbulence with a length scale of a few diameters and up to ten per cent in intensity has been shown to not have much effect on response amplitude [5], and will not be discussed further in this paper. When the term turbulence is used in this paper, it refers to unsteady fluctuations of wind speed with a frequency content lower than 0.4 Hz. This is because the maritime wind data used to calibrate the methods proposed in this paper were forty minute records sampled at 0.85 Hz. and only in mean wind speeds greater than or equal to 10.0 m/s [7]. Thus the minimum observed length scale was approximately 25 metres and the maximum frequency of use in the data was 0.42 Hz.

The proposed procedure is a convenient modification of accepted practice, which is to compute the fatigue damage rate that would result from steady state response of a structural member to the critical wind speed for that member. In the interest of keeping the total paper length within imposed limits, it is assumed that the reader is familiar with methods for determining the critical wind speed of the member in question as a function of reduced velocity and Reynold's number, and that the reader is able to estimate the steady state, lock-in, response amplitude, Y_{max}/D, and the resulting fatigue damage rate, D_{ss}. Typical example calculations are presented in references [2 & 8]. It remains to show how one obtains γ_0, γ_1, and γ_{bin}.

Estimation of γ_0

γ_0 accounts for the reduction in fatigue damage which results from the random fluctuation of the wind speed around its mean. In this study it has been assumed that the steady state response amplitude is a simple function of Reduced Velocity, which is defined as:

$$V_R = V/(f_n D) \quad \ldots\ldots (2)$$

For subcritical Reynold's numbers the peak steady state response amplitude, Y_{max}/D, corresponds to a reduced velocity of approximately 6.0 [3 & 5]. When the value of wind speed is such that the response is maximum, then the wind speed is said to be at the critical wind speed, V_c.

The wind tunnel data presented in references [2] and [5] revealed that response amplitude decreases from the peak value as an approximate linear function of reduced velocity. Such a model is used here for the purpose of computing γ_0. It is assumed that the response amplitude decreases linearly with reduced velocity until it reaches zero at winds speeds given by $V = V_c(1 \pm \alpha)$. For typical structural members in winds α is approximately 0.125. In other words the lock-in bandwidth is approximately $\pm$ 12.5% of V_c. When the reduced velocity at peak response is 6.0, this lock-in bandwidth corresponds to a reduced velocity range of $5.25 \leq V_r \leq 6.75$. Factors, such as small length scale turbulence, angle of incidence and Reynold's number may alter these values and other values may be substituted if better information is available. This point will be discussed further in the example calculation.

In wind tunnel measurements reported in [2 & 5] it was concluded that for subcritical Reynold's numbers the lock-in band extended over a range of reduced velocities from 5.0 to 6.5 with the peak at 6.0. This range is asymmetric with respect to V_c and has a lower limit at $V = V_c(1-\alpha)$, with $\alpha = .167$, and the upper limit is at $V = V_c(1+\alpha)$, with $\alpha = .083$. This asymmetric model has been evaluated in [5] and results in values for γ_0 and γ_1 which are equal to those obtained when using the proposed symmetric model with lower and upper values of α equal to the average of 8.3 and 16.7%; i.e. $\pm$12.5%. Other asymmetric lock-in ranges may be approximated by an equivalent symmetric one by the same averaging method.

If the steady state response amplitude is related to reduced velocity as described above, then natural fluctuations of the wind speed will cause the steady state response amplitude to rise and fall. Even if the mean wind speed equals the critical wind speed, the temporal variations of the wind speed will result in lower average response amplitudes than would be experienced if the wind speed stayed constantly at V_c. γ_0 accounts for the resulting reduction in fatigue life. The computation of γ_0 assumes that the response amplitude of vibration is always at the steady state value corresponding to the instantaneous value of the reduced velocity. In other words, the finite vibration response rise time is not taken into account in the computation of γ_0. That will come in the computation of γ_1.

γ_0 is a function of 'm', the slope of the S-N curve, and α/T_0, the ratio of half the lock-in bandwidth to the turbulence intensity, T_0. The mean wind speed of interest is equal to the critical wind speed, hence T_0 is defined as $T_0 = \sigma_v/V_c$. Where, σ_v is the standard deviation of the wind speed with respect to the mean. The data used to calibrate the computation of γ_0 and γ_1 were based on forty minute wind records. Turbulence data based on more conventional one hour samples are adequate for the purpose of applying the methods presented here. Wind speed magnitude but not direction is used in the computation of T_0. Typical values of α/T_0 vary from 0.3 to about 3.0 for values of turbulence varying from 30% down to about 4%. γ_0 is given by the following expression [5].

$$\gamma_0 = \sqrt{\frac{2}{\pi}}\,\frac{\alpha}{T_0}\int_0^1 x^m e^{-\left[\frac{\alpha^2}{2T_0^2}(x-1)^2\right]}dx \quad \ldots\ldots (3)$$

γ_0 is plotted in Figure 1, for various values of 'm' between 3 and 6, as a function of α/T_0. A very typical value of α/T_0 is 1.0. For this value, γ_0 varies from 0.1 to 0.2, depending on 'm'. In other words, just the natural variability of the wind typically reduces the fatigue damage rate by a factor of 5 to 10. γ_0 increases with the ratio α/T_0. This is intuitively satisfying, because, if α/T_0 is large, it means that the wind speed variations will be small compared to the lock-in bandwidth, resulting in long periods of large amplitude lock-in vibration. Conversely, as α/T_0 is made smaller the wind is more likely to stray outside of the lock-in region, resulting in smaller average response values.

Estimation of γ_1 and the Statistical Characteristics of Natural Winds

γ_1 accounts for the finite length of time required for a structural member to reach full steady state response, compared to the typical length of time that the wind speed stays in the lock-in band. In this paper this is expressed as a non-dimensional ratio of the "duration of visit" of the wind speed in the lock-in interval to the rise time of the structural member.

