Version 1.0
jproth@mit.edu

Viscoelastic Flows in Abrupt Contraction-Expansions

II. Relaxation Time and the Deborah Number

NB In what follows we use viscometric information from Note I (Fluid Rheology).

To determine the Deborah number for the axisymmetric contraction-expansion we need to select both a characteristic timescale for the flow and a representative measure of the spectrum of relaxation times of the fluid. A characteristic strain rate based on the local flow conditions near the plane of the contraction region is defined by $\dot{\gamma} \equiv \langle v_z \rangle_2 / R_2$ where $\langle v_z \rangle_2 = Q/\pi R_2^2$ is the average velocity into the contraction and R_2 is the radius of the contraction. A characteristic convective time of the flow can then be taken to be $\mathcal{T} \equiv R_2 / \langle v_z \rangle_2 = \dot{\gamma}^{-1}$. The simplest choice for the fluid time scale is of course the longest or Zimm relaxation time λ_z determined from the linear viscoelastic measurements. Even though the viscosity does not have a strong rate dependence, the first normal stress coefficient does. It is therefore important to note that the average relaxation time, which can be calculated from the viscometric properties of the fluid previously determined, is a function of shear rate [1].

Discrepancies between different estimates of the relaxation time have been discussed at length by Keiller *et al.* [2] and by Boger *et al.* [3]. In particular, they point out that numerical calculations based on the (constant) longest relaxation time need to achieve a very large values of Deborah number. For scaling purposes, in our present work the characteristic relaxation time of the 0.025% PS/PS solution is reported using the Maxwell relaxation time evaluated in the limit of zero shear rate. After substituting for the asymptotic value of Ψ_{10} obtained from the Rouse-Zimm model the characteristic relaxation time becomes

$$\lambda_0 \cong \frac{\Psi_{10}}{2\eta_0} = 0.588\ \lambda_z \frac{\eta_p}{\eta_0} = 0.147\ \mathrm{s}$$
$$\text{since}\ \Psi_{10} = 2\eta_p\lambda_z \sum_{j=1}^{N_m} \frac{1}{j^{2(2+\sigma)}} = 2\eta_p\lambda_z(0.590) = 6.72\ \mathrm{Pa\,s}^2 \qquad \textbf{(1)}$$

The zero-shear-rate Deborah number expressed with this choice of constant characteristic relaxation time becomes $De = \lambda_0\,\dot{\gamma}$. It is this number that we report in our paper [4]. Note that

$$\frac{De_0}{De_z} = \frac{\lambda_0}{\lambda_z} = 0.588\left(1 - \frac{\eta_s}{\eta_0}\right) = 0.049 \qquad \textbf{(2)}$$

This ratio is so small because the solution is so dilute. For a numerical simulation using a single mode model chosen to match the *longest relaxation* time in the fluid this corresponds to setting the model parameters to

$$De_z \equiv \lambda_z \dot{\gamma} = 3.08\,\dot{\gamma}$$
$$\frac{\eta_s}{\eta_0} = 0.92$$
$$L^2 \equiv b = 7744$$

In polymer melts it is customary to use a shear-rate-dependent relaxation time of the form

$$\lambda(\dot{\gamma}) \equiv \frac{\Psi_1(\dot{\gamma})}{2\eta(\dot{\gamma})} \qquad \textbf{(3)}$$

The dimensionless product $\lambda(\dot{\gamma})\dot{\gamma}$ is thus equivalent to half the stress ratio [5]. The actual *shear-rate-dependent Deborah number* for flow through a 4:1:4 axisymmetric contraction is then expressed as

$$De(\dot{\gamma}) = \lambda(\dot{\gamma})\;\dot{\gamma} = \frac{\Psi_1(\dot{\gamma})\,\langle v_z \rangle_2}{2\eta(\dot{\gamma})\;R_2} \qquad \textbf{(4)}$$

and may also be thought of as a *Weissenberg number* [6] or recoverable shear because it is a direct estimate of the ratio of the normal stress difference to the total shear stress in the fluid at a given deformation rate $\dot{\gamma}$. We refer to this generically as a *stress ratio* S_R which was used in early publications to report the magnitude of viscoelastic effects in flow through a contraction [5].

Figure 1 shows a plot of the stress ratio, $S_R \equiv N_1(\dot{\gamma}) \,/\, \tau_{12}(\dot{\gamma})$, as a function of strain rate to demonstrate the rate dependence of the elastic stress difference in the fluid. The filled circles represent the experimentally measured steady shear viscometric data and the broken line represents the linear approximation to the stress ratio if the constant relaxation time is used; $S_{R0} = 2\,\lambda_0\,\dot{\gamma}$. The non-monotonicity in the dynamic data is a consequence of the additional elasticity contributed from the polymeric solvent at high deformation rates. The short dashed line represents the same

