Local and Global Measures of the Stress Distribution in Abrupt Contraction-Expansions

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XIIIth International Congress on Rheology
August 23, 2000

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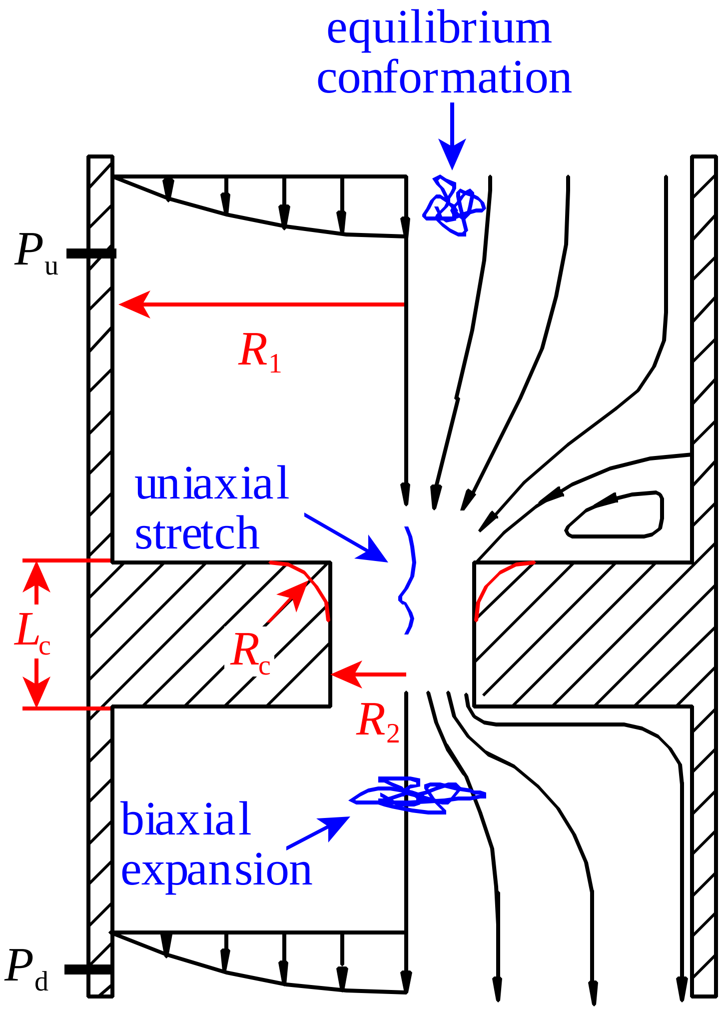
The Axisymmetric Contraction-Expansion

(Cartalos & Piau, 1992; Szabo *et al.*, 1997; Rothstein & McKinley,1999)



$R_1 / R_2 = \$ = 2, 4, 8$
$L_c / R_2 = 0.5, 1, 2$
$R_c = 0, 0.1R_2, 0.2R_2, 0.5R_2$

C Upstream contraction flow is numerical and experimental benchmark problem

C Complex flow containing mixture of shear near walls and extensional effects near contraction

C Combination of global and local flow measurements used to extensively characterize flow

