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## HIGH MODE NUMBER VIV EXPERIMENTS

**Abstract.** A simple equation is presented which provides the maximum achievable mode number for a flexible cylinder, towed by the top end with a weight at the bottom end. The maximum achievable mode number, while towing in still water, is shown to depend primarily on mass ratio, length to diameter ratio and the maximum allowable angle of departure from vertical at the top end. Modal overlap in lock-in regions is shown to depend strongly on mass ratio in uniform flow, but not in sheared flow. The reduced velocity bandwidth parameter is introduced to quantify the extent of lock-in regions in sheared flow. Two shear parameters are shown to be useful in characterizing low and very high mode number response. Finite length lock-in regions are described for cylinders with infinite length dynamic properties.

### 1. INTRODUCTION

As offshore oil and gas production pushes into deeper water, drilling, production and export risers as well as TLP tendons have become critical elements in the design of these facilities. All of these long cylinders are susceptible to significant vortex-induced vibration. Gulf of Mexico loop currents and detached eddies are the source of considerable concern to engineers having to design for fatigue resistance to VIV.

The prediction of VIV is currently based on data from relatively short cylinders tested in laboratory environments. The most successful VIV prediction programs for long cylindrical marine structures are empirically based and make use of the data from these laboratory models. Many assumptions are required to extend the simplified experimental results to the prediction of long cylinders in ocean currents. For the most part the prediction programs are intentionally quite conservative, sometimes resulting in the unnecessary use of VIV suppression.

Model tests with long flexible cylinders, responding at high mode number are highly desirable and are needed to help validate and fine-tune the prediction programs. There are many difficult challenges faced in conducting experiments at high mode numbers. The challenges range from the design of the experimental apparatus, to the identification of the key fluid and structural parameters essential to the understanding of the model test results.

This paper deals with some of the challenges posed in the experimental design and then addresses the important physical parameters that help one to understand the response of a long cylindrical structure to VIV. By carefully examining the dynamics of the expected behavior, several new parameters have been formulated. These parameters make it easier to understand the behavior of long cylinders in shear flows.

## 2. EXPERIMENT DESIGN

### 2.1 Objectives

High mode number experiments are expected to address a variety of questions, including:

- Does lock-in occur at high mode number?
- How much VIV suppression coverage is needed over the length of a riser to prevent high damage rate VIV events?
- What is the probability of single versus multi-mode response?
- What is the relationship between in-line and cross-flow response?

A Phase I testing program, sponsored by an industry consortium of companies, known as Deepstar, is being planned for the deep waters of Lake Seneca in Upper New York State. These tests will serve as a ‘proof of concept’ opportunity to test the sensors, the data acquisition system and the testing methods. The model will be towed behind a boat in a near vertical configuration with a large weight hanging off the bottom. The test matrix will address some of the issues mentioned in the questions above. Both uniform and sheared flow tests will be attempted.

Phase II testing is being discussed for a real, sheared current profile in the offshore environment, such as the Loop or the Gulfstream. The offshore environment introduces challenges and conditions beyond our control, such as the presence of surface waves.

### 2.2 Maximum mode number

In order to investigate the effects of high mode number VIV with model testing it is desired to design the model so that relatively high mode numbers are achievable. By performing some simple rearrangements of formula with approximations, the mode number,  $n$ , can be estimated giving insights into the important physical parameters required for the model.

For long cylinders where the tension dominates the bending stiffness effects in determining the natural frequencies, the natural frequencies may be approximated by

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{m_T}} \quad (1)$$

where  $L$  and  $T$  are the length and average tension, while  $m_T$  is the mass per unit length including added mass. For the lake experiments the tension is nearly constant along the length.

The highest shedding frequency,  $f_{v,max}$  is related to the highest velocity,  $U_{max}$ , the diameter,  $D$  and the Strouhal number,  $St$  as follows:

$$f_{v,\max} = St \frac{U_{\max}}{D} \quad (2)$$

The frequency of the highest mode number possibly excited will be approximately the same as the highest excitation frequency. Equating the frequencies and rearranging gives:

$$n_{\max} = 2St \frac{U_{\max}}{D} L \sqrt{\frac{m_T}{T}}. \quad (3)$$

In the case of a towed riser model, the horizontal component of the towing force required at the top end of the riser is equal to the total drag force, and is given by

$$F_D = T_{\max} \sin(\phi) \quad (4)$$

The total drag force,  $F_D$ , is the accumulation of local drag forces integrated over the length of the cylinder. Solving for  $T_{\max}$  yields

$$T_{\max} = \frac{F_D}{\sin \phi} = \frac{1/2 C_D \rho_f D \int_{z=0}^L U_z^2 dz}{\sin \phi} \quad (5)$$

Assuming the tension is approximately constant requires that  $\phi$  be small. By squaring expression (3) and substituting for  $T_{\max}$  from (5) yields an expression for  $n_{\max}^2$ , the highest achievable mode number for the specified conditions.

$$n_{\max}^2 = \frac{2\pi \cdot St^2}{C_D} \cdot \sin \phi \cdot \frac{U_{\max}^2}{\langle U^2 \rangle} \cdot \left( \frac{\rho_s}{\rho_f} + Ca \right) \cdot \frac{L}{D} \quad (6)$$

If  $T$  is not approximately constant, then Equation (6) is a lower bound. The expression  $\langle U^2 \rangle$  is the spatially averaged square of the velocity,  $U^2(z)$ , over the length of the cable.  $\rho_s$  and  $\rho_f$  are the densities of the structure and fluid, while  $Ca$  is the added mass coefficient.