jproth@mit.edu

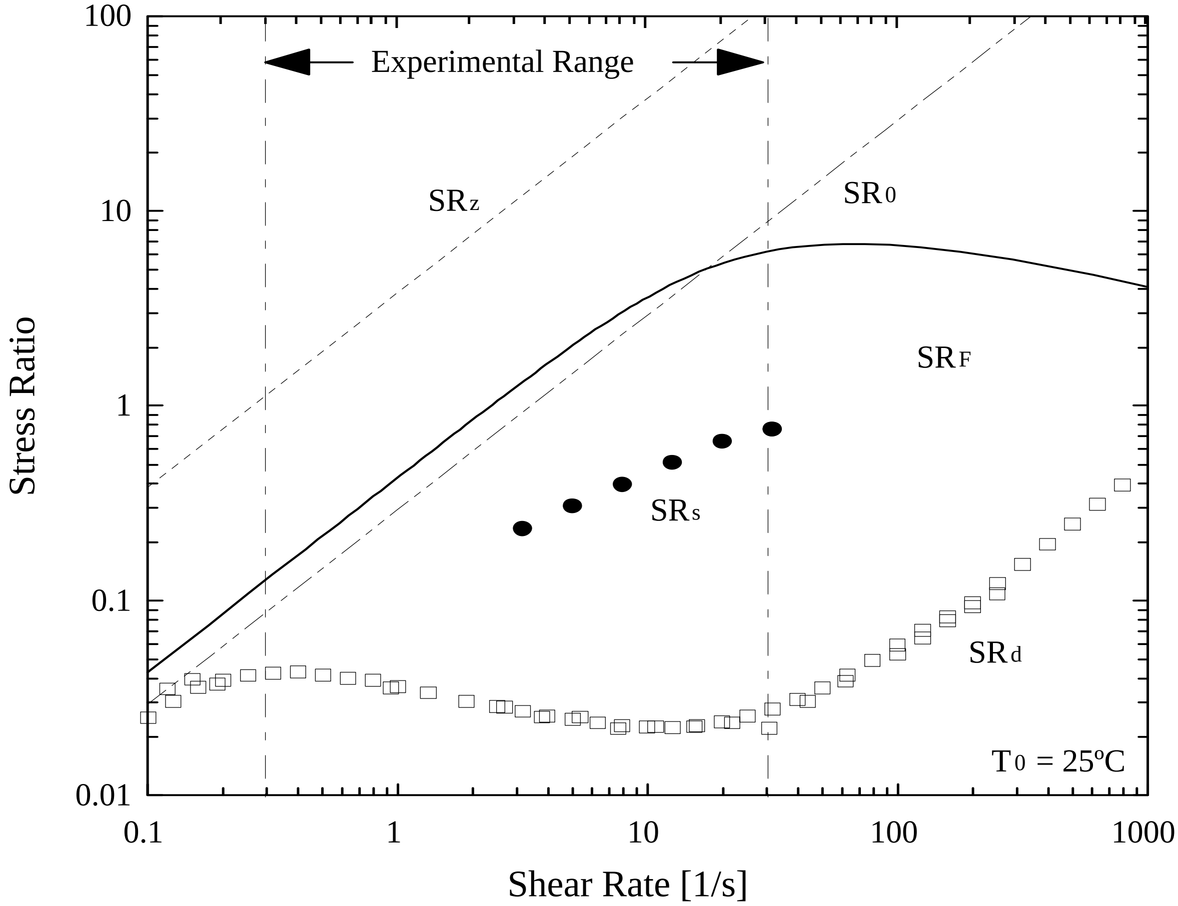

Figure 1: Various estimates of the stress ratio $SR \equiv \tau_{xx} / \tau_{yx}$ of the 0.025wt% PS/PS solution in steady shear. The data includes: '●', the experimental stress ratio from steady shear data $SR_s = N_1 / \tau_{yx} = \Psi_1 \dot{\gamma} / \eta$; '□', experimental stress ratio from dynamic data $SR_d = G' / G'' = \eta'' / \eta'$; '- -', the stress ratio predicted using the longest relaxation time of the Zimm model $SR_Z = 2\lambda_Z \dot{\gamma}$; '– –', the stress ratio predicted using the zero-shear-rate relaxation time $SR_0 = 2\lambda_0 \dot{\gamma}$; '—', the stress ratio SR_F predicted by the FENE-P model.

approximation if the longest "Zimm" relaxation time was used $SR_z = 2\,\lambda_z\,\dot{\gamma}$ and it clearly <u>over predicts the magnitude of elastic effects in the actual test fluid</u>. The solid line represents the stress ratio, SR_F, predicted by the shear thinning single mode FENE-P model. As expected, the experimental values of the stress ratio SR_S increase with strain rate and begin to approach a maximum around one. At low shear rates, the experimental values of SR approach the linear function SR_0, but throughout the range of deformation rates attained in the experiments, they remain up to an order of magnitude smaller. We hope to obtain additional first normal stress coefficient data at lower shear rates in the near future and will modify this plot as data is obtained. Even though SR_0 is clearly a better approximation than

SR_z it is by no means an ideal description of the actual magnitude of the elasticity in the flow of the 0.025% PS/PS solution and shear-thinning effects must be incorporated to achieve a quantitative description of the data. However, even with a shear-thinning first normal stress coefficient, the FENE-P model does not accurately predict the magnitude of the stress ratio.

Conclusion

We recommend that single mode simulations be made using the values for parameters given on page 2. The key conversion factor to remember when reporting data is Equation 2 on page 2.

References

[1] R.B. Bird, R.C. Armstrong and O. Hassager. Dynamics of Polymeric Liquids: Volume 1 Fluid Mechanics, John Wiley & Sons, New York, 1987.
[2] R.A. Keiller, J.M. Rallison and J.G. Evans. J. Non-Newtonian Fluid Mech., **42** (1992) 249-266.
[3] D.V. Boger, M.J. Crochet and R.A. Keiller. J. Non-Newtonian Fluid Mech., **44** (1992) 267-279.
[4] J.P. Rothstein and G.H. McKinley. J. Non-Newtonian Fluid Mech., **86** (1999) 61-88.
[5] C.J.S. Petrie and M.M. Denn. AIChE J., **22** (1976) 209-236.
[6] M. Gupta, C.A. Hieber and K.K. Wang. International J. Numerical Methods in Fluids, **24** (1997) 493-517.