C Characteristic Deborah number

$$De = l\dot{g} = l\, Q / \pi R_2^3$$

C Creeping flow regime

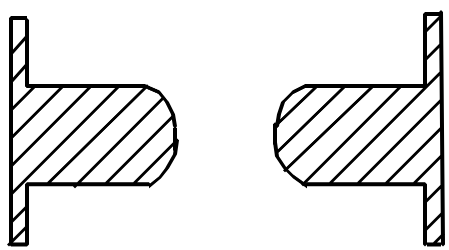
$$Re < 10^{-3}$$

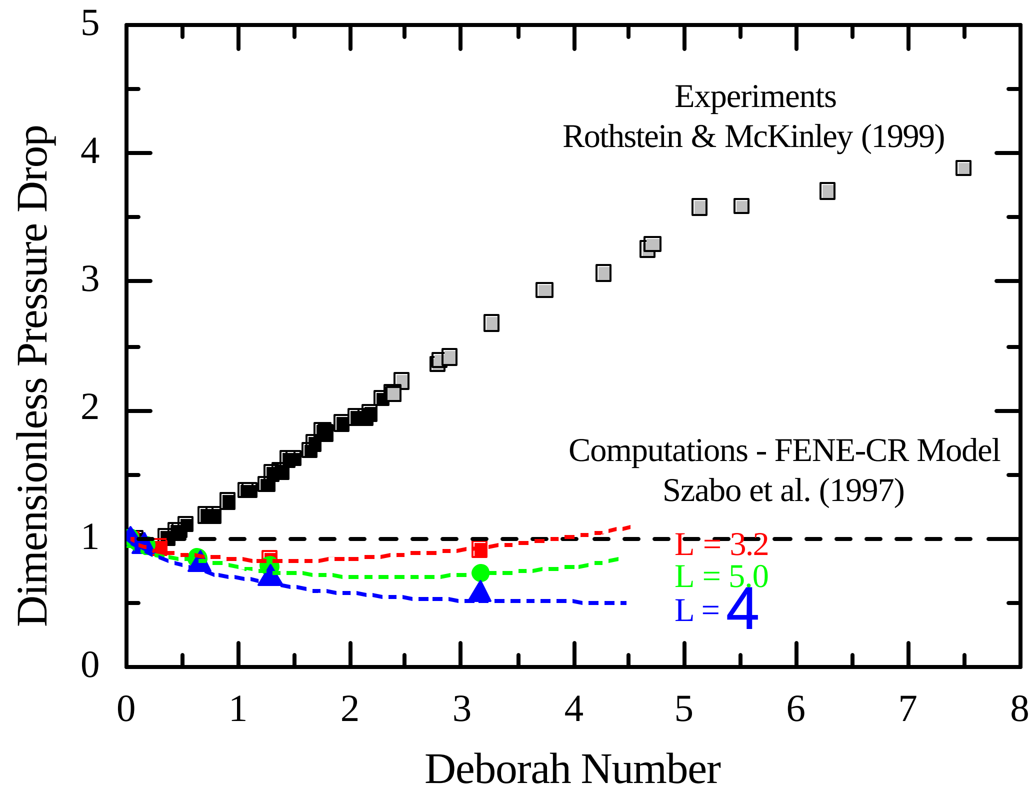
Motivation

- There is a lack of agreement between experiments and computations in non-homogeneous flows

4:1:4 Contraction-Expansion

with Rounded Corners





- These discrepancies may be due to the internal, purely-dissipative stresses arising from non-equilibrium molecular conformations.

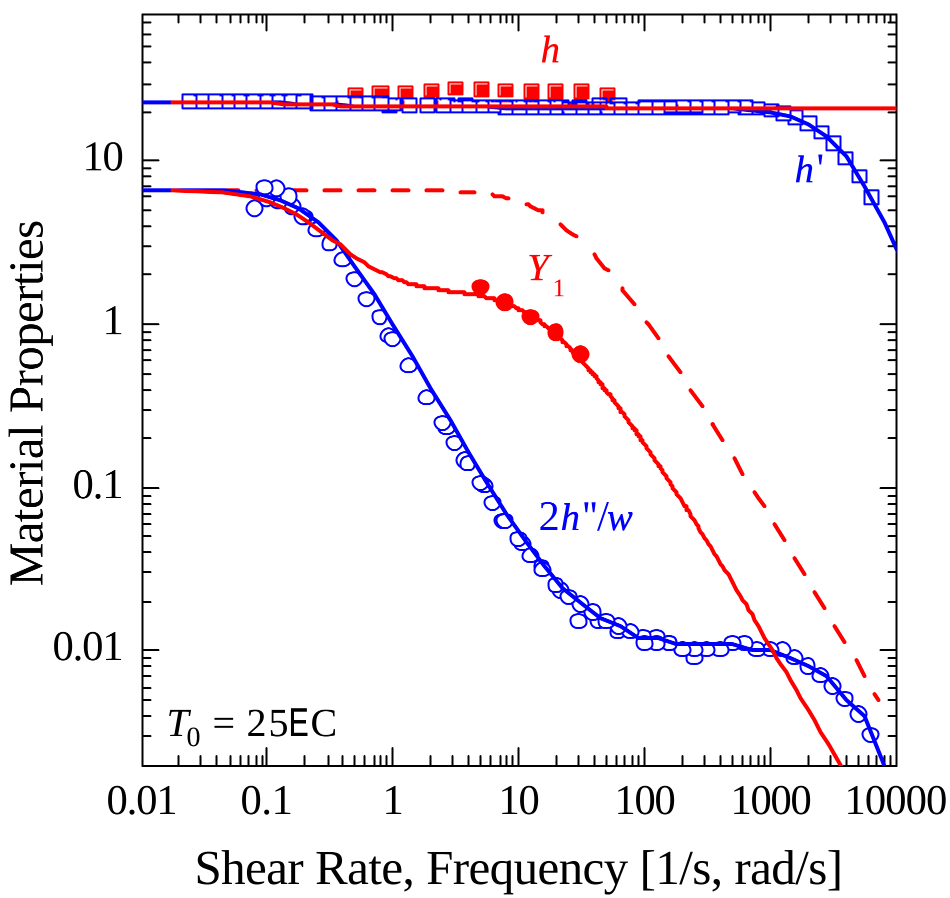
- Presence of stress-conformation hysteresis in transient uniaxial elongation suggests need for *global* (pressure drop) and *local* (birefringence) probes of conformation and stress.