Expression (6) has several different parameter groups. For uniform flows the ratio of the maximum velocity to the average velocity equals one, for sheared flows

the ratio is higher and thus the maximum mode number achievable is higher. The equation also reveals that the maximum mode number increases with top angle, mass ratio and aspect ratio. It may be desirable to limit the top angle so that the incident inflow remains close to perpendicular to the pipe axis. The mass ratio should be near to realistic prototype risers. The easiest parameter to systematically vary is limited to the aspect ratio. Larger aspect ratios lead to higher responding mode numbers.

For the proposed experimental model the aspect ratio is about 4100. By limiting the top angle to 20 degrees, the achievable cross-flow mode number, as estimated from Equation (6) is 18. This is for a cylinder length of approximately 427 feet (130m), a diameter of 1.25 inches (0.0318m), a specific gravity of 1.5, an  $S_t$  of 0.17, a  $C_D$  of 2.0, and for a  $Ca$  of 1.0. In-line frequency content is approximately a factor of two greater than the dominant cross-flow frequencies. For a tension dominated system this means that if the cross-flow maximum mode number is 18 then the maximum in-line mode will be approximately Mode 36.

### 2.3 Resolving in-line and cross-flow components

Maximizing the aspect ratio naturally results in choosing as small a diameter as practical. With a small diameter, the ability to install instruments becomes more difficult. Additionally smaller diameters have lower torsional stiffness. One of the objectives of the testing program is to be able to resolve both in-line and cross-flow motion components at every location that there is a sensor. At each sensor one must also know the orientation. This requires that the torsional stiffness be kept reasonably high. The torsional stiffness  $K_\theta$  is given by

$$K_\theta = \frac{GJ}{L} \quad (7)$$

where  $G$  is the shear modulus and  $J$  is the polar area moment of inertia.

The proposed model is Aluminum tubing (6061) with a 1.25 inch O.D. x 0.120 inch wall thickness. The length will be 130 m for the Phase I tests and approximately 400 m for the Phase II tests. The respective values of torsional stiffness are 9.1 and 3.0  $N\cdot m$  per radian. This is adequate for the shorter model, but might be a problem at longer lengths.

### 2.4 The Number of required Sensors

A current topic under evaluation is the number of sensors required to describe the vibration in space and time. If the mode shapes are approximately sinusoidal in shape, then a spatial Nyquist criterion would suggest a minimum of two measurement points per wavelength. The number of wavelengths in each mode shape is  $n/2$ . Therefore, a rough estimate of the number of required sensors is the maximum mode number. In the example computation in Section 2.2 that number is 36 biaxial sensors for the 130-meter long model. For a fixed maximum top towing

angle the number of required sensors increases approximately in proportion to the square root of the length.

The large number of sensors required to resolve the response at high mode numbers is both a financial and technical challenge. Current technology provides a choice among strain sensors, accelerometers and angular rate sensors. The installation of a large number of sensors requires a serial, digital, data transfer. Otherwise there would be too many conductors running down the center of the pipe. Additionally if there is too much information to be passed along the serial connection, as would happen with a high sampling rate and high number of sensors, then the information must be stored at each sensor location. Synchronous data acquisition among all sensors is extremely important if modal analysis is to be performed. For this reason a common trigger has to be designed into the acquisition system so that the sensors sample together.

### 3. DIMENSIONLESS PARAMETERS IMPORTANT TO THE PREDICTION OF VORTEX-INDUCED VIBRATION OF LONG, FLEXIBLE CYLINDERS IN OCEAN CURRENTS

The heading above is the title from a paper (Vandiver, 1993) which discusses several important parameters including the reduced damping or mass-damping parameter, mass ratio, shear fraction,  $(\Delta V / V_{\max})$ , the number of modes within the shear bandwidth  $(N_s)$ , and the wave propagation parameter  $(n\zeta_n)$ . Some additional discussion and some new parameters follow.

#### 3.1 *The Reduced Velocity Bandwidth*

Also introduced in that paper is the concept of “the lock-in bandwidth of the wake”. This is a measure of the ability of the wake to synchronize with the motion of a vibrating cylinder in a sheared flow. It is based on the concept that at a specific vibration frequency and amplitude there is a flow velocity,  $V_c$ , which is ideally suited to supporting lock-in. Furthermore, this ideal flow velocity is centered in a range of flow velocities which make up the lock-in region. At this center velocity an ideal reduced velocity may be defined as

$$V_{Rc} = \frac{V_c}{f_v D} \quad (8)$$

It is important to note that this reduced velocity is defined in terms of the vibration frequency, and not any fixed natural frequency.

