The Influence of Transient Extensional Viscosity on Flow Through an Axisymmetric Contraction-Expansion

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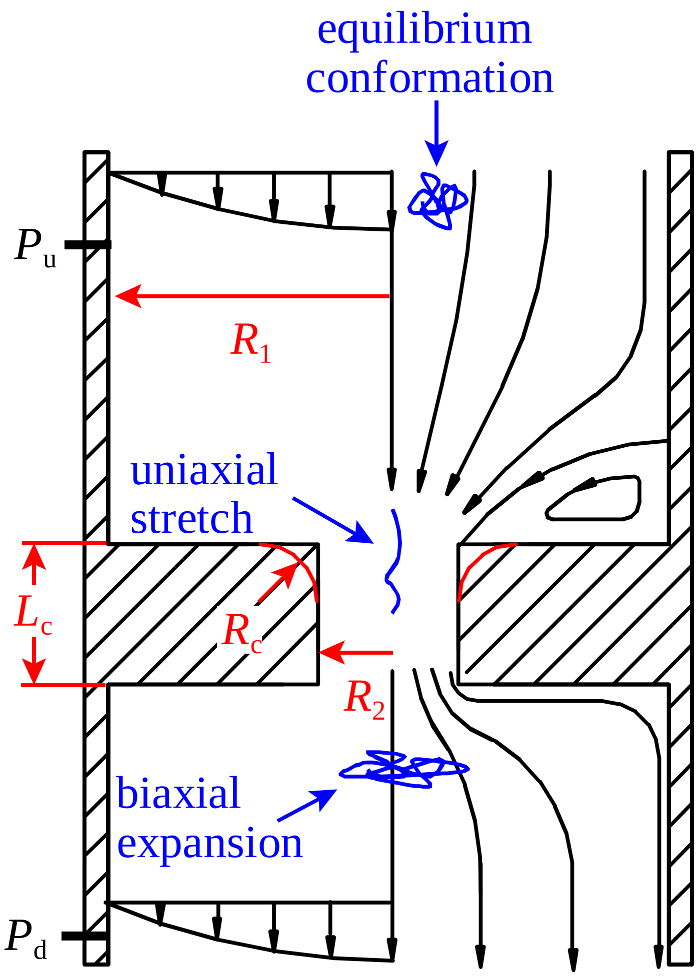
The Axisymmetric Contraction-Expansion

(Cartalos & Piau, 1992; Szabo *et al.*, 1997; Rothstein & McKinley,1999)



$R_1 / R_2 = \beta = 2, 4, 8$
$L_c / R_2 = 0.5, 1, 2$
$R_c = 0, 0.1R_2, 0.2R_2, 0.5R_2$

- Analog of ‘orifice plate’ in Newtonian fluid mechanics!

Sampson (1891)
Dagan et. al (1982)

$$\Delta P_N = \frac{3\eta Q}{\pi R^3}\left(1 + \underline{\frac{8}{3}\frac{L_c}{R_2}}\right)$$

- Upstream contraction flow is numerical and experimental benchmark problem
- Complex flow containing mixture of shear near walls and extensional effects in vicinity of contraction
- Characteristic Deborah number

$$De = \lambda\dot{\gamma} = \lambda Q / \pi R_2^3$$

- Determine extra pressure drop ΔP_{ext} across orifice
  - Insight into existence of purely dissipative ‘internal’ stress

Motivation

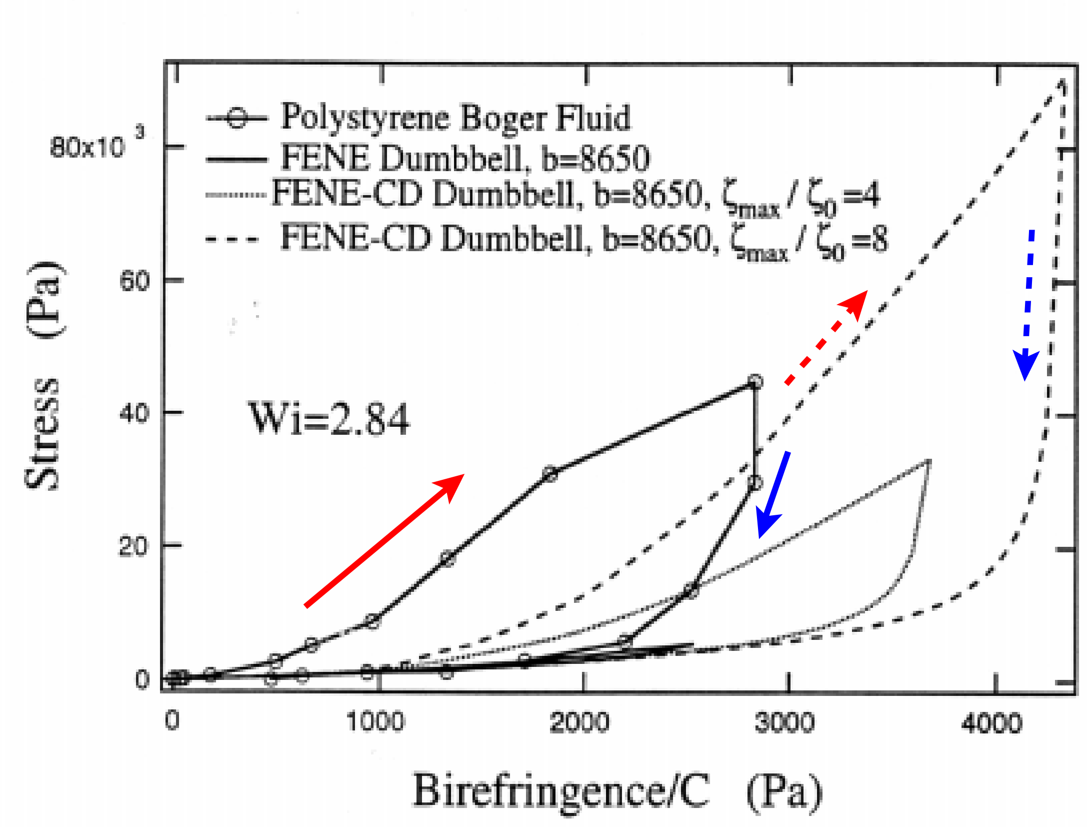
- Existence of a purely dissipative ‘internal’ stress in homogeneous uniaxial elongation resulting in a *stress-conformation hysteresis* proposed by Ryskin(1987) and Doyle *et al.*(1998)

- Stress increases along one pathway as fluid filament is extended and relaxes down a very different pathway once flow is removed

- Result of *stress-conformation hysteresis* is large energy loss

- FENE dumbbell models demonstrate little hysteresis

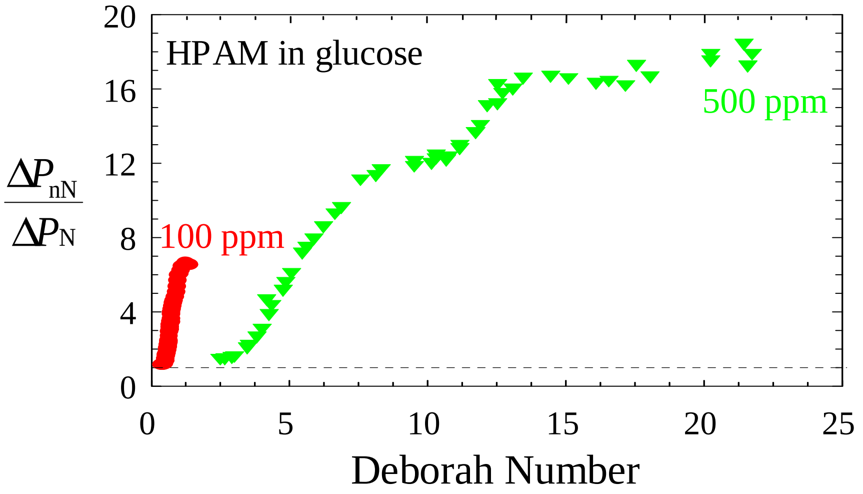
- FENE dumbbell models with additional conformation dependent drags can increase hysteresis but cannot quantitatively match experimental data