The duration of visit, E[τ]. The concept of the expected duration of visit of the wind speed is depicted in Figure 2, in which time varying wind is shown crossing into and then out of the lock-in wind speed band, defined by the levels 'a' and 'b'. E[τ] is the expected value of the length of time the wind spends in the interval. The duration of visit is shown in [4] to be strongly dependent on the statistics of the wind speed and in particular on the turbulence intensity, $T_0 = \sigma_v/V_m$, and $\sigma_v/\sigma_{\dot v}$, the ratio of the standard deviation of the wind speed, to the standard deviation of the time derivative of the wind speed. T_0 has been widely used to describe wind characteristics in the literature. $\sigma_v/\sigma_{\dot v}$ is new to the literature and when this study began data was unavailable.

Natural wind statistics. The authors conducted an in depth statistical study of approximately 500 hours of maritime wind data from the Statoil Joint Industry Project on Maritime Turbulent Windfield Measurements [7]. The essential results of this study are summarized in Figures 3 and 4. Figure 3 is a plot of T_0 versus mean wind speed, V_m. Figure 4 is $\sigma_v/\sigma_{\dot v}$ versus T_0. The plot of T_0 versus mean wind speed is very disordered. This is because turbulence is sensitive to atmospheric stability and in this plot no attempt has been made to account for variations in stability. The turbulence levels above 20% mostly come from days with highly unstable temperature profiles, which resulted in considerable vertical convection. The low turbulence cases correspond to stable conditions.

Figure 4 shows $\sigma_v/\sigma_{\dot v}$ versus T_0 as computed from the same 500 hours of data. In contrast to Figure 3, which appears disordered, this data is clearly height dependent with an apparent linear relationship between $\sigma_v/\sigma_{\dot v}$ and T_0. Straight line fits to this data are shown in equation 4, which has a slope which is given by a logarithmic function of 'h'. The height above the sea, 'h', is expressed in metres.

$$\sigma_v/\sigma_{\dot v} = 26\log(1.35h)T_0 \text{ in seconds} \quad \ldots\ldots\ldots\ldots (4)$$

This function is plotted in Figure 4 for two values of 'h', 5 and 46 metres. Equation 4 is for North Sea maritime wind data. We believe it to be useful as an approximate model for other maritime locations. The North Sea wind data from the Statoil project show that $\sigma_v/\sigma_{\dot v}$ varies from 2 to 15 seconds, as shown in Figure 4.

In order to model natural winds mathematically it has been necessary to assume a probabilistic model for the random fluctuation of the wind speed with respect to the mean wind speed. The model for the wind that has been assumed here is a Gaussian one. Winds are known not to be truly Gaussian. However, it has been shown that the approximation of a Gaussian probability model for the wind provides conservative estimates for fatigue damage rates [4 & 5]. E[τ] may be expressed as:

$$E[\tau] = (\sigma_v/\sigma_{\dot v}) * G(\alpha/T_o) \text{ -- } (seconds)$$

where

$$G(\alpha/T_o) = \sqrt{2\pi}\,\frac{\int_0^{\alpha/T_o} e^{\frac{-x^2}{2}}dx}{e^{-\frac{\alpha^2}{2T_0^2}}} \quad \ldots\ldots\ldots\ldots (6)$$

and is plotted in Figure 5.

Response rise time, t_r: When a structural member, initially at rest, is excited by a steady sinusoidal lift force at the natural frequency of the member, the envelope of the positive response peaks A(t) has the following mathematical form.

$$A(t) = Y_{max} * (1 - e^{-\zeta\omega_n t}), \text{ where } \omega_n = 2\pi f_n \quad \ldots\ldots\ldots (7)$$

A lightly damped structural member will take considerable time to reach steady state response amplitude. The rise time, t_r, is defined as the time taken for the amplitude of vibration to reach 63% of its steady state value. In the function shown in equation 7 this occurs when the exponent $-2\pi\zeta f_n t = -1$, implying that $t_r = 1/(2\pi\zeta f_n)$. An additional interpretation is possible by noting that $t_r * f_n = t_r/T = 1/(2\pi\zeta)$, which is the number of vibration cycles or periods that it takes the structure to reach 63% of the steady state value. When the damping ratio is .002, this number is 80. The same structure would reach 86% of steady state after 160 cycles and 95% after 240 cycles. Note further that the lower the value of damping the longer it takes the structure to reach steady state.

A typical duration of visit of the wind speed to the lock-in band is about 24 seconds. A structural member with a damping ratio of 0.002 and a natural frequency of 10 Hz would reach 95% of the full steady state value in that time, whereas, a structural member with the same damping but a 4 Hz natural frequency would require 60 seconds to reach the same level. The member with the 4 Hz natural frequency would rarely reach full steady state response levels and would tend to sustain less fatigue damage during the exposure period.

The ratio of duration of visit to rise time, r. In this study the dimensionless parameter 'r' is the ratio of the expected value of the duration of visit to the rise time of the member.

$$r = E[T]*(2\pi\zeta f_n) \qquad (8)$$

If 'r' is much larger than 1.0 then the member may approach steady state vibration levels. If this number is smaller than 1.0 then the structural member will never reach full steady state response.

The computation of γ_1. It has been shown in [5] that γ_1 is a monotonically increasing function of 'r' as shown in Figure 6 for various values of 'm', the slope of the S-N curve. The function for γ_1 is given by:

$$\gamma_1 = [1-\exp(-\beta r^{\delta})] \qquad (9)$$

The parameters β and δ depend on 'm', the slope of the S-N curve. These parameters are tabulated with common values of 'm' varying from 3 to 6 in Table 2.

Estimation of γ_{bin}

γ_{bin}, accounts for the use of discrete wind speed and direction scatter diagrams rather than smooth wind speed probability density functions, and adjusts for errors that would be caused by using finite wind speed bin widths.

If ΔV, the wind speed bin width, is very small then γ_{bin} must be larger than 1.0 to account for the occurrences of lock-in outside of the wind speed bin that includes V_c. Conversely, if ΔV is large then γ_{bin} will be smaller than 1.0 to reduce the probability of occurrence of the critical wind speed to the appropriate level. γ_{bin} is a function of T_0, α/T_0 and V_c as shown below in equation 10. The theory behind the estimation of γ_{bin} is given in [5].

$$\gamma_{bin} = (V_c/\Delta V)*[2.6T_0 + 0.015\alpha/T_0] \qquad (10)$$

Typical values for γ_{bin} range for 0.2 to 8.