Fluid Shear Rheology

C Monodisperse polystyrene M_w = 2.03×10^6 g/mol dissolved in oligomeric polystyrene

C Dilute solution with concentration c = 0.025wt% Y c/c^* = 0.23

C Model viscoelastic fluid to probe flow in the absence of polydispersity and inertial effects



C Small amplitude oscillatory shear rheology:

6 well fit by Rouse-Zimm bead-spring model

h^* • 0.1 Y dominant hydrodynamic interactions

8_1 = 3.08s

$8_i = 8_1 / i^{1.77}$

$0_s/0_0$ = 0.92

C Steady shear rheology:

6 poorly fit by FENE-P model ‘&&’ (L = 88)

6 well fit by Bird-DeAguiar model ‘&’

(L = 88, F = 0.62, $\$$ = 1.0)

C Extensional rheology also **very** important

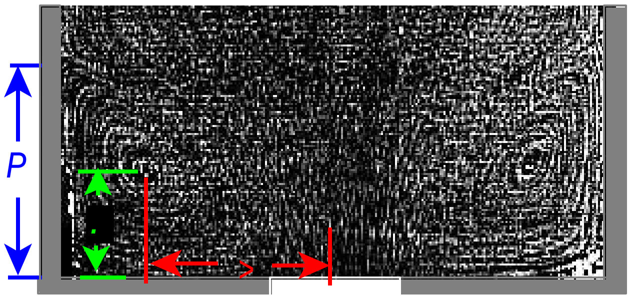
Vortex Growth Dynamics

- At moderate Deborah numbers, two distinct patterns of vortex growth exist
  - For *b* $ 4, vortex grows out from salient corner, grows upstream and eventually becomes unstable
  - For *b* = 2, 'lip' vortex emerges near re-entrant corner, grows toward salient corner and upstream