In the presence of a shear, the velocity varies in the axial direction along the cylinder in both directions from this most favorable position. Lock-in or wake

synchronization is able to persist over a limited range of velocity values, which is defined as  $\Delta V$ . This variation when divided by the center velocity provides a definition of the lock-in bandwidth,  $dV_R$ .

$$dV_R = \frac{\Delta V}{V_c} = \frac{V_{R,U} - V_{R,L}}{V_{Rc}} \quad (9)$$

The second part of this equation is in terms of the upper and lower reduced velocities corresponding to the limits of wake synchronization. This lock-in bandwidth parameter has also been referred to as the “reduced velocity bandwidth” in the user guide to the response prediction program SHEAR7 (Vandiver et al, 2002). This is a parameter used in design computations to predict the extent of a potential lock-in region. A commonly prescribed design value for this number is 0.4. This specific value is to be interpreted as meaning lock-in may exist in a sheared flow over a range of flow velocities, which may vary  $\pm 20\%$  around the most favorable velocity.

Figure 1 is a conceptual sketch borrowed with permission from Prof. Michael Triantafyllou of MIT (2003). It shows how the lock-in bandwidth and the strength of the excitation might vary with reduced velocity, defined in terms of the measured vibration frequency and cylinder vibration amplitude. Outside of the positive energy region the lift coefficient will be negative and will produce hydrodynamic damping. Outside of the wake capture region, the wake will not be correlated to the motion of the cylinder. Experimentally such information has been obtained by driving 2D rigid cylinders, at constant frequency and amplitude in a constant speed uniform flow.

A very useful dimensionless frequency has been described by (Govardhan and Williamson, 2000). It is  $f_v/f_{vo}$  where  $f_v$  is the vibration frequency and  $f_{vo}$  is the stationary cylinder Strouhal frequency associated with the ideal center velocity,  $V_c$ , and is given by

$$f_{vo} = \frac{S_t V_c}{D} \quad (10)$$

If  $f_U$  and  $f_L$  are the upper and lower bounds of the lock-in range, then an experimentally determined estimate of the lock-in bandwidth is given by

$$dV_R = \frac{\Delta f}{f_{vo}} = \frac{f_U - f_L}{f_{vo}} \quad (11)$$

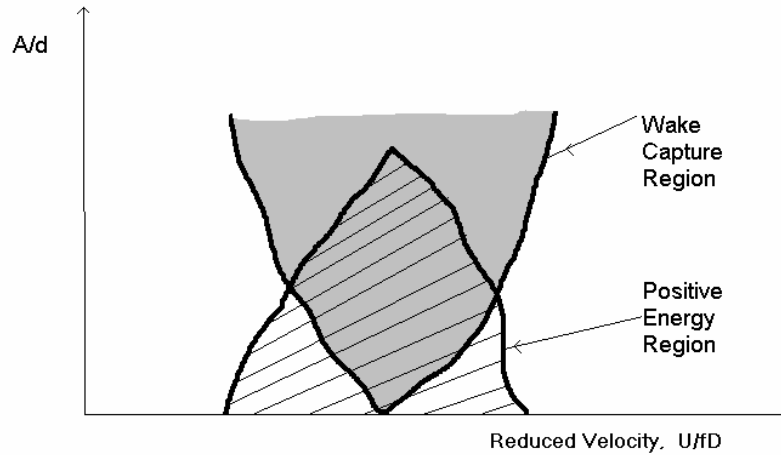


Figure 1. Wake capture range in terms of  $A/D$  and reduced velocity. From Triantafyllou, 2003.

This way of expressing the bandwidth has the advantage that  $S_i$  and therefore  $f_{vo}$  may be allowed to be a function of Reynolds number. Equations 9 and 11 are equivalent if within a lock-in region the spatial velocity variation along a vibrating cylinder in a sheared flow is equivalent to a slowly changing frequency variation over time in a uniform flow. This cannot be strictly true, but there is considerable experimental evidence to suggest that, as an engineering model of the real world, it is adequate for use in designing risers to resist VIV.

In section 3.3 the role of reduced velocity bandwidth in determining response of a flexible cylinder in sheared flow is addressed. Before engaging that discussion it is important to address the role of added mass, because it is often not correctly understood.

### 3.2 The Role of Added Mass in Uniform Flow

The reduced velocity bandwidth discussion above made no reference to added mass. The effect of added mass on lock-in of spring-mounted cylinders in uniform flow is well understood. (Vandiver, 1993) describes it quite carefully. In brief, the most significant aspect is that in a uniform flow added mass decreases dramatically as the reduced velocity is increased through the lock-in range. A decrease in added mass results in an increase in the natural frequency of the cylinder. A very common form of the reduced velocity is one defined in terms of a fixed frequency, such as the natural frequency in vacuo or in still water. The symbol used for this form in this paper is  $V_{Rn}$ .