The Estimation of the Probability of Occurrence of the Critical Wind Speed as a Function of Wind Incidence Angle

VIV may occur when the wind is not perpendicular to the member. In such cases lock-in occurs when the perpendicular component of the wind is sufficiently close to the critical wind velocity. Let θ be the angle between the perpendicular to the member and the mean wind direction. The critical wind speed for the member with a wind incidence angle of θ may then be expressed as

$$V_c(\theta) = V_c(0)/\cos(\theta) \qquad (11)$$

This formula is valid up to approximately 45 degrees, the typical resolution of wind compass roses and scatter diagrams. VIV in winds at greater than 45 degrees is small and not considered further in this analysis. In the example calculation to follow, the Ekofisk data is divided up into sixteen directional bins of 22.5 degrees each. Reference to this data will be made in the forthcoming discussion.

Recalling equation 1 for fatigue damage rate,

$$D_a = \gamma_0*\gamma_1*D_{ss}*(\gamma_{bin}*\Sigma P_{V,\theta}), \qquad (1)$$

we are reminded that we must sum the probabilities over both wind incidence angle and velocity bins. For vertical members, the appropriate summation is over all 16 wind directional bins in mean wind speed bins which include V_c. For horizontal members, the appropriate summation is over ten wind direction bins. Thus, for example, for a member with its axis aligned east-west, one would sum the probabilities $P_{V,0}$ corresponding to the north and south direction bins and the associated velocity bins which include V_c, plus $P_{V,22.5}$ at ±22.5 from north and south (NNW,NNE,SSW,& SSE) in those velocity bins which include $V_c/\cos(\pm 22.5) = 1.08V_c$, plus $P_{V,45}$ at ±45 degrees from north and south (NW,NE,SW,& SE) in those velocity bins which include $V_c/\cos(\pm 45) = 1.41V_c$.

The Effect of Wind Incidence Angle on the Product of γ_0, γ_1, and γ_{bin}

An exhaustive study was conducted which included evaluating wind statistics such as T_0 and σ_v/σ_v for the component of the wind normal to the member, as a function of wind incidence angle varying from 0 to 45 degrees from the normal. The result is that at $\theta = 22.5$ degrees for wide ranges of 'm', 'r' and α/T_0 the product $\gamma_0*\gamma_1*\gamma_{bin}$ varied less than ±10% from the same product at normal incidence. At 45 degrees the variation of this product was as much as ±20%.

The authors believe that increases in this product at large incidence angles will be more than offset by factors, such as shielding from other members and reduction in the lock-in bandwidth, α. Such factors need further experimental investigation. At this time the authors recommend that the product

$\gamma_0 * \gamma_1 * \gamma_{bin}$ as computed for the normal incidence case be used for all incidence angles. Thus, to include the effect of wind incidence angle in the computation of fatigue damage rate requires only that one account for the correct probability of occurrence of the critical wind speed as a function of incidence angle.

Example Computation

Member properties. A typical structural member on a flare boom in use in the North Sea might have the following properties: Length = 12.5 m, diameter = 0.324 m, wall thickness = 0.00953 m, and an end fixity of 0.7 [9].

We adopt for this example the "T-curve" [10] for tubular nodal joints, for which $N*S^m = K_2 = 1.46*10^{12}$, where S, the stress range, is specified in N/mm^2, and "m" is taken as 3.0. For the purpose of simplicity the stress concentration factor is set for this example at 1.0. Any desired factor of safety may be applied at the end of the fatigue life calculation.

Compute V_c. Assuming that the peak response occurs at a reduced velocity of 6.0, the critical velocity of the member is given from Equation (2) by $V_c = 6.0*f_n*D$. A lower reduced velocity might be appropriate at super-critical Reynold's numbers and would result in a correspondingly lower critical velocity. This in turn may have dramatically different probabilities of occurrence, which one may evaluate by consulting Table 1. To compute V_c requires the natural frequency, f_n, which may be computed from the following equation [2].

$$f_n = \frac{[1.59\phi+\pi]^2}{2\pi L^2}\sqrt{\frac{EI}{m_e}} \quad \ldots\ldots (12)$$

Where m_e is the mass per unit length of the member (73.47 kg/m), E is the Young's modulus ($2.11*10^{11}$ N/m^2), I is the area moment of inertia of the member (0.0001164 m^4), and ϕ is the degree of joint fixity, which may vary from 0.0 to 1.0 for pin ended and fully fixed members respectively and is commonly assumed to be 0.7 for fully welded tubular joints. For this example, ϕ = 0.7 or 70% fixity. The natural frequency for this member is 10.64 Hz. At a reduced velocity of 6.0 this results in a value of the critical velocity of 20.68 m/s. This corresponds to a Reynold's number of 462,000 which is in the transition range from subcritical to super-critical flow. At these Reynold's numbers the peak response reduced velocity is not well known, but is in the range of 5 to 6. 6.0 is used in this example.

Compute response and steady state damage rate. Using, for example, the method described in [2 or 8], the steady state response amplitude may be computed from the following equation:

$$\frac{Y_{max}}{D} = \frac{3.82\gamma_i C_L}{\left[1+0.19\left(\frac{2\pi S_t^2 K_s}{C_L}\right)\right]^{3.35}} \quad \ldots\ldots (13)$$

where, C_L the lift coefficient is Reynold's number dependent. The C_L recommended in [2] is 0.4 for 1000 < Re < 300,000, 0.3 for Re > $2*10^6$ and varies linearly between 300,000 and $2*10^6$. For this example the Reynold's number at the critical velocity is 462,000 and C_L = 0.39. S_t is the Strouhal number which is fixed in this equation at 0.2.