4:1:4 Contraction/Expansion

Corner Vortex

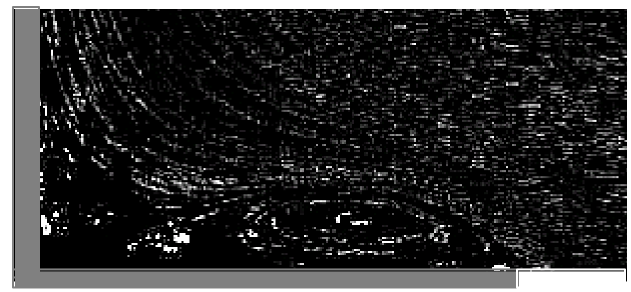
De = 2.6

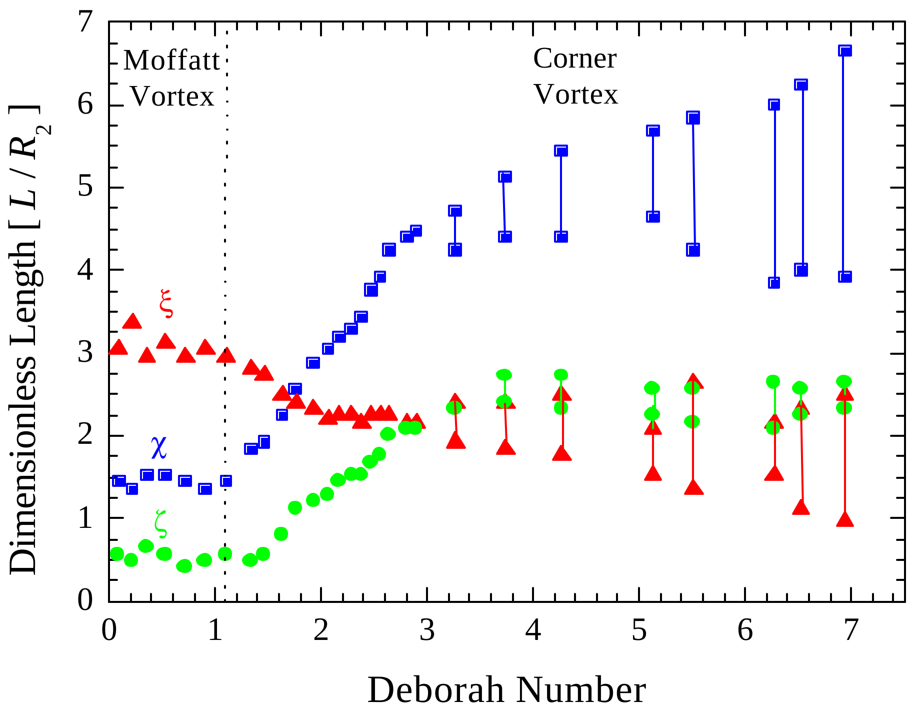


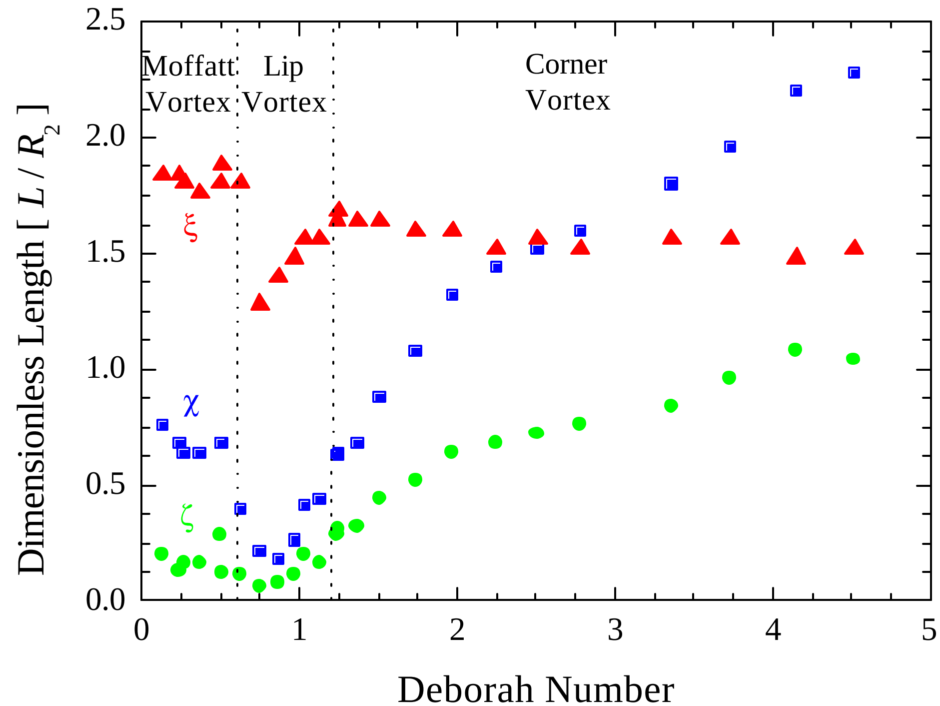
2:1:2 Contraction/Expansion

Lip Vortex

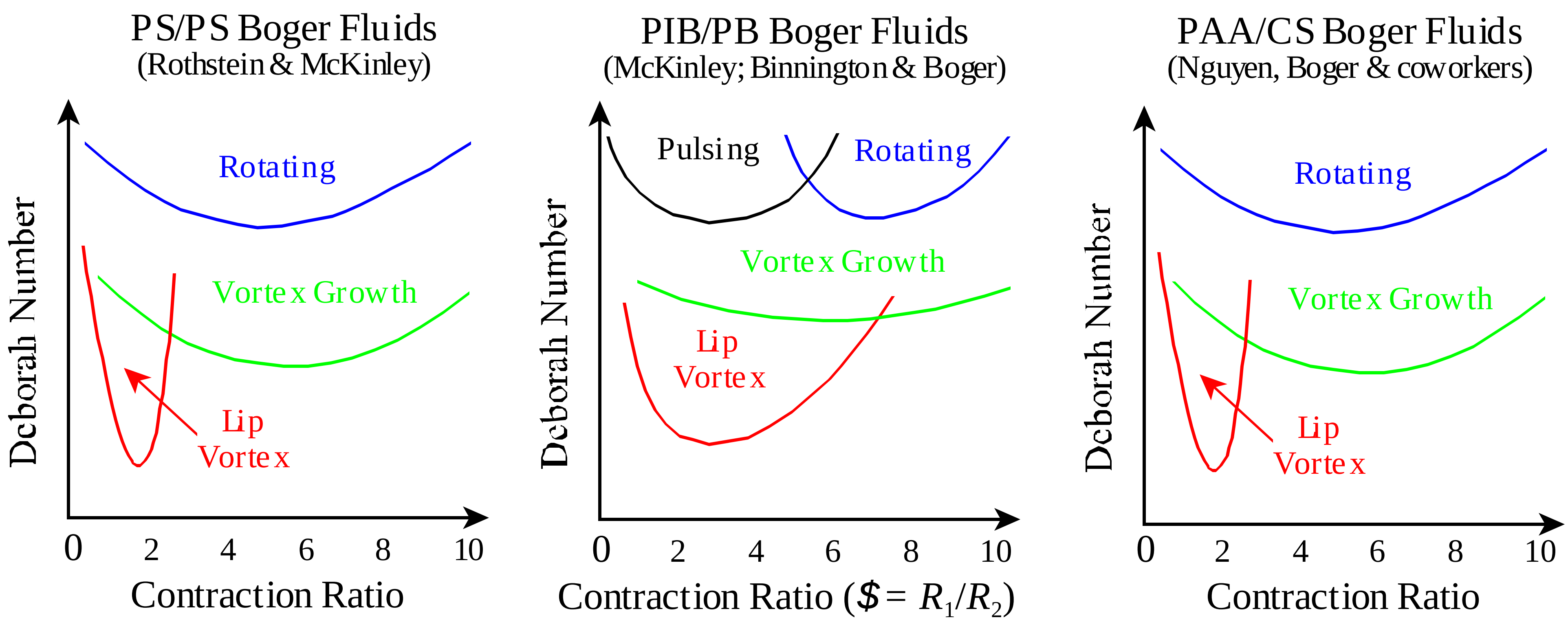
De = 0.9







Flow Stability Diagram for PS/PS Boger Fluid



- Flow stability diagram of PS/PS Boger fluid is very similar to PAA/CS Boger fluid and dissimilar to PIB/PB Boger fluid
- Why do vortex growth dynamics depend on both contraction ratio and test fluid used?