A plot of response  $A/D$  versus  $V_{Rn}$ , for a flexible cylinder vibrating in first mode with pinned ends is shown in Figure 2a.

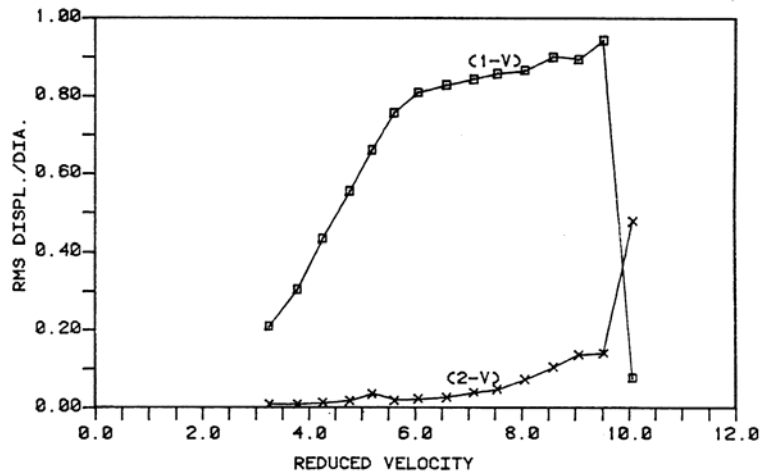


Figure 2a. RMS response/D versus reduced velocity,  $V_{Rn}$  based on the natural frequency in air. First and second mode cross-flow vibration of a flexible cylinder From a 1981 experiment conducted by Exxon Production Research. Published with permission from ExxonMobil.

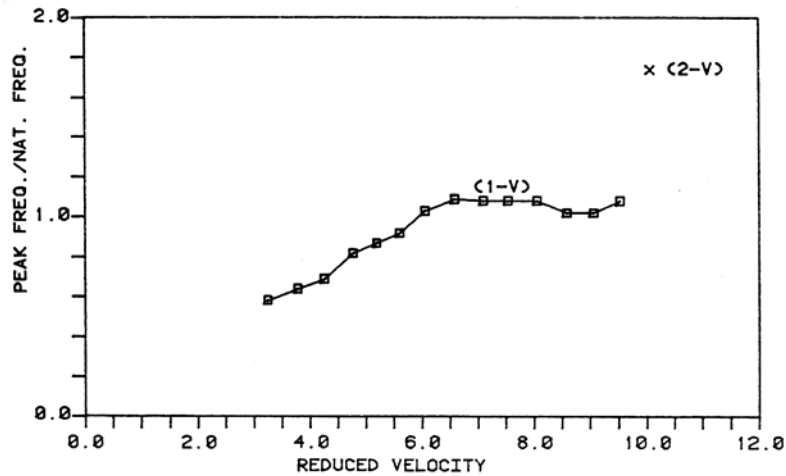


Figure 2b. Vibration frequency versus reduced for the 1981 EPR experiment on a flexible cylinder in water.  $V_{Rn}$  defined using the  $f_n$  in air. Published with permission from ExxonMobil.

This is data from an experiment conducted by Matt Greer of Exxon Production Research in 1981 at the Skibstekmisk Laboratorium in Denmark. Ten hollow aluminum segments (outside diameter=0.12m, inside diameter=0.11m) were



mounted on a central tensioned steel rod. Plastic spheres were placed inside the segments to achieve the desired mass ratio, when flooded. The model was 9.93 meters in length and had an external diameter of 0.120 meters. The composite cylinder had a specific gravity of 1.0 and a mass per unit length, including trapped water, but not including added mass of 11.3 kg/m. The cylinder was tensioned between struts beneath a towing tank carriage. The damping ratio of the first mode in air was approximately 1.0 to 1.5% of critical and the natural frequency in air was 1.936 Hz. The tension was 16.7 kN. The tension increased by as much as 8% due to drag force during the tests. The value of  $f_n$  used to compute  $V_{Rn}$  in Figures 2a and 2b included the variations caused by tension variation, but does not include the variation in natural frequency caused by added mass variation.