γ_i is the cross flow mode shape parameter and is computed using equation 14, as a function of the assumed mode of vibration. Table 3 presents typical values of this parameter for a range of typical boundary conditions and fundamental modes of vibration. Also included in Table 3 are the corresponding maximum values of the strain response parameter, F_i, which may be computed using equation 15 and relates the proportionality between the strain at some location on the member to the maximum modal deflection.

It is important to note that $F_i(x)$ should be evaluated at the location of the peak hot spot stress, S_{hss}. This location may not be coincident with the maximum value of F_i, since it is also a function of the appropriate stress concentration factor. The general formulas for γ_i and F_i are given below.

$$\gamma_i = Y_{max}\left[\frac{\int_0^L [y^2(x)]dx}{\int_0^L [y^4(x)]dx}\right]^{0.5} \quad \ldots\ldots (14)$$

$$F_i = \frac{|(y'')|L^2}{Y_{max}} \quad \ldots\ldots (15)$$

K_s is the response parameter or reduced damping. It is given by:

$$K_s = 4\pi\zeta m_e/(\rho_a D^2) = 13.64 \quad \ldots\ldots (16)$$

where m_e is the mass per unit length and ρ_a is the mass density of air(1.29 kg/m^3). The damping ratio for the first mode is taken to be 0.002. Substituting the above information into equation (13) results in an estimate of the steady state mid span maximum response:

$$Y_{max}/D = 0.065$$

Once Y_{max}/D is known and the strain response parameter for the location of interest is evaluated (In this case at the member end F_i = 22.4), then the corresponding stress range is given by

$$S_p = (Y_{max}/D)*E*F_i*SCF*D^2/L^2 \quad \ldots\ldots (17)$$
$$= 2.064*10^8 \text{ N/m}^2 = 206.4 \text{ N/mm}^2$$

Equation (17) above is also in reference [2] as equation (15). In reference [2] equation (15) should have the factor S_p and not $2S_p$, where S_p is the stress range, the same as here. The error in reference [2] propagates through the example calculation in that paper. A summary of the errata may be obtained from the authors of reference [2].

Knowing the form of the T-curve, allows one to find the number of cycles to failure as follows:

$$N{*}S^m = N_p{*}S_p{}^m = 1.46{*}10^{12} \text{ (S in N/mm}^2\text{)} \quad (18)$$

This can be solved for N_p(166,000 cycles), the number of cycles to failure at the steady state response level Y_{max}/D. From this the steady state damage rate, D_{ss}, is simply given as,

$$D_{ss} = f_n/N_p = 0.000064 \text{ s}^{-1} \quad (19)$$

Steady state vibration at this level would lead to failure in 4.3 hours. Even if the mean wind speed equals the critical wind speed the actual damage rate would be less by the factor $\gamma_0{*}\gamma_1$ due to the variability of the winds and finite response rise time of the structure. This will be further reduced by the probability of encountering the critical wind speed. The next step is to compute γ_0 and γ_1.

Compute γ_0. Assuming a value of $\alpha/T_0 = 1.0$ and $T_0 = 0.125$ then the value of γ_0 may be computed from equation 3 or taken from Figure 1 at $\alpha/T_0 = 1$. From Figure 1 the value is approximately $\gamma_0 = 0.19$.

Compute $\sigma_V/\sigma_{\dot{V}}$, $G(\alpha/T_0)$, $E[\tau]$, r, Rise time and γ_1. γ_1 may be computed in order from equations 6, 5, 4, 8 and 9. Assume the member is forty five metres above the ground (h = 45 m). Then from equation 6, $\sigma_V/\sigma_{\dot{V}} = 5.8$ seconds. From Figure 5, $G(\alpha/T_0 = 1) = 3.5$, and from equation 4, the expected duration of visit, $E[\tau] = 19.8$ seconds. The rise time of this member is $1/2\pi\zeta f_n = 7.5$ seconds and from equation 8, 'r', the ratio of the duration to rise time is 2.65. If 'm' the slope of the S-N curve is taken as 3.00, then, using Table 2 and equation 9, $\gamma_1 = 0.7$.

Damage rate reduction factor. The product of γ_0 and γ_1 is 0.133. The fatigue damage rate is reduced by this factor due to the effects of the unsteadiness of the wind and the finite rise time required by the structure to respond to changes in wind speed. This is a factor of 7.5 increase in fatigue life.

Evaluate the probability of encounter of V_c. It remains to evaluate the probability of encounter of the wind speed. In this example this is done using the wind speed and direction scatter diagram, for example, from Ekofisk in the North Sea as shown in Table 1. This is wind data gathered over 15 years at Ekofisk. It represents 10 minute mean wind speeds and directions. The increment in wind speed, herein defined as the bin size ΔV, is 1.0 m/s. The angular bin size is 22.5 degrees or 16 compass headings. The total number of wind conditions is 42436.

Table 1 is shows the probability of occurrence of the wind speed and direction at 10.0 metres above the sea. Our example member is 45 m above the sea. A boundary layer profile must be assumed for the wind and used to determine which wind speed bin at 10 m above the sea corresponds to $V_c = 20.68$ m/s at 45 m above the sea. A typical industry standard profile is given by:

$$V(z) = V_{10}(z/10)^p \quad (20)$$

where V_{10} is the velocity at 10 metres above the sea and 'p' is an exponent which depends on the averaging period for the wind data being used. 'p' is 0.125 for 1-hour mean wind speed data, 0.120 for 10-minute mean wind speed data , 0.113 for 1-minute mean wind speed data, and 0.1 for 3 second gust data. The data presented in Table 1 is 10-minute mean wind speed data and therefore we will use p = 0.120.

Let θ be defined as the angle between the mean wind direction and the perpendicular to the member. In order to calculate the probability of occurrence of the critical wind speed, it is necessary to identify the relevant wind speed and direction bins in table 1. Since the directional discretization is in steps of 22.5 degrees, then the appropriate directional bins are those for which $\theta = 0$, ±22.5 and ±45 degrees incidence angle to the member. The corresponding critical wind speeds at these incidence angles are from equation 11, 20.68, 22.38 and 29.24 m/s, respectively. Substituting these values into equation 20 for V(z=45 m) and solving for V_{10} yields 17.26, 18.68 and 24.41 m/s respectively. This allows us to enter Table 1 for $P(V,\theta)$.