Manifestation of Hysteresis in Complex Flows

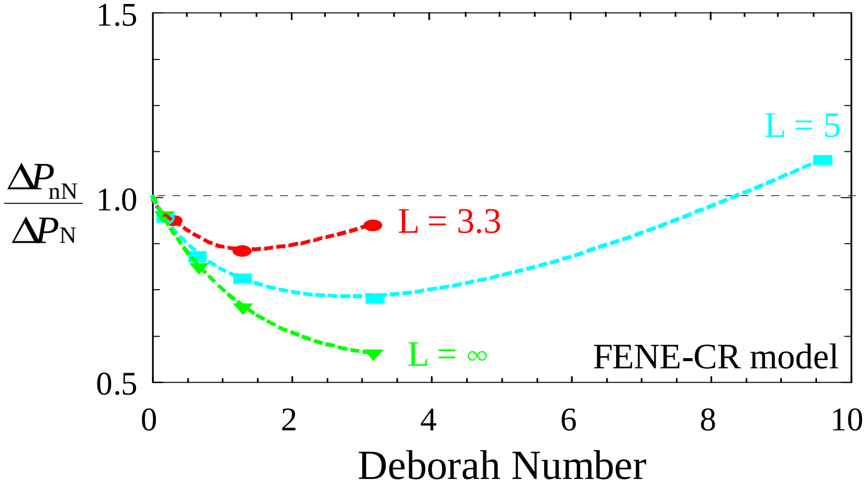
- We wish to observe impact on prototypical non-homogeneous flow

Observation [Cartalos & Piau (1992)]



- ○ Large Enhancement in pressure drop

Computation [Szabo et al. (1997)]



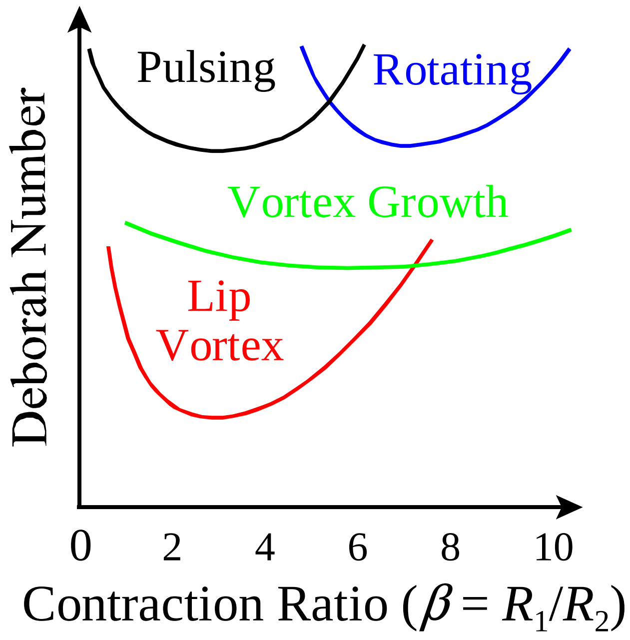
- ○ Entropic elasticity and energy storage
  ⇒ Reduced pressure drop

Question: Is this discrepancy due to ‘dissipative’ or ‘internal’ stresses arising from
non- equilibrium conformation?

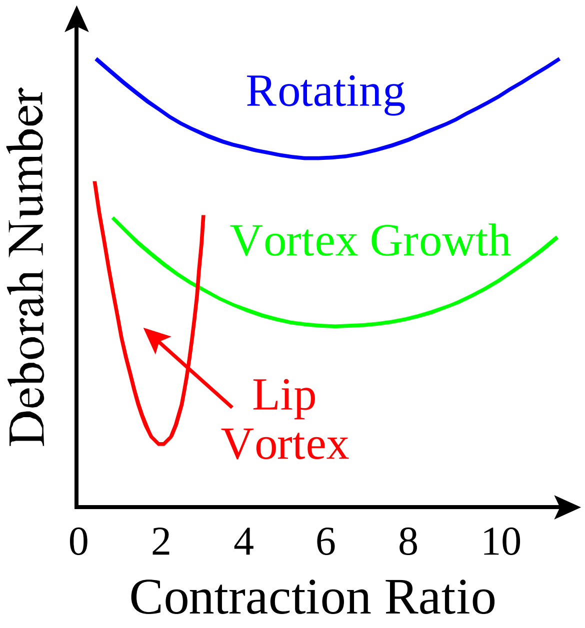
- Similar discrepancies between observations and computation present in other complex flows
  - ○ C_D of spheres [Solomon & Muller (1996)]
  - ○ Pressure drop across banks of cylinders [Khomami & Moreno (1997)]

Overview of Flow Stability in Abrupt Contractions

PIB/PB Boger Fluids
(McKinley; Binnington & Boger)



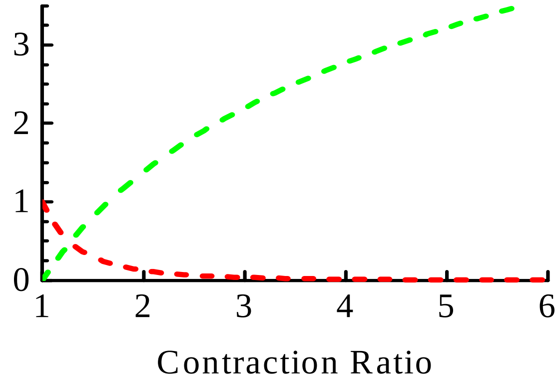
PAA/CS Boger Fluids
(Nguyen, Boger & coworkers)



PS/PS Boger Fluids
(Rothstein & McKinley)

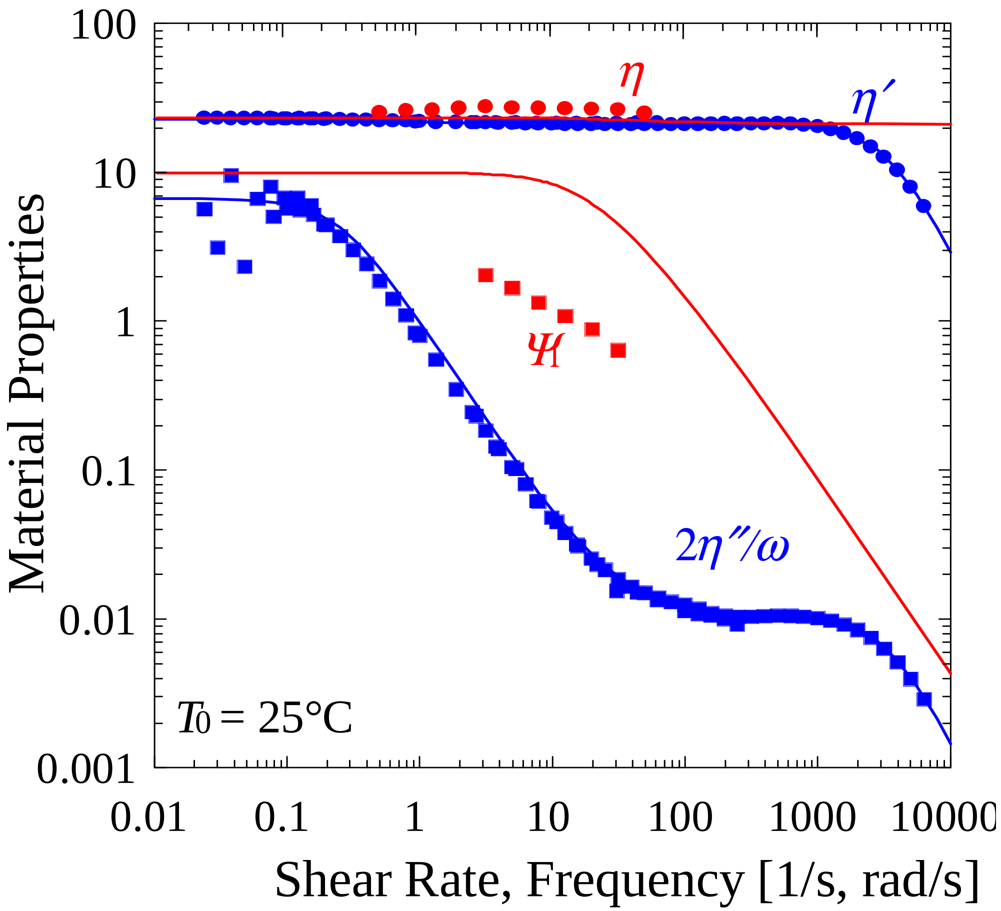
?