When reduced velocity is defined in terms of a fixed natural frequency such as the natural frequency in vacuum, air or still water, the extent of the lock-in range depends on the mass ratio. When this definition of reduced velocity is used, low mass ratio cylinders have a much greater value of the upper limit of the reduced velocity range than high mass ratio cylinders. This is quite evident in Figure 2a. The upper end of the lock-in range is 9.5. Williamson has shown that the maximum upper limit value of  $V_{Rn}$  is given by

$$V_{Rn,u} = 9.25 \sqrt{\frac{m^* + C_a}{m^*} - .54} \quad (12)$$

where the mass ratio,  $m^*$ , is defined as

$$m^* = \frac{m}{(\pi \rho_f D^2 / 4)} \quad (13)$$

The upper limit of the reduced velocity in Figure 2, for a  $C_a$  of 0.0, is predicted by equation (12) to be 13.6. The actual upper bound shown in the figure is approximately 9.5. The reason the upper bound is not achieved is that the second mode of vibration asserts control of the wake synchronization. The second mode amplitude is also shown in Figure 2a. The second mode's potential lock-in range overlaps that of the first, and for reasons not yet fully comprehended, the second mode becomes dominant at this particular reduced velocity. This phenomenon is central to the behavior of high mode number response. For typical risers with mass ratios of 1.0 to 3.0, lock-in regions will always overlap at high mode number.

Describing high mode number response under controlled uniform flow conditions is a high priority objective for the experiments and understanding mode to mode transitions is a particular focus.

Thus far two kinds of reduced velocity bandwidth have been described.  $V_{Rn}$  is based on a fixed natural frequency and is relevant to determining the lock-in bandwidth and extent of modal overlap for uniform flow conditions only. As the flow speed changes with time the natural frequency changes with the change in added mass. Equation (12) reveals a significant disadvantage of using  $V_{Rn}$ . The

values depend on the  $C_a$  value the user prescribes. Using an added mass coefficient of 0 or 1 to define the natural frequency used to compute  $V_{R,n}$  shifts the value.

$V_R$ , which depends on the actual vibration frequency, has a bandwidth defined as  $dV_R$ . This bandwidth will be observed in a uniform flow experiment in which the cylinder is forced to vibrate over a range of frequencies and it will also determine the range of the lock-in region on a cylinder in a sheared flow. In uniform or sheared flow reduced velocity defined in terms of the actual vibration frequency does not change with added mass, added mass coefficient or natural frequency. This topic is expanded upon in the next section.

### 3.3 The Role of Reduced Velocity Bandwidth

The reason that low mass ratio cylinders have such a broad lock-in range in uniform flows is that the natural frequency increases with flow speed. This is because the fluid added mass decreases as the flow speed and reduced velocity increase. If the mass ratio is less than 0.54, there is no upper bound of lock-in-reduced velocity (Govardhan and Williamson, 2000). This is also evident from Equation (12).

If the reduced velocity is defined in terms of the actual observed vibration frequency,  $f_v$ , then the dependence on mass ratio is removed.

$$V_R = \frac{U}{f_v D} \quad (14)$$

When plotted this way the bandwidth of the lock-in range is a function only of the ability of the wake to synchronize with the motion of the cylinder, and is not a function of added mass. Plotted in Figure 2b is  $V_{R,n}$  versus the ratio of the actual vibration frequency to the natural frequency in air for the same experiment that produced the data in Figure 2a. In Figure 2a the upper and lower bounds of the lock-in range in terms of  $V_{R,n}$  for Mode 1 are 3 and 9.5. The same range in terms of  $V_R$  defined in Eq. (14) is 5.2 and 9.5. This is a variation of  $\pm 30\%$  around a center value of 7.35. Alternatively it could be said that the reduced velocity bandwidth,  $dV_R$ , was 0.6.

If we define the ideal reduced velocity for the occurrence of lock-in as the inverse of the Strouhal number,

$$V_R = \frac{1}{S_i(\text{Re})} = \frac{V_c}{f_{vo} D} \quad (15)$$

then a reduced velocity lock-in bandwidth may be defined as

$$dV_R = \frac{V_{R,U} - V_{R,L}}{V_{R,i}} = \frac{f_U - f_L}{f_{vo}} \quad (16)$$

which is another way of arriving at Equation (11).

Two phenomena influence the range of flow speeds over which a natural mode of response of a flexible cylinder may exhibit lock-in. One is added mass variation and the other is the ability of the wake formation process to synchronize with the motion of the cylinder. Both may affect the range of flow speeds that define the lock-in region, but at high mode number in sheared flow, it is only the ability of the wake to synchronize with the motion of the cylinder that is important.

The principle focus of this paper is to understand the vibration of flexible cylinders in sheared flow. It is helpful to pose the problem as a question. Over what spatial variation in flow speed is the wake able to synchronize with the vibration of a cylinder at one of its natural frequencies?