This example computation is for a horizontal member that has a perpendicular in the north-south direction. Therefore the direction bins corresponding to $\theta = 0$ degrees are the N and S bins. Similarly, for $\theta = \pm 22.5$ degrees, the corresponding bins are NNW, NNE, SSW AND SSE and at ±45 degrees the bins are NW,NE,SW AND SE. Table 1 shows the total number of occurrences for each speed and direction combination. To get the correct probability one sums the number of occurrences and divides by the total number, 42436.

$$\Sigma P(V,\theta) = \Sigma P(V_c,\theta{=}0) + \Sigma P(1.08V_c,\theta{=}\pm 22.5) + \Sigma P(1.41V_c,\theta{=}\pm 45)$$

$$\Sigma P(V,\theta) = \Sigma P(17.26,N{+}S) + \Sigma P(18.68,NNW{+}NNE{+}SSW{+}SSE) + \Sigma P(24.41,NW{+}NE{+}SW{+}SE) \quad (21)$$

$$= [(17+30) + (38+3+35+21) + (5+0+1+1)]/42436 = 0.00356$$

Compute γ_{bin}. It remains only to compute γ_{bin} from equation (10). In this case $V_c/\Delta V$ = 17.26 and γ_{bin} = 5.87. Where V_c is taken as 17.26 m/s, the value of the wind speed at 10 m which corresponds to the true V_c at 45 m above the sea surface. Returning to equation (1), the adjusted damage rate is

$$D_a = \gamma_0 * \gamma_1 * D_{ss} * (\gamma_{bin} * \Sigma P_{V,\theta}) \quad \ldots\ldots (1)$$

$$= 0.19*0.7*0.000064(5.87*.00356) = 1.78*10^{-7}\ s^{-1}$$

$$1/D_a = 5.62\mathrm{x}10^5\ s = 65\ \text{days} \quad \ldots\ldots (22)$$

This is 7.5 times longer than would have been computed using D_{ss}, γ_{bin} and $\Sigma P_{V,\theta}$, but with no correction for γ_0 and γ_1. Even with the 7.5 times increase, this member is unsuitable for use.

Lengthening the fatigue life of the member

The member evaluated above is unsuitable. Had a more conservative SCF been applied, the resulting fatigue would have been much lower. There are three approaches to solving the problem. The first is to reduce the L/D of the member. If L/D is reduced sufficiently, the critical wind speed will increase to the point that the probability of encountering the wind speed becomes very small(i.e. $P_{V,\theta}$ =0). If this is not feasible or too costly, then one must increase the damping or decrease the lift coefficient. Both of the latter methods requires attachments to the member. Increasing the damping may be accomplished by adding tuned mass dampers, such as Stockbridge or AR dampers [1,10]. Reducing the lift coefficient may be accomplished by helical strakes or by putting sleeves on the member, which changes the diameter at select locations [2,9].

Conclusions

This paper has shown how one may account for unsteady wind speeds in estimating fatigue damage rates for vortex-induced vibration of structural members. It has also shown how one accounts for the discrete nature of the wind speed and wind direction scatter diagram in estimating the probability of occurrence of the critical wind speed for the member.

Two previously unrecognized dimensionless parameters have been identified as being important in characterizing VIV in unsteady situations. They are α/T_0, the ratio of the bandwidth of the lock-in interval to the turbulence intensity, and 'r' the ratio of the duration of visit to the rise time of the structural member. There are areas needing further investigation. There is inadequate data on the lock-in bandwidth parameter, α, as a function of Reynold's number, especially in super critical regimes. The knowledge of turbulence intensity, T_0, as a function of geographic region and height above the sea, and the relation to the wind scatter diagram is at present not adequate. Both of these factors affect α/T_0.

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wind speed bin [m/s]	wind direction in DEG AZIMUTH																Total
	N	NNE	NE	ENE	E	ESE	SE	SSE	S	SSW	SW	WSW	W	WNW	NW	NNW	
0 0.9	311	7	10	11	28	7	13	6	17	15	24	22	28	14	19	25	557
1 1.9	114	29	50	50	55	30	14	27	50	23	42	54	115	45	63	59	820
2 2.9	258	70	81	86	135	47	48	75	136	63	101	141	243	110	80	113	1787
3 3.9	397	104	119	102	127	94	100	173	279	113	188	218	410	154	152	161	2891
4 4.9	503	129	113	128	156	104	136	219	409	227	250	252	441	255	265	173	3760
5 5.9	548	145	105	99	164	120	160	204	388	330	294	290	429	310	300	203	4089
6 6.9	534	111	106	129	163	132	142	215	377	367	305	335	451	308	249	253	4177
7 7.9	500	106	89	115	182	165	150	236	371	290	328	316	488	267	211	220	4034
8 8.9	449	85	62	75	215	176	174	211	381	318	335	313	395	261	214	231	3895
9 9.9	310	48	40	73	188	159	173	163	298	290	302	258	370	212	157	175	3216
10 10.9	235	39	51	88	164	183	161	170	262	245	296	263	348	190	183	182	3060
11 11.9	195	22	41	71	167	126	130	111	166	192	240	206	306	144	98	127	2342
12 12.9	150	27	23	48	116	105	100	109	168	145	211	198	274	129	81	138	2022
13 13.9	95	9	21	31	79	76	74	73	103	121	155	145	216	94	73	75	1440
14 14.9	60	7	13	28	72	76	85	42	72	118	108	143	188	82	62	65	1221
15 15.9	31	20	10	13	64	67	49	35	80	82	97	89	142	89	60	57	985
16 16.9	11	2	3	5	39	27	17	29	25	34	47	48	66	25	24	30	432
17 17.9	17	0	2	15	65	47	18	15	30	52	57	43	92	30	27	33	543
18 18.9	8	3	3	13	36	34	20	21	23	35	36	38	79	40	24	38	451
19 19.9	5	4	1	2	19	24	11	12	12	19	30	23	45	23	13	13	256
20 20.9	3	0	0	1	15	13	6	9	15	12	18	16	39	24	7	16	194
21 21.9	4	0	1	0	4	6	3	0	6	6	5	8	20	6	10	1	80
22 22.9	1	0	1	1	0	3	1	4	5	2	2	4	14	5	4	9	56
23 23.9	1	0	0	0	4	0	5	1	2	4	1	3	12	6	7	9	55
24 24.9	0	0	0	0	1	5	1	0	0	1	1	1	5	6	5	3	29
25 25.9	1	0	0	0	0	0	2	2	1	1	0	3	4	2	4	2	22
26 26.9	0	0	0	0	0	0	0	0	0	0	0	1	3	0	1	2	7
27 27.9	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	3
28 28.9	0	0	0	0	0	0	1	0	0	0	0	0	3	0	1	0	5
29 29.9	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2
30 30.9	0	0	0	0	0	0	2	0	0	0	0	0	0	1	0	0	3
31 31.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
32 32.9	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
33 33.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
≥ 34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL	4741	967	945	1184	2258	1826	1796	2162	3676	3105	3473	3431	5228	2834	2397	2413	42436