- Why do vortex growth dynamics change with contraction ratio and Boger fluid?
- Competing roles of upstream shear rate $\dot{\gamma}_1 / \dot{\gamma}_2 = \beta^{-3}$ and accumulated strain $\varepsilon = \ln(\beta^2)$
- Measurements by Shelley Anna (SF3) of transient extensional viscosity may yield insights



Fluid Rheology

- Monodisperse polystyrene Mw = 2.03×10^6 g/mol dissolved in oligomeric polystyrene
- Dilute solution with concentration $c = 0.025$wt% ⇒ $c/c^* = 0.23$
- Model viscoelastic fluid to probe hysteresis in the absence of polydispersity effects



- Small amplitude oscillatory shear rheology well fit by Rouse-Zimm bead-spring model

  $h^* \simeq 0.1$ ⇒ dominant hydrodynamic interactions

  $\lambda_1 = 3.08\text{s}$ $\beta \equiv \eta_s/\eta_0 = 0.92$

  $\lambda_i = \lambda_1 / i^{1.77}$

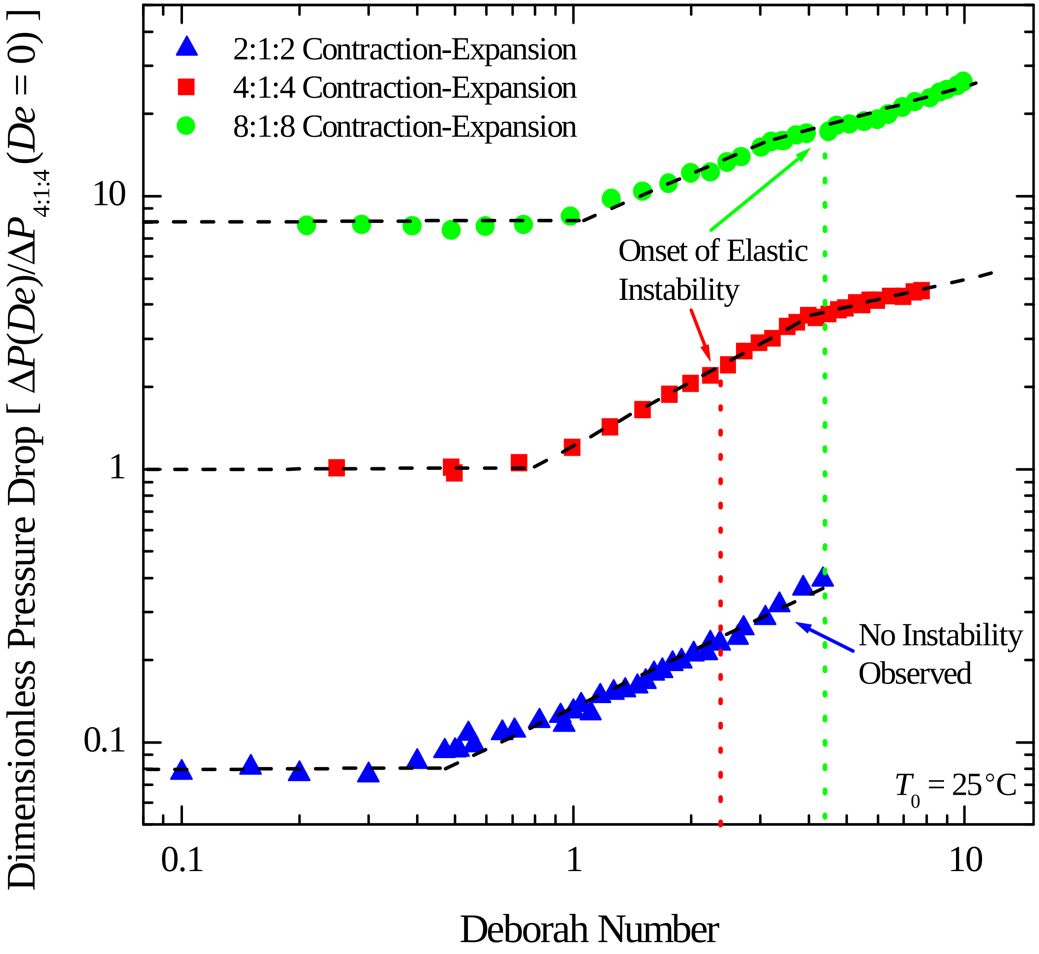
- Weakly elastic polymeric solvent

  $\lambda_{ps} = 2.5\times10^{-4}$ s

- Steady shear data poorly fit by FENE-P model with a finite extensibilty of L = 88
- Temperature variations described by shift factor of WLF theory.

Pressure Drop Across Contraction/Expansions of β=2,4 and 8

- All contraction ratios show similar pressure growth trends.
- Pressure drop increase with *De* before onset of elastic flow instability.



- Newtonian-like response

$$\Delta P_{\mathrm{nN}} \sim Q$$

$$0 < De \lesssim 1$$

- Pressure drop increases quadratically with flow rate

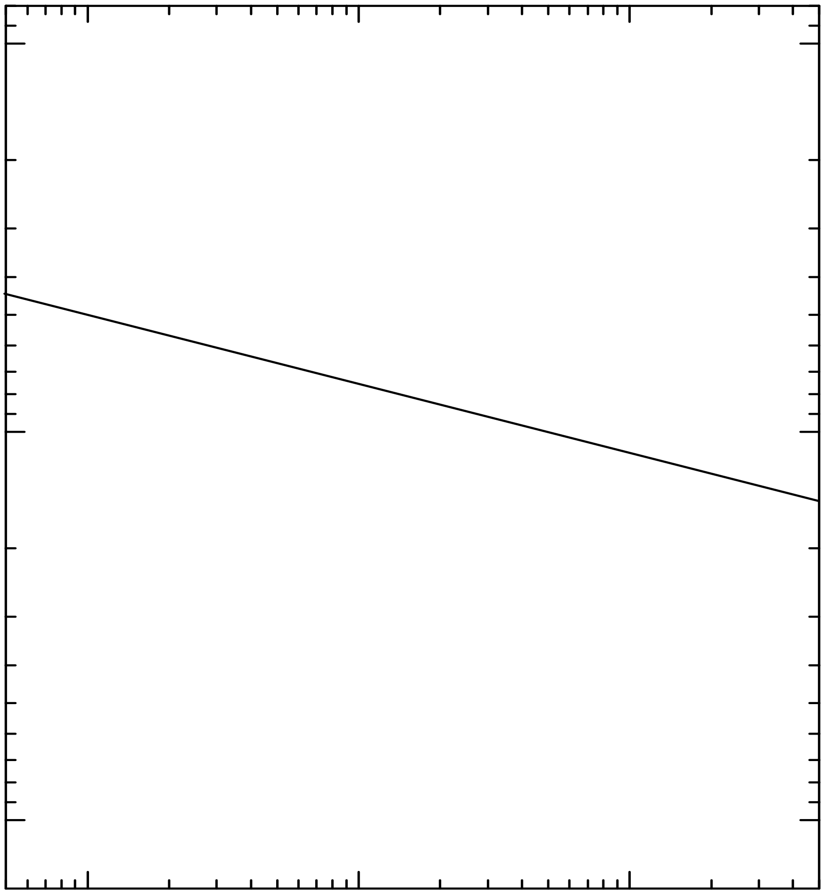
$$\Delta P_{\mathrm{nN}} \sim Q^2$$

$$1 \lesssim De \lesssim 5$$

- Pressure drop increases more slowly approaching a linear dependence on flow rate