  - Many have speculated that fluid dependence of flow structure arises from differences in extensional viscosity of different Boger fluids.

Transient Extensional Rheology

- PIB/PB and PS/PS Boger fluids exhibit similar stress growth

- Results are approximately independent of $De_Z = \lambda_Z \dot{\varepsilon}$ (for $De_Z > 1$)

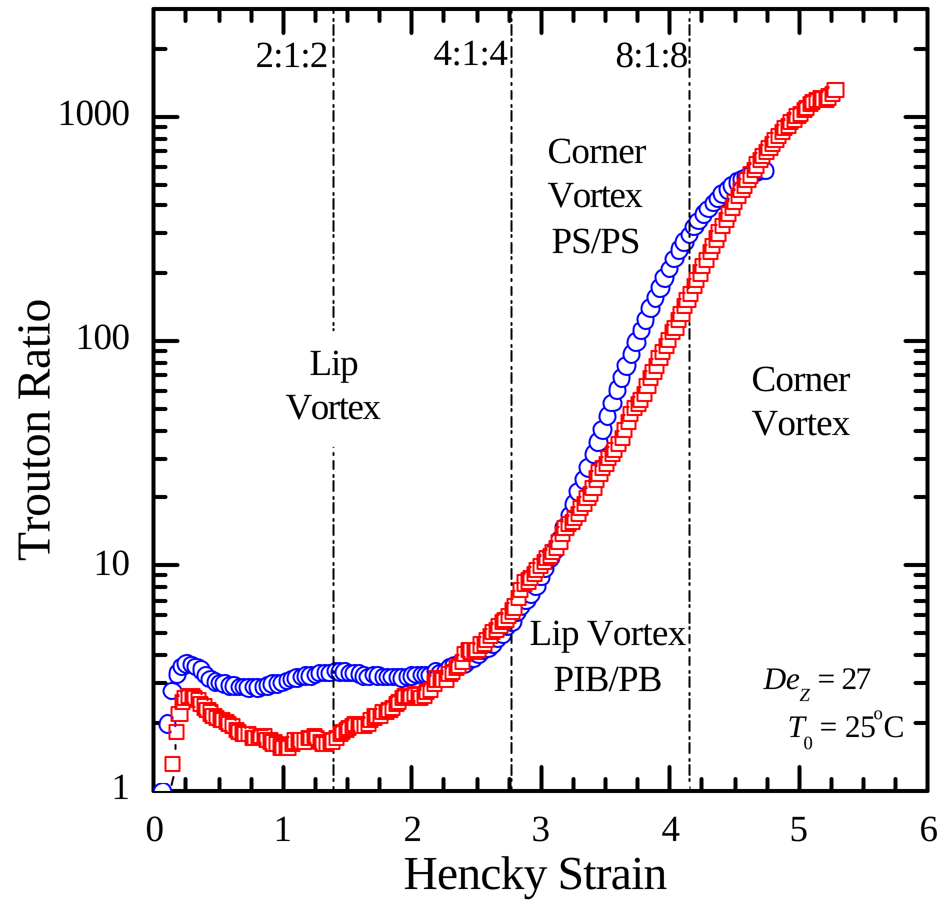
Accumulated strain along centerline

$$\varepsilon = \int_0^{t_1} \dot{\varepsilon}\, dt = \int_{v_z(z=-\infty)}^{v_z(z=0.5L_c)} \frac{dv_z}{v_z} = 2 \ln \beta$$