Table 1. Number of Occurrences of Wind Speed and Direction at 10 [m] based on 15 year measurements. Wind speeds are from 10 minu' averages recorded every 3 hours. Total Number of Wind Occurrences is 42436.

m	β	δ
3.0	.9309	.2583
3.5	.7721	.2773
3.74	.7093	.2859
4.0	.6488	.2952
4.38	.5718	.3085
5.0	.4693	.3302
5.5	.4023	.3478
6.0	.3462	.3657

Table 2. Parameters governing the relationship between γ_1 and 'r' the ratio of the duration of visit to the rise time.

Boundary Conditions	γ_i	F_i	*Strain location*
Free-Fixed	1.304	3.52	Fixed end
Pinned-Pinned	1.155	9.87	Mid span
Fixed-Pinned	1.161	20.4	Fixed end
70% Fixity	1.163	22.4	Either end
Fixed-Fixed	1.167	28.2	Either end

Table 3. Mode shape parameters and mid-span strain respon parameters for various boundary conditions

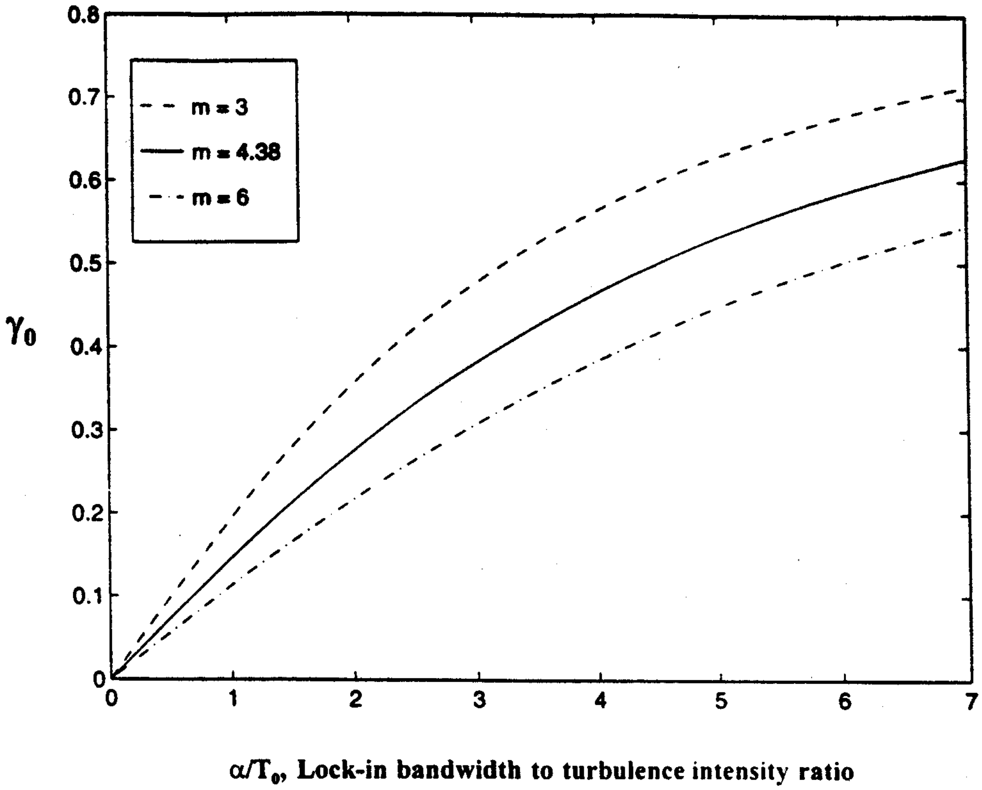

Figure 1. γ_0 versus α/T_0, the ratio of lock-in bandwidth to turbulence intensity.