$$\Delta P_{\mathrm{nN}} \sim Q$$

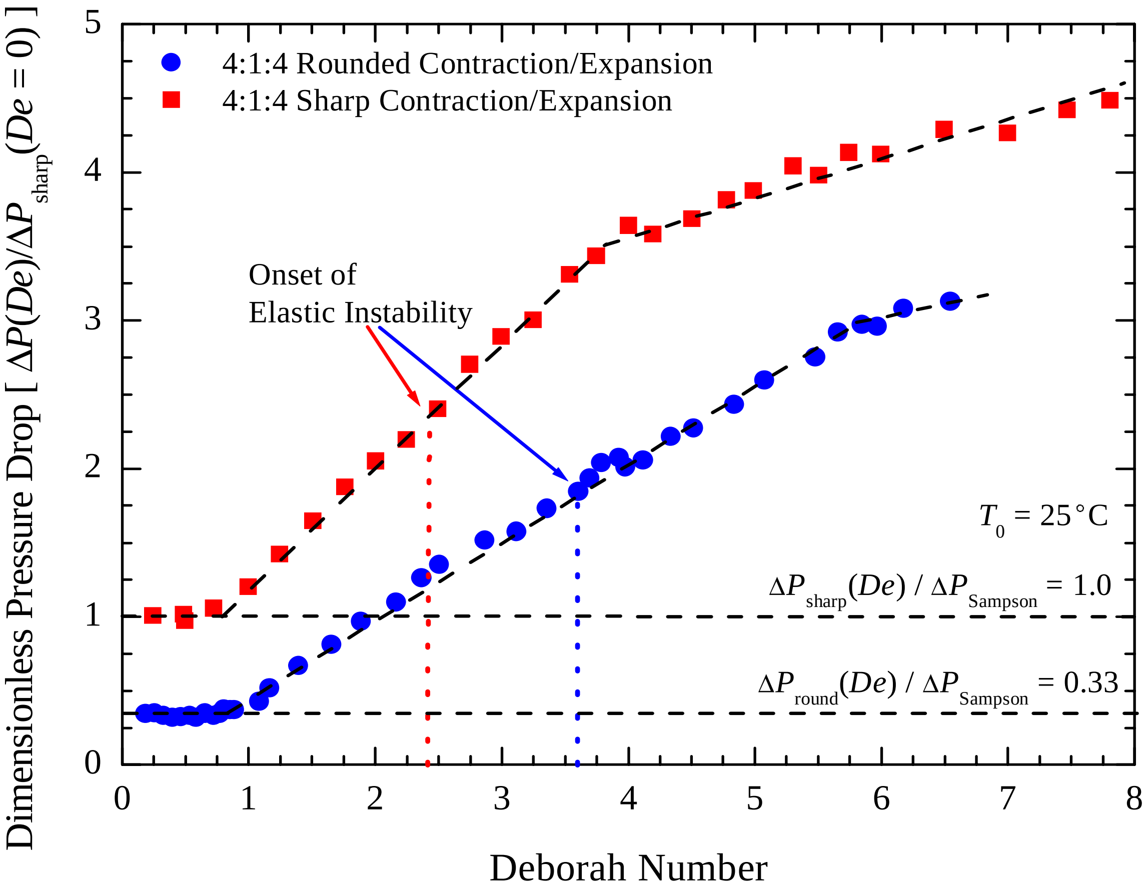
$$De \gtrsim 5$$



Effect of Re-entrant Lip Curvature on Pressure Drop

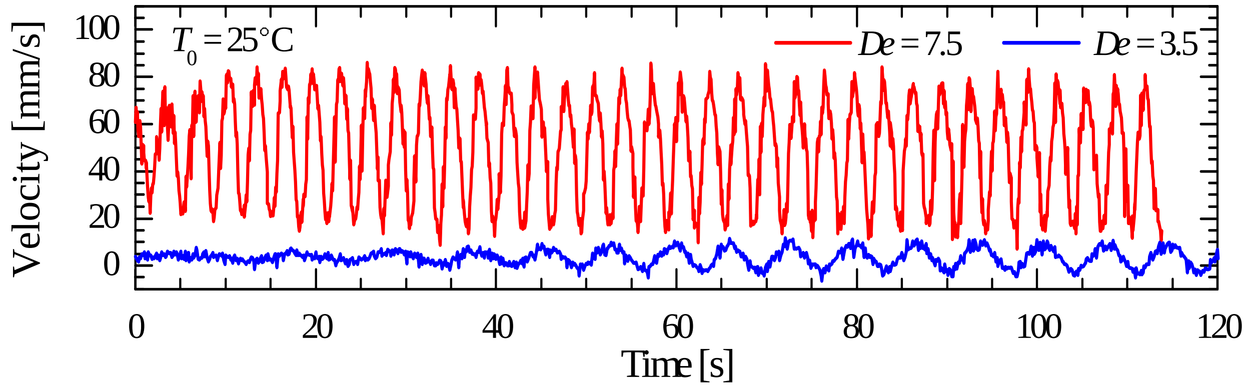
- Sharp and rounded entrance lip pressure drop data follow same trends

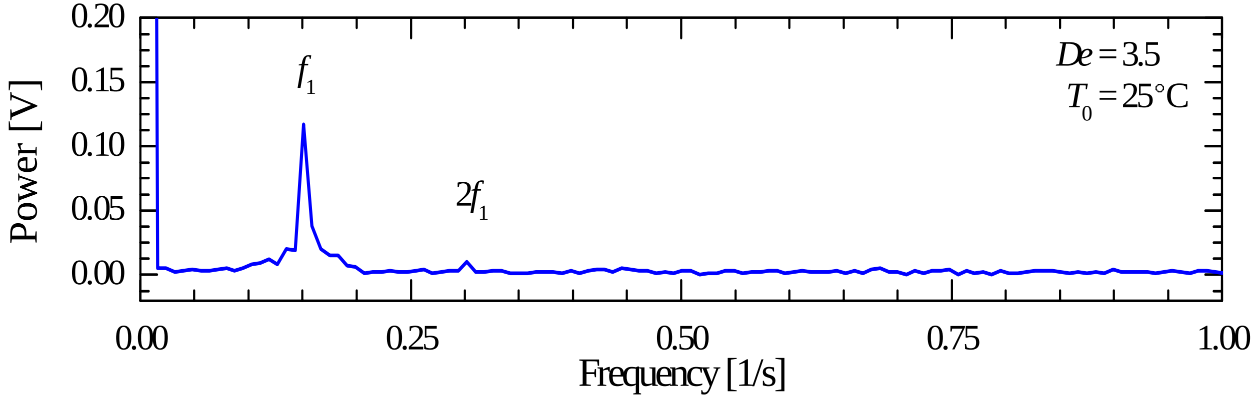
- Small changes in lip curvature have huge effect on pressure drop

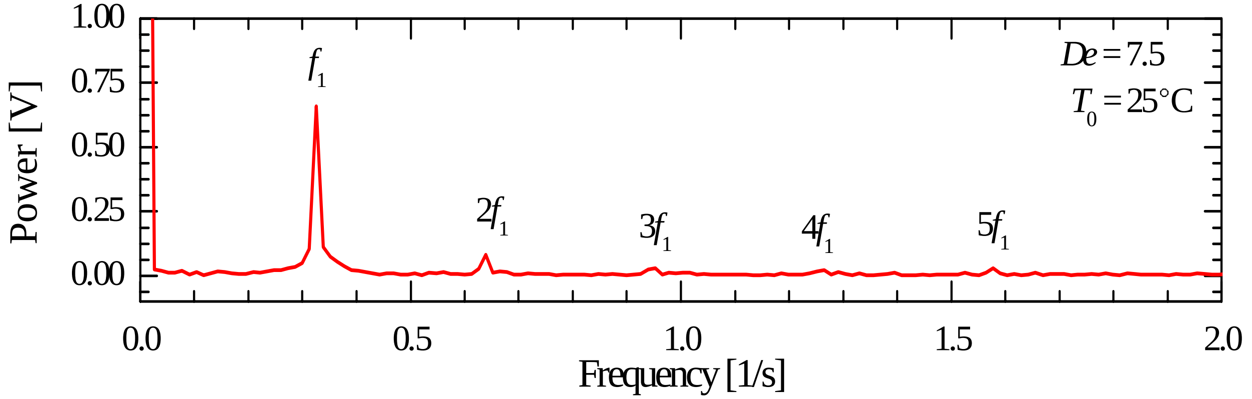


- Rounding lip entrance delays onset of each pressure drop transition and elastic flow instability

Onset of Elastic Instability



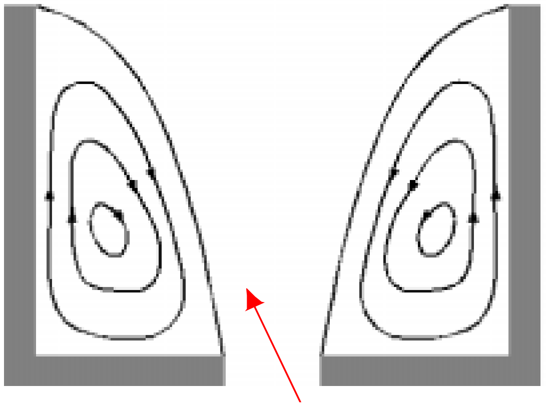




- Small amplitude oscillations seen in Laser Doppler Velocimetry (LDV) measurements at large *De*