- Lip vortex present for

  PS/PS 6 , < 2.77 Y $Tr = (\tau_{zz} - \tau_{rr})/\eta_0 \dot{\varepsilon} < 6$

  PIB/PB 6 , < 4.16 Y $Tr < 200$



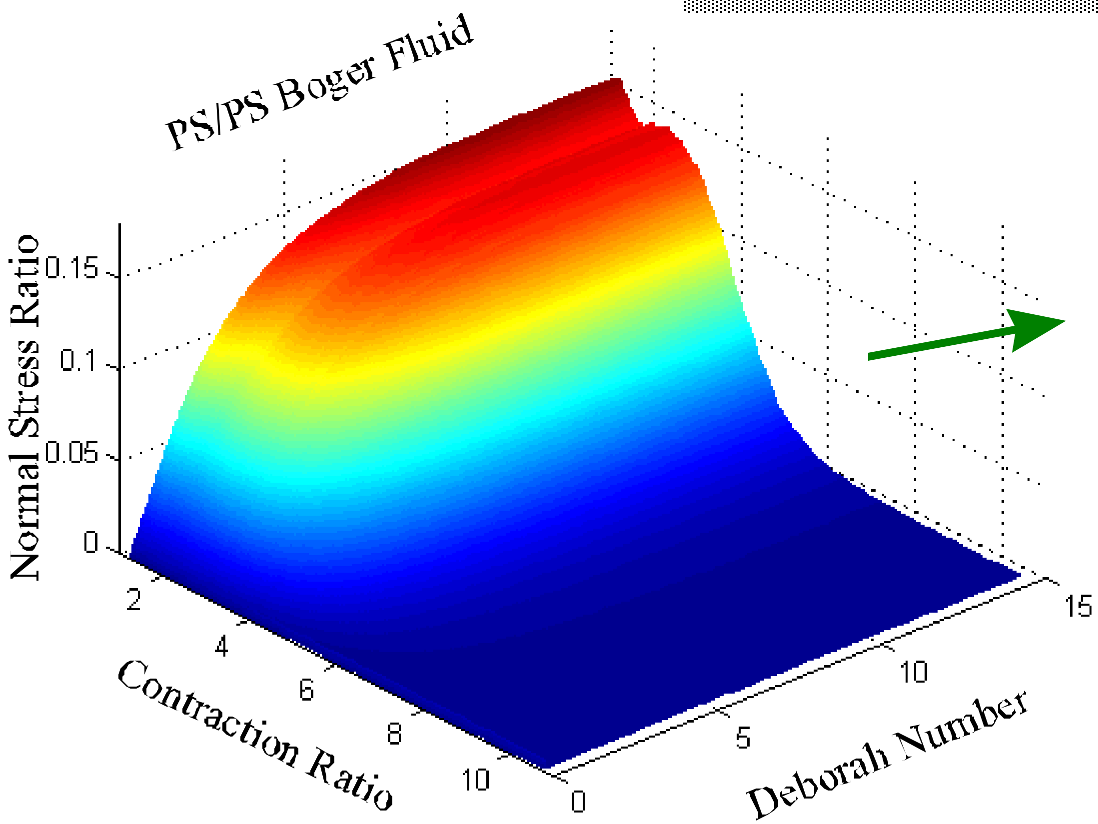
- Independent knowledge of extensional and shear rheology alone does not explain choice of lip or corner vortex!

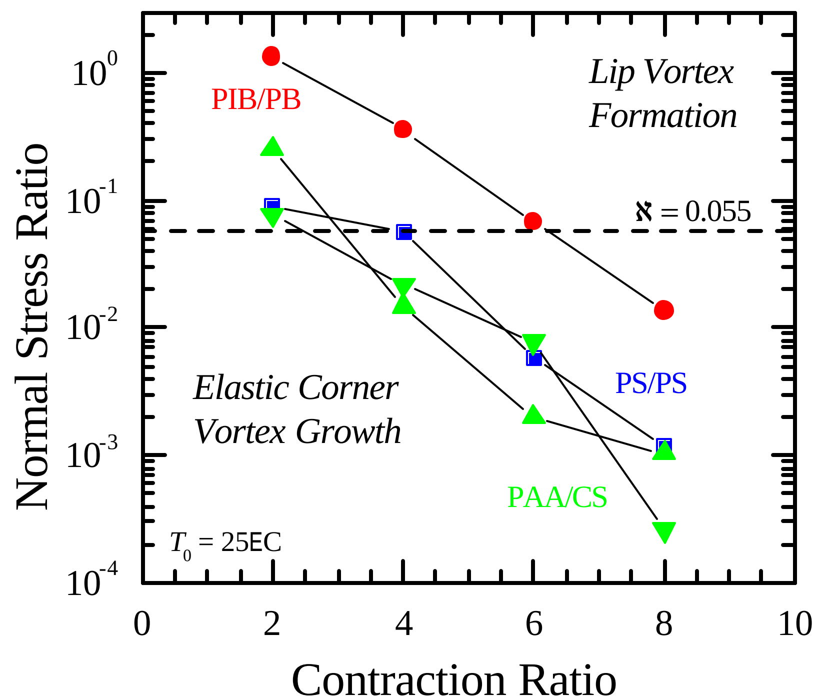
- Need to understand how upstream shear flow affects extensional flow entering contraction.