Figure 3 shows a linear sheared flow. Mode ‘ $n$ ’ of a riser exposed to the flow is likely to experience flow-induced excitation at its natural frequency in regions where the flow speed will give rise to cross-flow lift forces whose frequencies are at or near the natural frequency of the mode. Such a region is shown in the figure. The region is characterized by upper and lower velocities which define the limits of the wake synchronization region. Between these limits it is assumed that the wake is synchronized with the cylinder motion. The length of this region can be described in terms of a reduced velocity bandwidth as given in Equation (9).

$$dVr = \frac{\Delta V}{V_i} = \frac{V_U - V_L}{V_c} \quad (17)$$

The extent of the lock-in region in a sheared flow is not dependent on mass ratio or added mass. It is principally dependent on the ability of the wake to synchronize with the cylinder vibration. It is not dependent on added mass variation because that effect would require the natural frequency to vary with position along the riser, whereas there can be only one natural frequency for each mode in a given sheared profile. In a shear the added mass does vary with reduced velocity all along the riser, but at steady state the modal mass including added mass is a constant, given by

$$M_n = \int_0^L (m(x) + m_a(x)) \psi_n^2(x) dx \quad (18)$$

where  $m(x)$  is the riser material mass per unit length including contents and  $m_a(x)$  is the added mass/length. The added mass distribution along the riser for a given shear profile and vibration frequency is fixed and so also is the modal mass and the associated natural frequency.

If the sheared flow profile were to change, for example as with a tidal variation, the position of the power in region would move for each mode. This would cause the added mass distribution for each mode on the riser to vary slowly because outside of the lock-in region the added mass at high and low reduced velocities

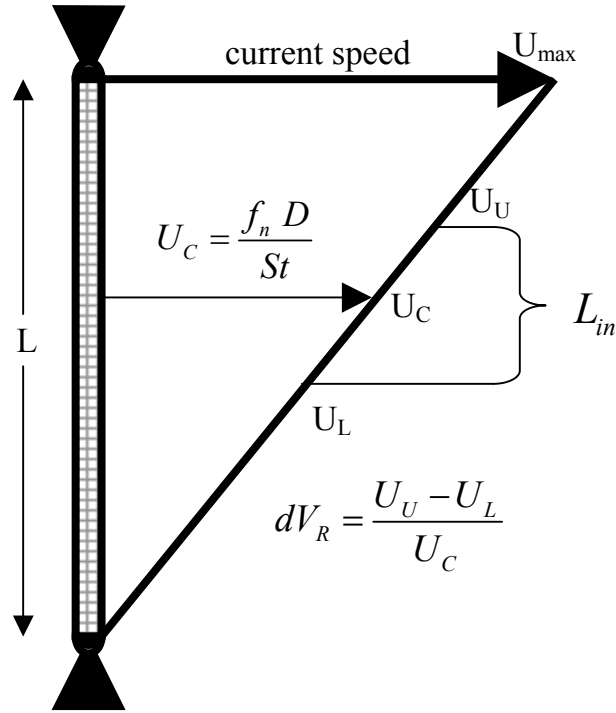


Figure 3. Power-in region for mode 'n' in a linear sheared flow.

changes slowly. Therefore the modal mass and the natural frequency of each mode changes slowly with incremental changes in the sheared profile.

Added mass effects on the natural frequencies and therefore on lock-in behavior are only associated with actual changes in the flow profile. For a steady state sheared profile the extent of power-in regions is governed only by the wake synchronization bandwidth as quantified in the parameter  $dV_R$ . When reviewing drilling riser data, this author has found that as the velocity profile changes slowly with time, only the first two or three modes show significant variation in natural frequency due to added mass variation. The first mode shows the most variation, followed by the second and then others in descending order. It is predicted that higher mode number response in sheared flows will show little variation in natural frequency due to variation in added mass.

#### 3.4 Shear Parameters and Correlation Length

Another way to think about the power-in region defined by the bandwidth  $dV_R$  is as the length over which vortex-induced lift forces are correlated with the vibration of

the cylinder at its natural frequency. This length is assigned the symbol  $L_{in}$ . In a linear shear the change in velocity over a length  $\Delta X$  is given by

$$\Delta V = \frac{dV}{dx} \Delta X \quad (19)$$

When  $\Delta X$  is the length  $L_{in}$ , then  $\Delta V$  is the variation of velocity over the wake-synchronized power-in region. Dividing both sides of Equation (19) by  $V_c$ , the center velocity for the lock-in region, provides a new expression for  $dV_R$  in terms of the shear gradient.

$$dV_R = \frac{\Delta V}{V_c} = \frac{L_{in}}{V_c} \frac{dV}{dx} \quad (20)$$

Solving this equation for  $L_{in}$  yields

$$L_{in} = \frac{dV_R}{\left[ \frac{1}{V_c} \frac{dV}{dx} \right]} \quad (21)$$

A useful expression for vibration of a riser, which exhibits standing wave vibration over its entire length, is the ratio of the power-in length to the total length. Equation (21) is easily modified to yield such an expression.

$$\frac{L_{in}}{L} = \frac{dV_R}{\left[ \frac{L}{V_c} \frac{dV}{dx} \right]} \quad (22)$$

The quantity in square brackets is a shear parameter, which is conceptually useful when considering the structural dynamic response prediction problem. For example, consider a linear velocity profile specified by

$$V(x) = \frac{V_{\max}}{L} x \quad (23)$$

then

$$\frac{dV}{dx} = \frac{V_{\max}}{L} \quad (24)$$

and from Equation (22)

$$\frac{L_{in}}{L} = \frac{dV_R}{\left[ \frac{V_{\max}}{V_c} \right]} = \frac{V_c}{V_{\max}} dV_R \quad (25)$$

This tells us that the fraction of the length occupied by the power-in region is larger at higher flow velocities. This expression fails as the center velocity approaches  $V_{\max}$  because the lock-in region cannot extend above  $V_{\max}$  and  $L_{in}$  will be only half as long as predicted by Equation (25) when  $V_c = V_{\max}$ .