- Higher harmonics of fundamental frequency appear as *De* increases

Location of LDV Probe

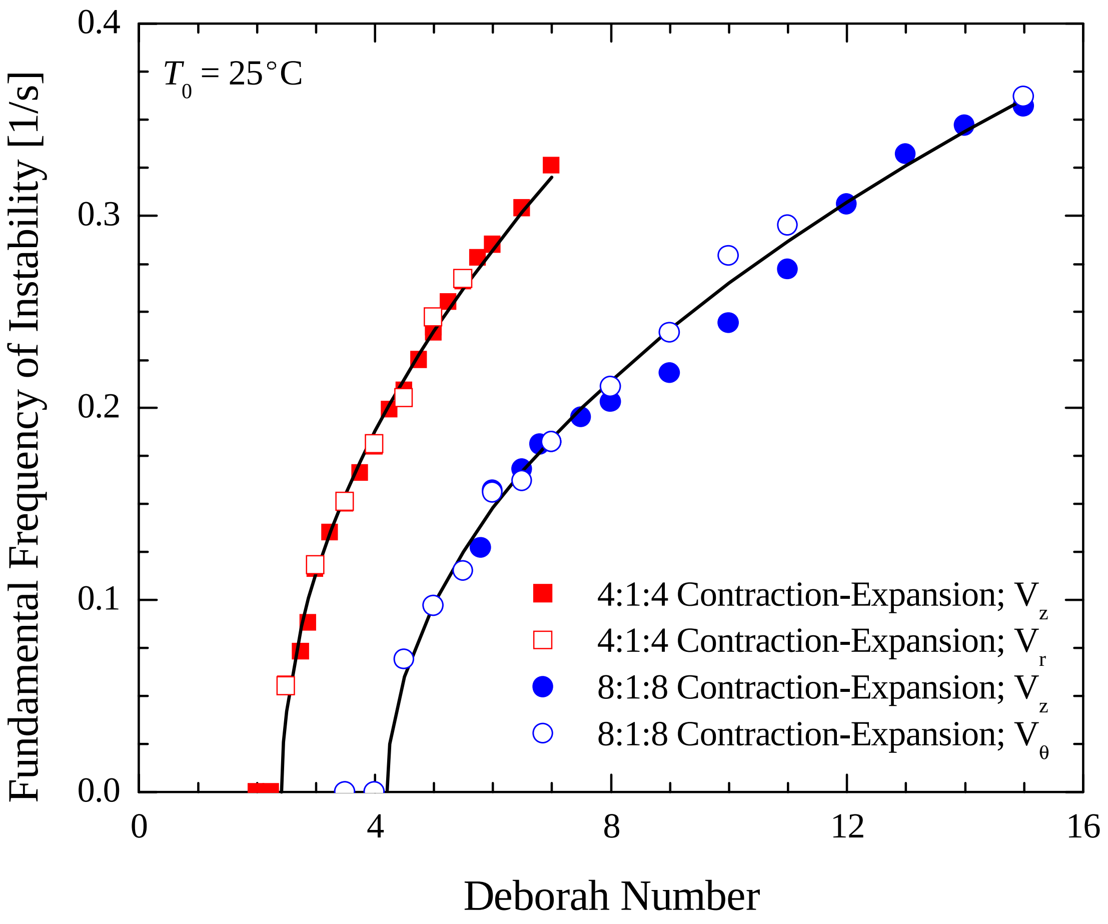


($r/R_2 = 0.63$, $z/R_2 = -1.26$)

Characterization of Elastic Instability

	Critical Deborah Number for Onset of Instability
4:1:4 Sharp Contraction/Expansion	2.4 ± 0.1
4:1:4 Round Contraction/Expansion	3.6 ± 0.2
8:1:8 Sharp Contraction/Expansion	4.2 ± 0.1

- Lip curvature delays the onset of the elastic instability



- Solid lines denote theoretical result for supercritical Hopf bifurcation

$$f_1 \propto (De - De_{crit})^{½}$$

- Dimensionless frequency of elastic flow instability

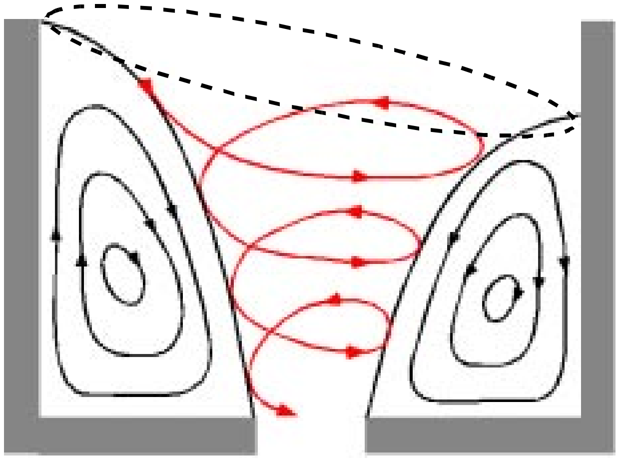
$$\lambda_z f_1 = \frac{\lambda_z \omega}{2\pi} \sim 1$$

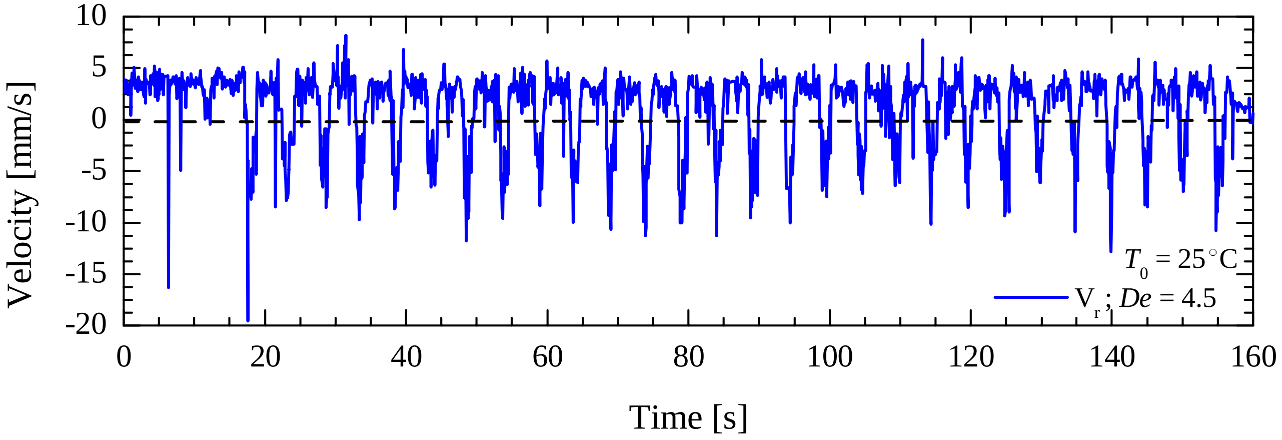
$$\lambda_z = 3.08s$$

Nonlinear Dynamics: Jetting

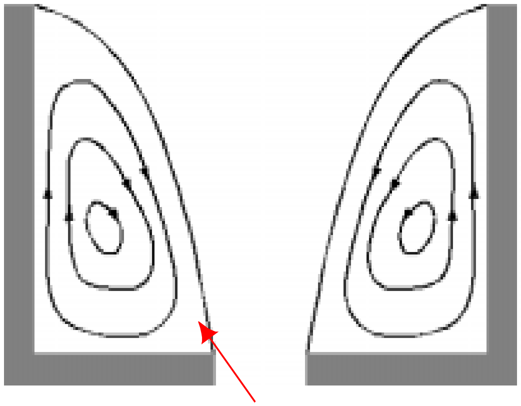
- Localized elastic jetting instability seen in 4:1:4 contraction-expansion after onset of vortex asymmetry

- We postulate that instabilityis helical jet of high speed fluid traveling down the interior of vortex structure