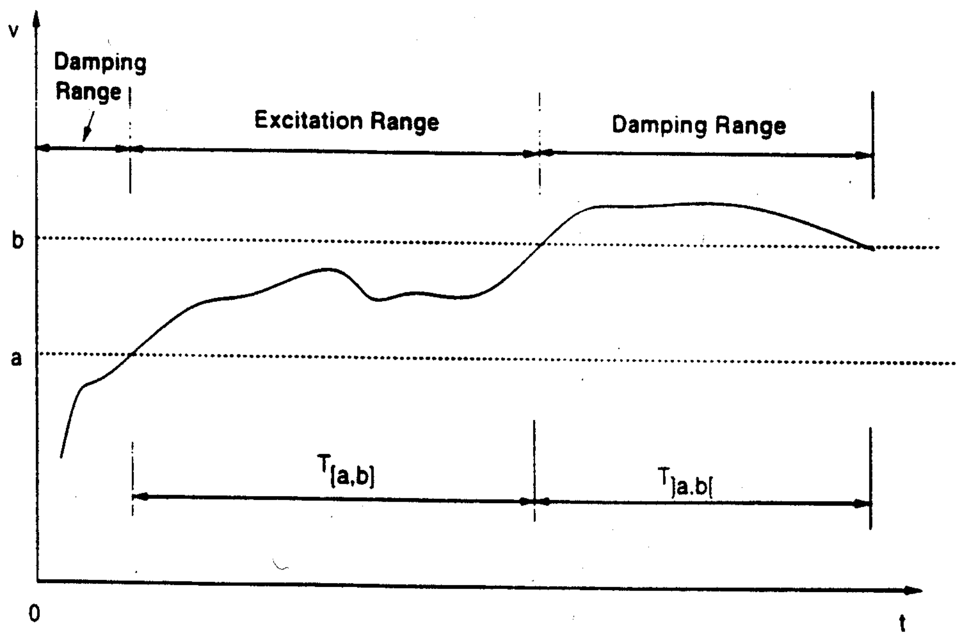

Figure 2. Duration of visit, $T_{[a,b]}$, of the wind speed, V, to an interval [a,b]

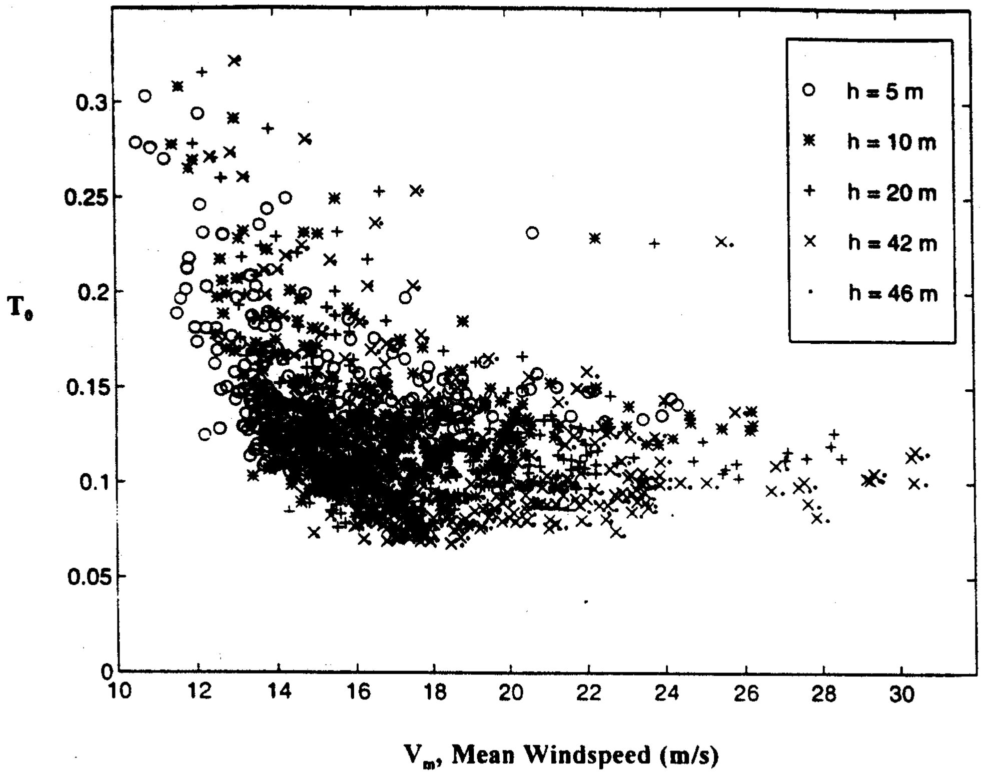

Figure 3. Turbulence intensity, T_0, versus mean wind speed, V_m.

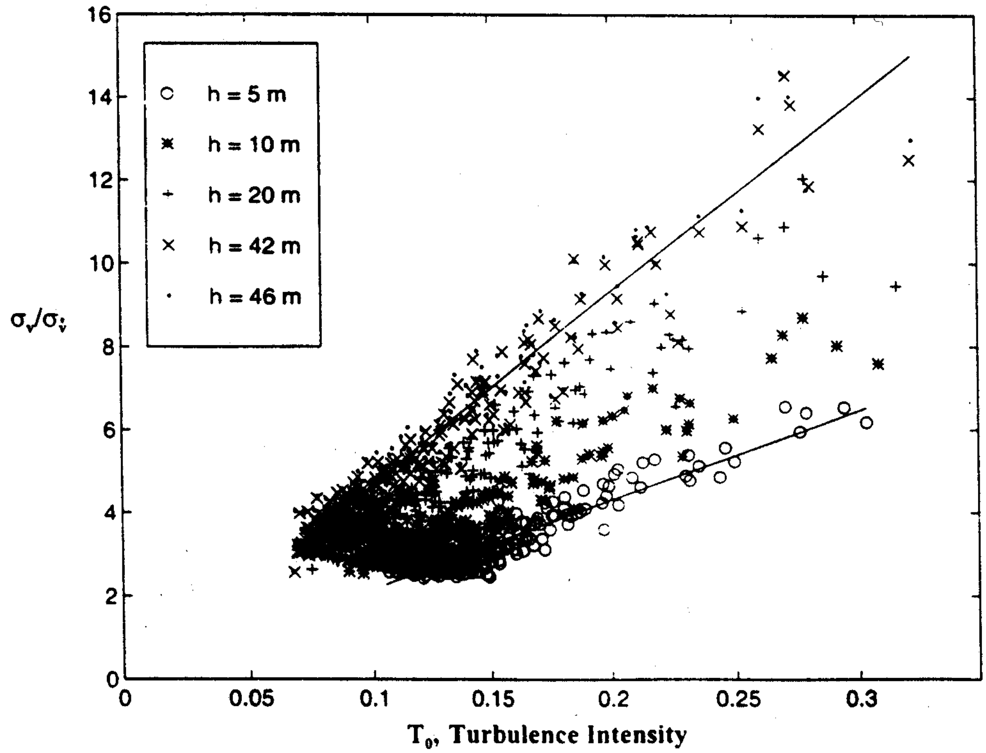

Figure 4. $\sigma_v/\sigma_{\dot{v}}$(in seconds) versus turbulence intensity, T_0, as a function of height above the sea, h-meters. Slope of straight line fit is given by 26log(1.35h) for h=5 & 46m.