Normal Stress Ratio

- Systematic differences between fluids can be understood by considering relative importance of elastic normal stresses generated in shear to elastic normal stresses generated in transient uniaxial extension.

$$\aleph = \frac{N_1/\eta_0\dot{\gamma}}{(\tau_{zz}-\tau_{rr})/\eta_0\dot{\varepsilon}} = \frac{S_R(\dot{\gamma})}{T_R(\varepsilon)}$$





- For each fluid, a lip vortex does not develope for normal stress ratios (ℵ < 0.055).

Need for Birefringence Measurements

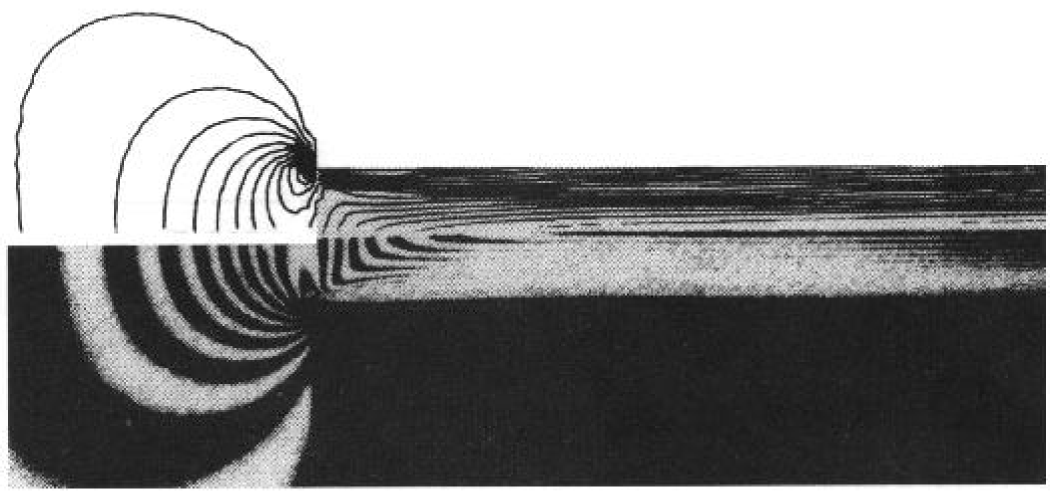
- Normal stress ratio formed with rheological data from homogeneous transient uniaxial extension and simple steady shear flow.

...but entry flow is a complex flow with nonhomogeneous extensional kinematics

- Would be nice to be able to form normal stress ratio from *in situ* stress measurements.

  Flow Induced Birefringence (FIB) Y) nN, P] $(J_{zz} - J_{rr}), J_{rz}$

- FIB measurements are also an excellent comparative tool for evaluating the ability of constitutive models to capture small scale physics.

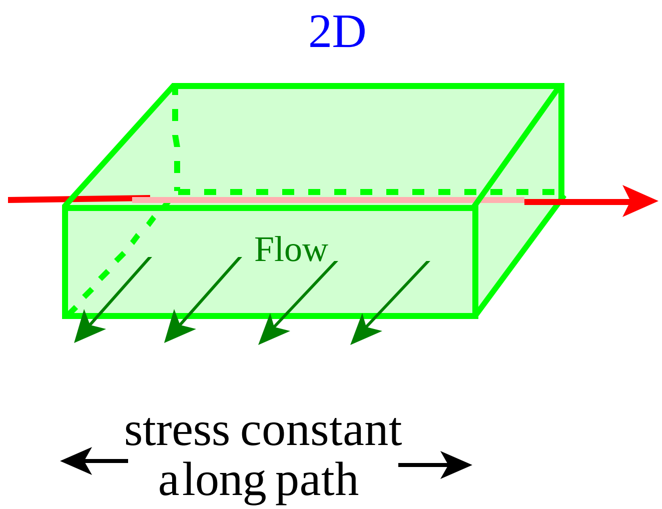


Bernaudo et al. (1998) - Flow of a LLDPE melt into a 8:1 planar contraction

Axisymmetric Flow Induced Birefringence

Li and Burghardt (1995)