In summary Equation (25) is a conceptually useful tool in understanding the relationship between shear and the correlation length which is available to excite each mode of vibration. It is probably true that the reduced velocity lock-in bandwidth,  $dV_R$ , is itself a function of the shear gradient, but that level of understanding must await future experimental research.

### 3.5 Reduced Damping Parameter

The mass-damping or reduced damping parameter as defined for uniform flows is given by

$$S_g = \frac{r\omega}{\rho_f U^2} = 8\pi^2 S_t^2 \frac{m}{\rho_f D^2} \zeta_s \quad (26)$$

The expression on the right is from Griffin (1998) and that on the left is from Vandiver (1993).  $r$  is the structural damping constant per unit length,  $\omega$  is the vibration frequency,  $U$  is the flow velocity,  $\zeta_s$  is the structural damping ratio, and  $\rho_f$  is the fluid density. The expression on the right is shown in Vandiver (1993) to reduce to that on the left. It is well known that this parameter may be used to predict resonant response  $A/D$ . Response decreases with increasing values of  $S_g$ . At values less than about 0.1, cross-flow VIV reaches limit cycle amplitudes of one to two diameters depending on the mode shape. This parameter has been extended by Vandiver (2002) to the sheared flow case, and may be written as

$$S_u = \frac{R_n \omega_n}{\rho_f U^2 L_{in}} \quad (27)$$

where  $R_n$  is the modal damping,  $\omega_n$  is the natural frequency, and  $L_{in}$  is the power-in length as defined earlier. Vandiver (2002) may be checked for further details of the derivation and examples of its use.

### 3.6 The Response of Dynamically Infinite Cylinders

Infinite cylinders are defined in Vandiver (1993) as ones in which vibration waves excited in one region die out due to structural and hydrodynamic damping before reaching boundaries. Thus, standing waves do not dominate the response as with low mode number short cylinders. Vandiver (1993) provides a parameter,  $n\zeta_n$ , for assessing if a cylinder is likely to behave as of infinite length with travelling waves or of short length with standing wave mode shapes.  $n$  is the mode number and  $\zeta_n$  is the modal damping ratio. Above  $n\zeta_n \approx 1$  the vibration approaches that of an infinite cylinder.

In a recent doctoral dissertation by Jung Chi Liao (2001), a theoretical solution is presented for an infinite cylinder excited over a finite region by an harmonic force applied in a standing wave pattern. This is shown in Figure 4. The excitation is of the form  $f(x, t) = F_o \sin\left(\frac{2\pi x}{\lambda}\right) \cos(\omega t)$  and is applied over a finite length,  $L_{in}$ . If it is assumed that the force is due to cross-flow VIV,  $F_o$  may be expressed as

$$F_o = \frac{1}{2} \rho_f U^2 D C_L \quad (28)$$

$C_L$  is the local lift coefficient, which is  $A/D$  and reduced velocity dependent. Liao's solution is shown in Figure 4. Within the power-in region a standing wave appears. Outside of the power-in region the solutions are traveling waves, which decay due to structural and hydrodynamic damping as they travel away from the power-in region.

The maximum amplitude of the standing wave at the center of the excitation region is given by Liao as

$$\frac{A(x=0)}{D} = \frac{C_L}{2} \frac{\rho_f U^2}{r \omega} \left( 1 - e^{-\frac{\pi}{2} N_m \zeta} \right) \quad (29)$$

where  $N_{in} = L_{in}/\lambda$  and  $\lambda$  is the wave length of vibration waves in the cylinder with frequency  $\omega$ . The damping,  $\zeta$ , is the structural damping only inside of the power-in region and is assumed to be reasonably small, less than 0.15. This equation tells us that given a sufficiently long power-in region ( $N_{in}\zeta > 2$ ) maximum amplitude standing waves may be achieved.

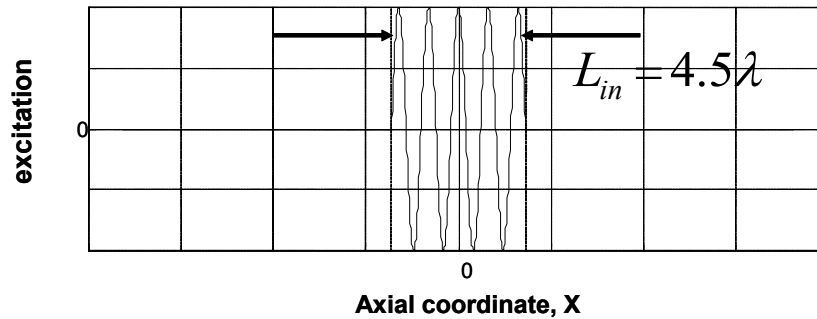


Figure 4. Harmonic excitation in a finite length power-in region on an infinitely long cylinder. The wavelength of the excitation matches that of the standing waves in the cylinder at that frequency.