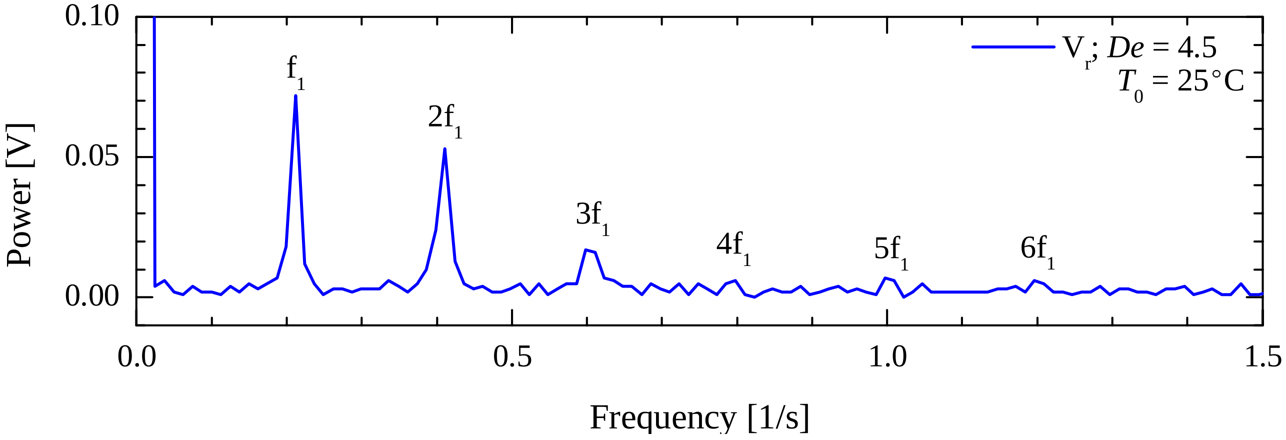




Location of LDV Probe

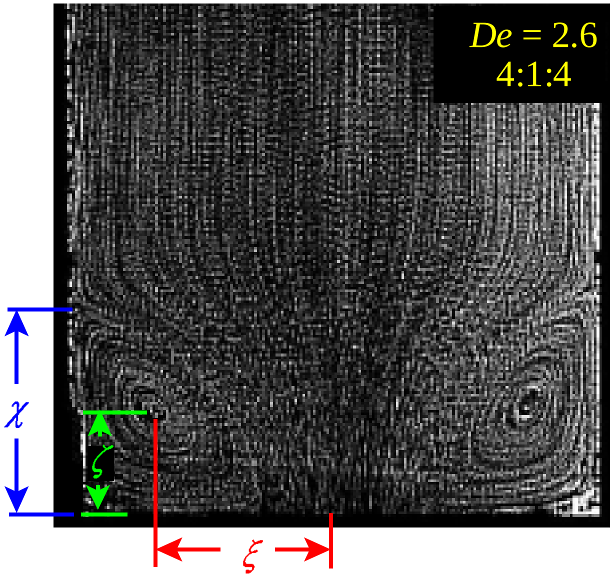


(r/R_2 = 1.57, z/R_2 = -0.16)

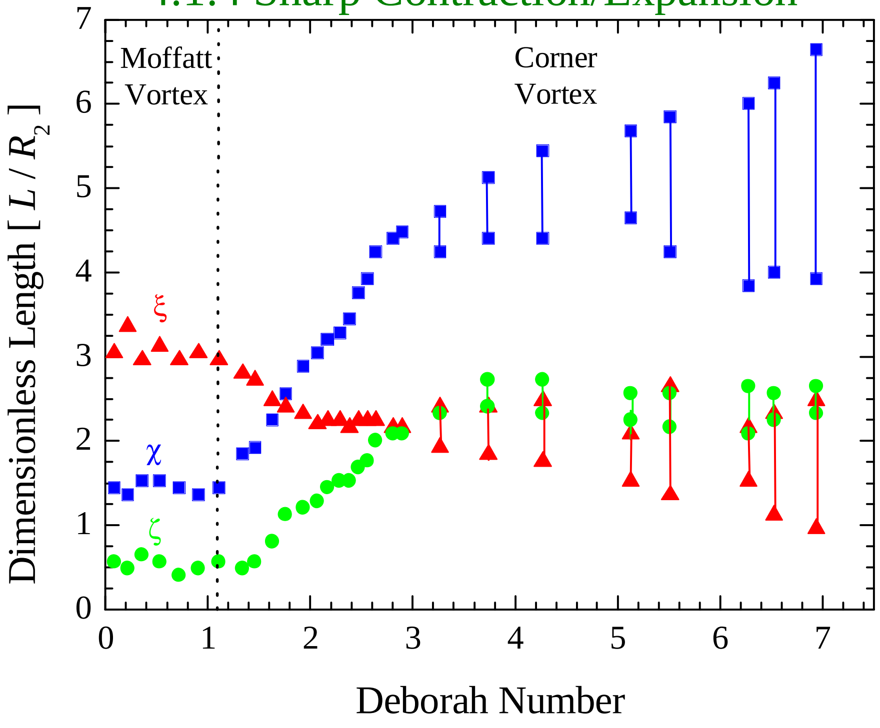


Vortex Growth Dynamics

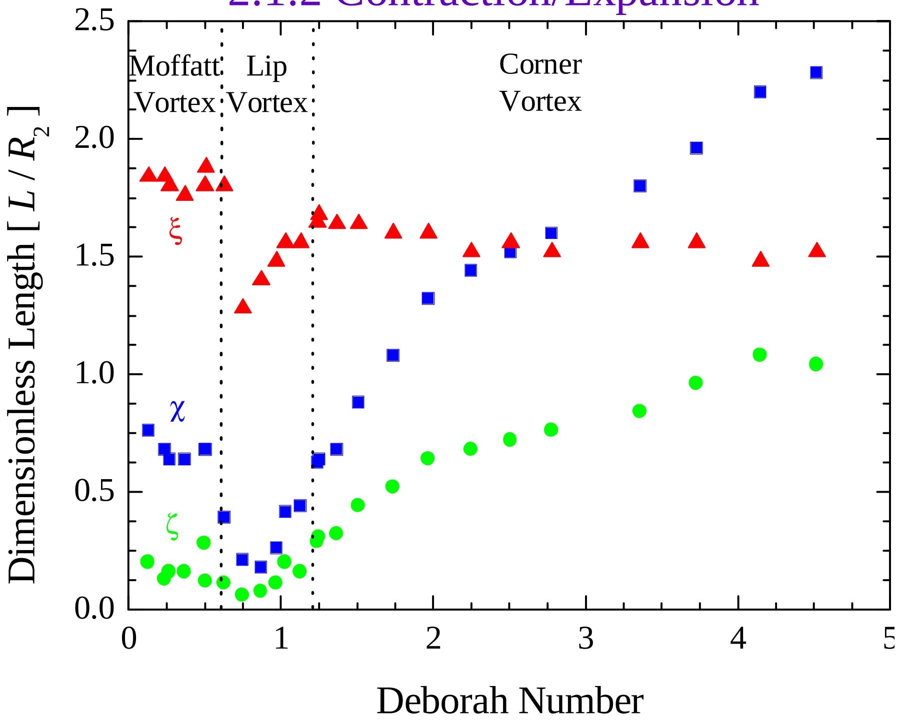
- At low *De*, Newtonian-like Moffatt vortex exist
- At moderate *De*, two distinct patterns of vortex growth exist
  - For $\beta \geq 4$ vortex grows out from salient corner, grows upstream and eventually becomes unstable
  - For $\beta = 2$ new lip vortex emerges near re-entrant corner, grows toward the salient corner and then proceeds to grow upstream