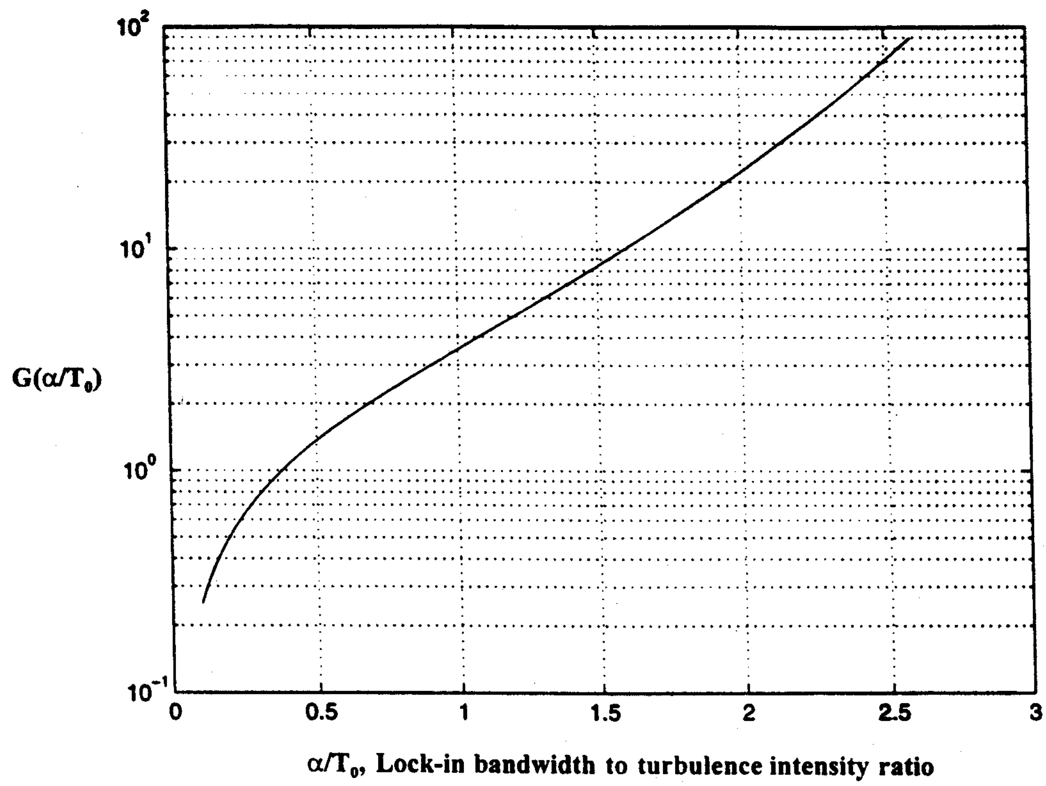

Figure 5. $G(\alpha/T_0)$ versus α/T_0, the ratio of the lock-in bandwidth to turbulence intensity.

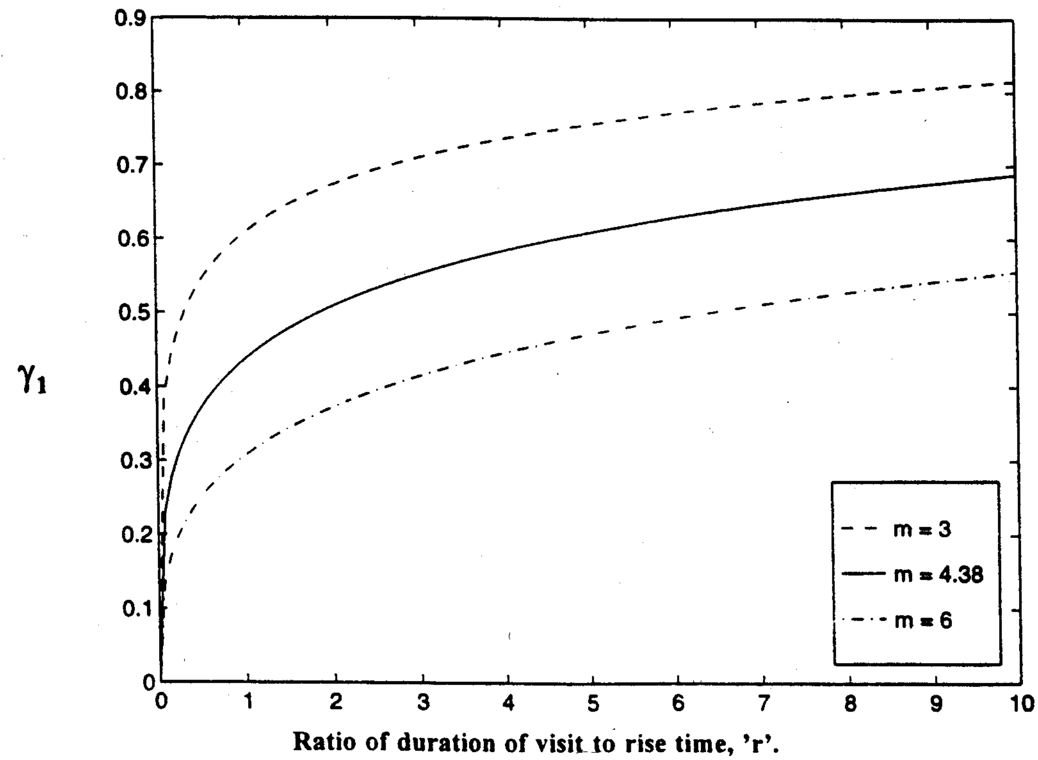

Figure 6. γ_1 versus the ratio of duration of visit to rise time,'r', as a function of the SN curve slope 'm'.