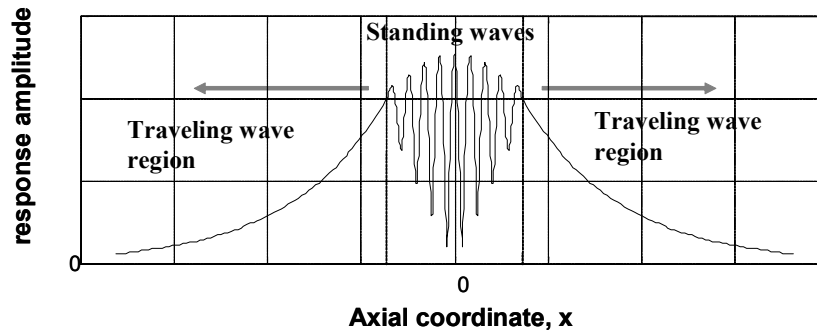


Figure 5. Magnitude of the response of the infinitely long cylinder, given the excitation shown in Figure 4.

The lift coefficient is of course dependent on many factors including reduced velocity and  $A/D$ . If the power-in region is not long enough for the waves to reach the maximum value, Equation 7(29) tells us that the maximum amplitude response in the power-in region will be smaller.



Such behavior is unlikely to be seen on typical offshore petroleum industry risers. However, it is very likely to occur on cables used to tow heavily weighted deep survey vehicles behind ocean vessels. The tow cable typically enters the water almost horizontally due to total system drag as given in Equation (4) but then curves downward, eventually becoming nearly vertical at the survey vehicle.

The survey vehicle serves as a reflecting boundary condition. The near vertical region of the cable near the vehicle will have passing by it a nearly uniform flow crossing the cable perpendicular to its axis. This flow will excite standing wave VIV near the boundary. This is a semi-infinite cable problem. The waves generated in a power-in region of length  $L_{in}/2$ , starting at the attachment point on the vehicle are equivalent to those that would be generated in a power-in region twice as long on a cable extending to infinity in both directions. Half of the wave energy travels in each direction for the infinite cable. By symmetry arguments only half the power-in length is needed to supply the wave energy traveling in only one direction in the semi-infinite case.

Waves generated in the power-in regions propagate along the cable never to return. At any equilibrium amplitude the power going into creating waves in the power-in region is equal to the sum of the power lost to damping in the power-in region and the power radiated away from the power-in region as traveling waves.

There is a convenient expression for  $N_{in}$  in terms of the local velocity gradient.

As before  $L_{in}$  may be expressed as

$$L_{in} = \frac{dV_R}{\left[ \frac{1}{V} \frac{dV}{dx} \right]} \quad (30)$$

since  $N_{in} = \frac{L_{in}}{\lambda}$ , then

$$N_{in} = \frac{dV_r}{\left[ \frac{\lambda}{V} \frac{dV}{dx} \right]} \quad (31)$$

From this expression it is easy to see that the stronger the shear, the shorter the power-in region as measured in wave lengths. From Equation (29) it is easy to see that as  $N_{in}$  increases so does the response amplitude in the power-in region.

$\frac{\lambda}{V} \frac{dV}{dx}$  is a very useful dimensionless parameter which expresses the shear gradient in a way which is significant to structural dynamic response on very long cylinders.

There is one situation applicable to offshore risers where the previous results may be appropriate. Consider the example of a very long riser which is covered by

fairings over most of its length, except for one bare section. How long must the bare section be in order for VIV to result in significant response amplitudes in the bare region? An approximate answer is given by Equation (29). The faired region of the riser will have significant hydrodynamic damping for any traveling waves created in the region without fairings. Due to the large damping in the faired region the riser will behave as if it were of infinite length. No standing waves will occur outside of the bare region. As before a bare region at one end of a riser of length  $L_{in}/2$  is equivalent to a region of length  $L_{in}$  found in the middle of an infinite cable.

#### 4. CONCLUSIONS

The purpose of this paper was to

- identify areas of weakness in our understanding of high mode number VIV,
- describe experiments that might help resolve some of the questions, and
- identify dimensionless parameters which might help in the planning of experiments and help us to better understand the results.

Four particularly useful parameters have been discussed: a reduced velocity bandwidth parameter, a reduced damping parameter appropriate for sheared flows and, finally, two shear parameters which have significance in structural dynamic response prediction. A simple formula has been provided for estimating the maximum achievable mode number. It shows that  $L/D$ , mass ratio and top towing angle are the key parameters that control maximum mode number.

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