4:1:4 Sharp Contraction/Expansion



2:1:2 Contraction/Expansion



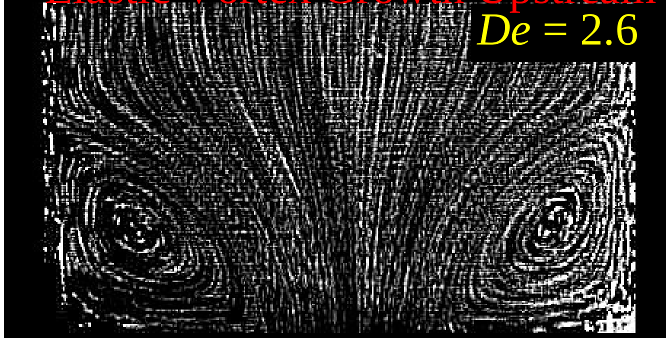
A Proper Taxonomy of Vortex Structure

$\beta = 4$

Elastic Vortex Enhancement

$De = 1.6$

Elastic Vortex Growth Upstream



$De = 2.6$

Unstable Elastic Vortex

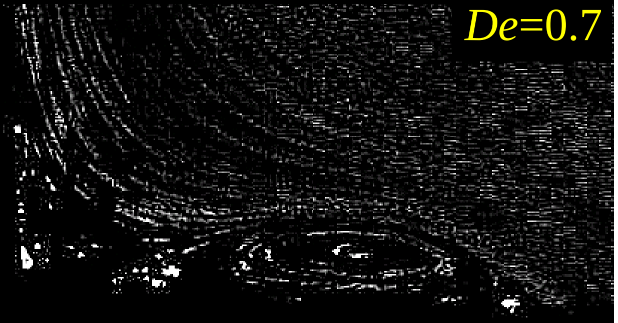
$De = 3.6$

$De=0.5$

Moffatt Corner Vortex

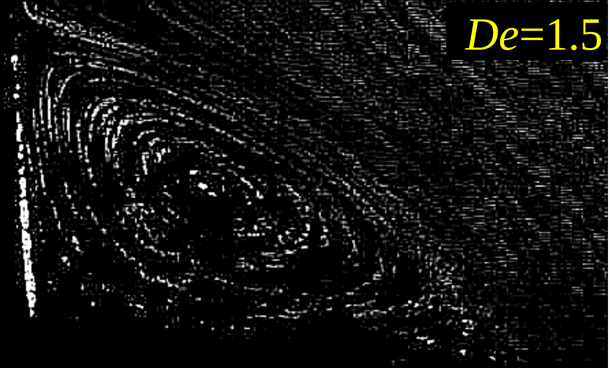
$\beta = 2$

Elastic Lip Vortex



$De=0.7$

Elastic Vortex Enhancement



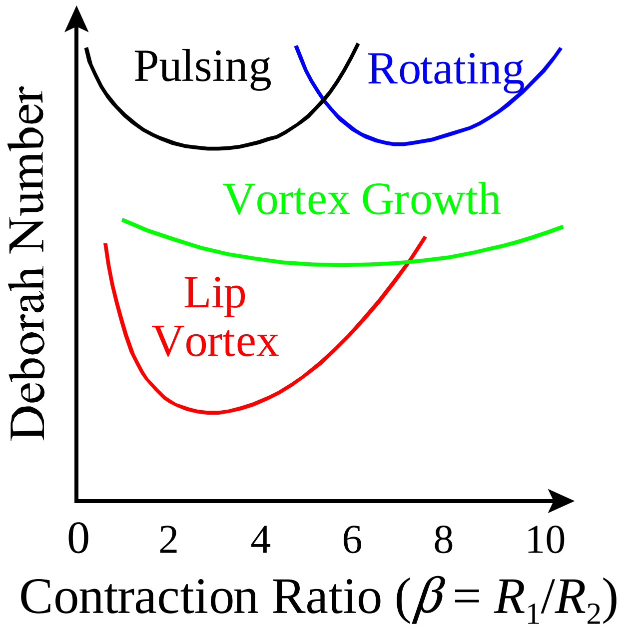
$De=1.5$

Vortex Remains Stable

$De=2.7$

Flow Stability Diagram for PS/PS Boger Fluid

PIB/PB Boger Fluids
(McKinley; Binnington & Boger)



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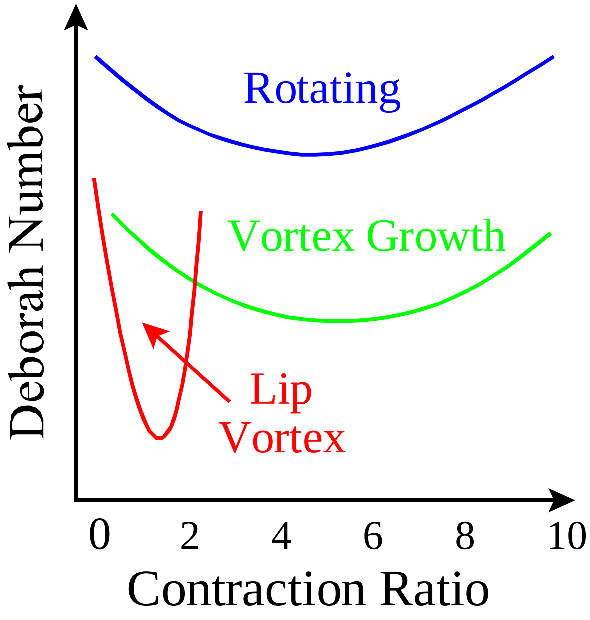
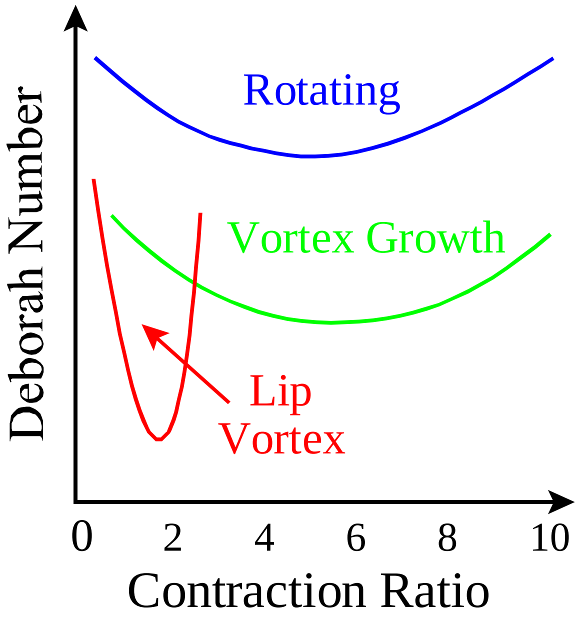


PS/PS Boger Fluids
(Rothstein & McKinley)



- Flow stability diagram of PS/PS Boger fluid is very similar to PAA/CS Boger fluid and dissimilar to PIB/PB Boger fluid

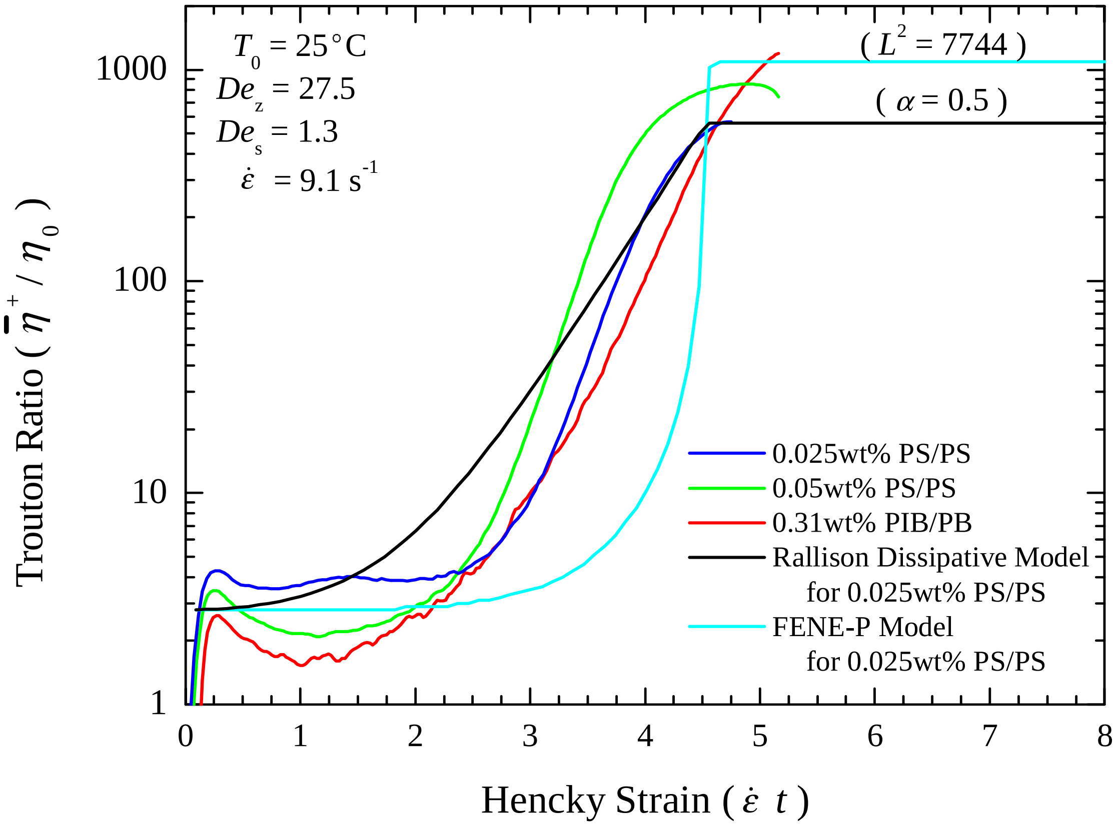
Transient Extensional Rheology (In collaboration with Shelley Anna)

- Why do two fluids with similar shear properties behave so differently in complex flows?

- PIB and PS Boger fluids exhibit markedly different stress growth

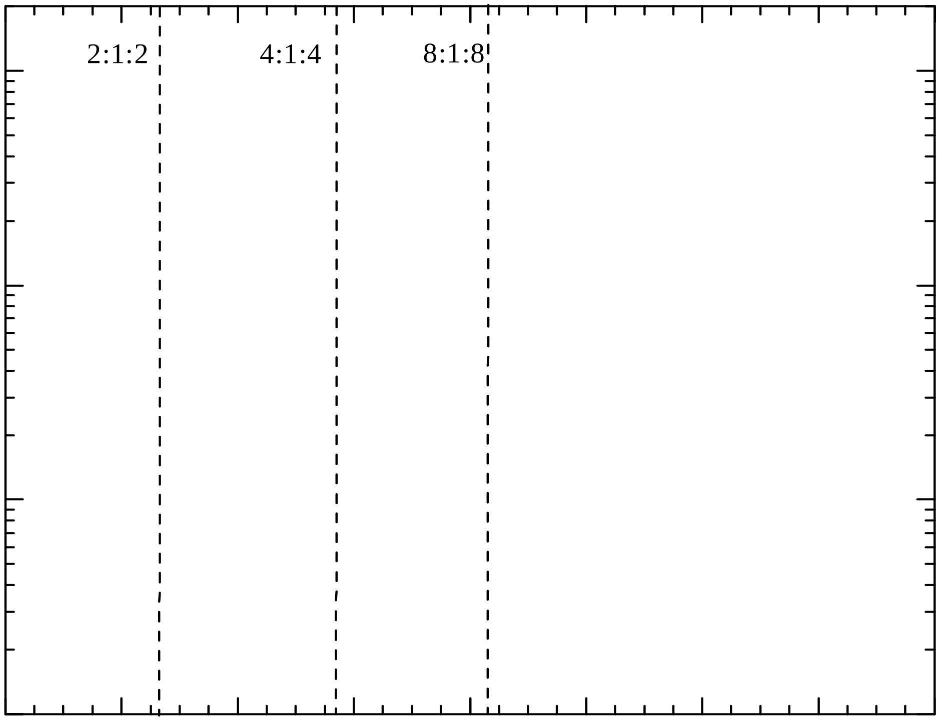
- Results $\bar{\eta}^{+}(\dot{\varepsilon}_0)\,/\,\eta_0$ are approximately independent of $De_z = \lambda_z\,\dot{\varepsilon}_0$ (for $De_z > 1$)

- How much stress growth do we get in an axisymmetric contraction?



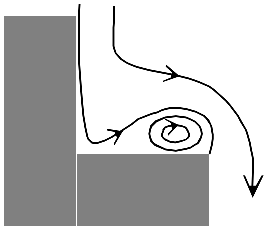
Accumulated Strain Along Centerline

$$\varepsilon \equiv \int_0^{t_1} \dot{\varepsilon}\,dt = \int_{v_z(-\infty)}^{v_z(z=1/2L_c)} \frac{dv_z}{v_z} = \ln\left(\frac{R_1}{R_2}\right)^2 = 2\ln\beta$$


2:1:2
4:1:4
8:1:8

Comparison of PS and PIB Boger Fluids

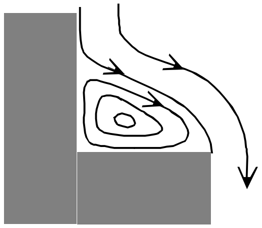
Lip Vortex



PIB β ≤ 4

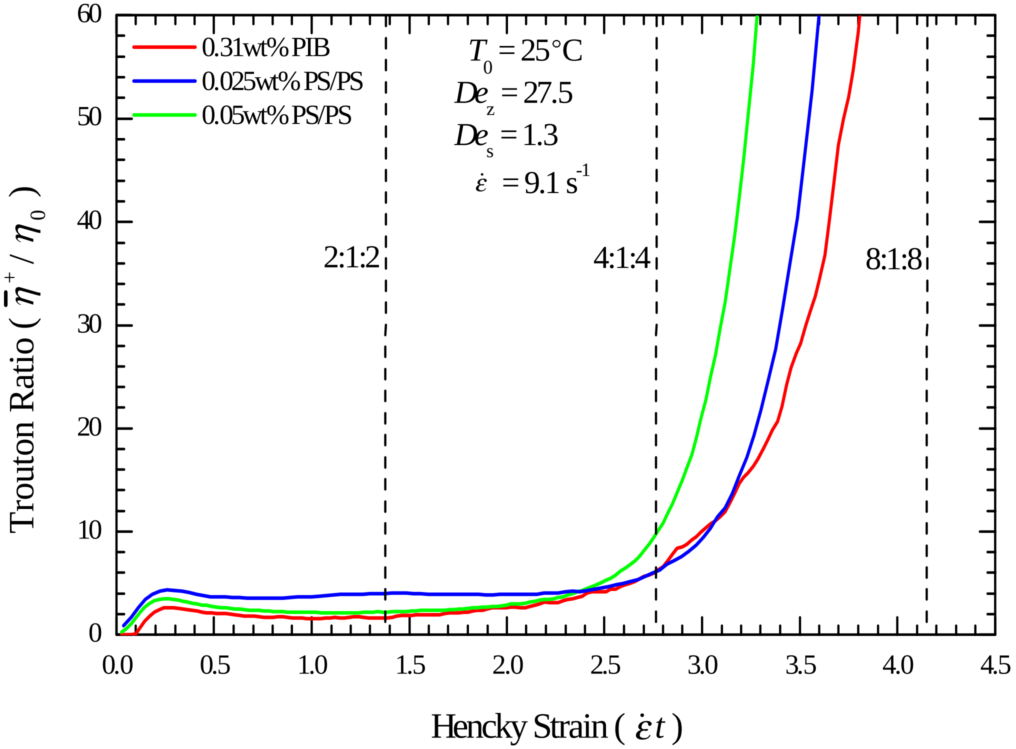
PS β = 2
PAA

Corner Vortex



PIB β > 4

PS β > 2
PAA



- Experiments suggest that the lip vortex is present for smaller contractions (with larger shear component) while the corner vortex is present in flows dominated by extension.
- However, extensional viscosity alone still cannot differentiate between the PS and PIB Boger fluids if we consider only the pure extension along the centerline.
- Does the shearing near the wall increase the effective Hencky strain? Can we investigate this by preshearing the fluid before stretching it?

Concluding Remarks

- Observed large monotonic increase in pressure drop not correlated with the onset of elastic flow instability and not qualitatively affected by changes in lip curvature

  ⇒ Is this the result of the *stress-conformation hysteresis* observed in transient uniaxial extension?

- To answer this question more concretely, molecular simulations in transient inhomogeneous flows need to be performed

- Evolution in flow structure of PS/PS Boger fluid similar to PAA/CS Boger fluids and dissimilar to PIB/PB Boger fluids

  ⇒ Are variations in vortex growth dynamics the result of differences in transient extensional rheology of fluids?

- Results of simple transient uniaxial extension are inconclusive

- Preliminary results of preshear to transient uniaxial extension show considerable impact on Trouton ratio.