- FIB typically used in two-dimensional flows because it is a line-of-sight technique.



birefringence — stress-optical coefficient

$$\Delta n' \sin 2\chi = 2C\, n_m k_B T ?_{12}$$

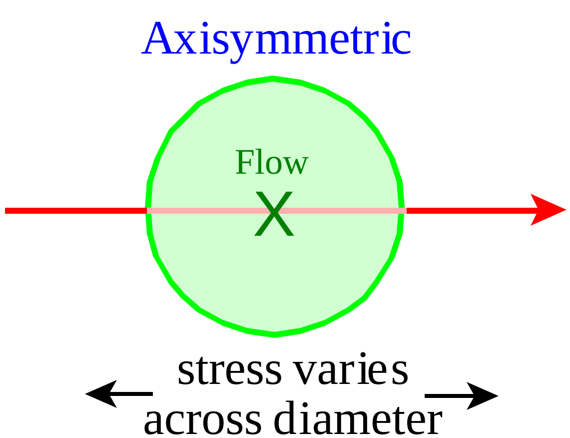
$$\Delta n' \cos 2\chi = C\, n_m k_B T \left(?_{11} - ?_{22} \right)$$

extinction angle

$$\delta = \frac{2\pi \Delta n' d}{\lambda} \qquad \mathbf{A} = \langle \mathbf{QQ} \rangle$$

retardation — chain end-to-end vector

- Axisymmetric flow result in an integrated measure of FIB and polymer chain conformation.

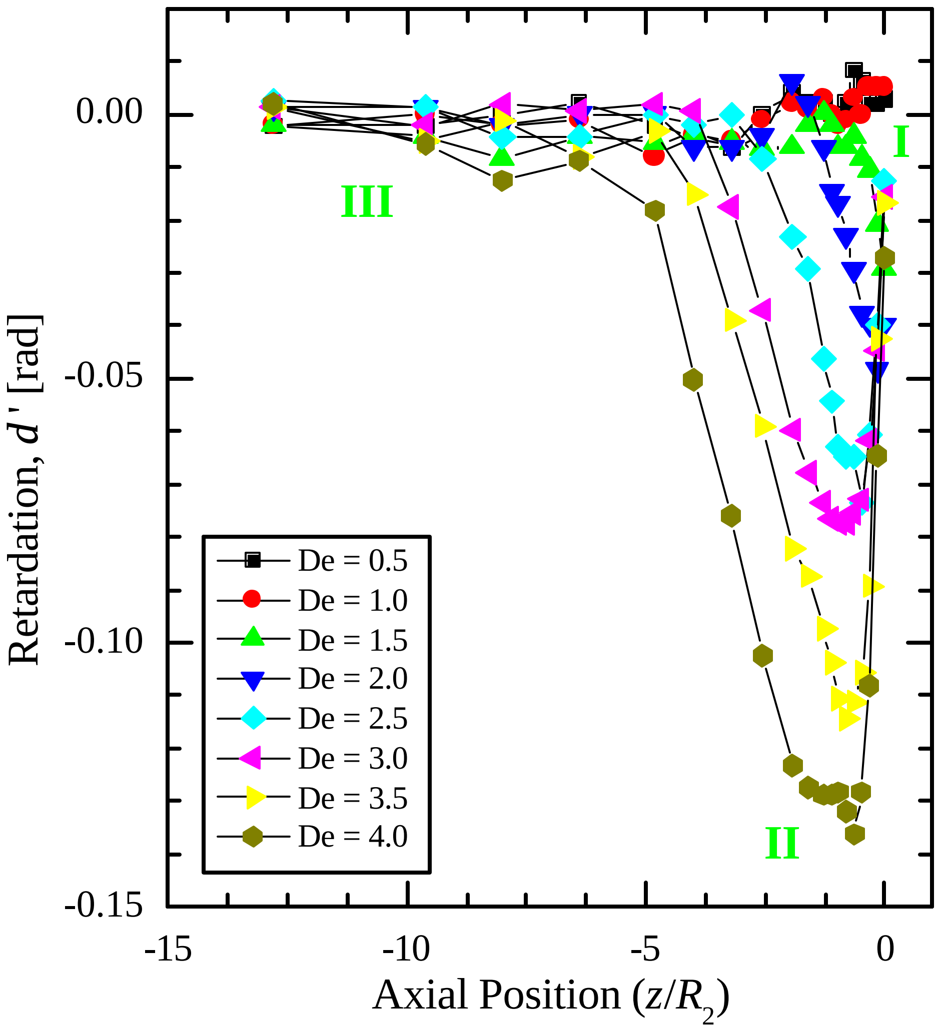


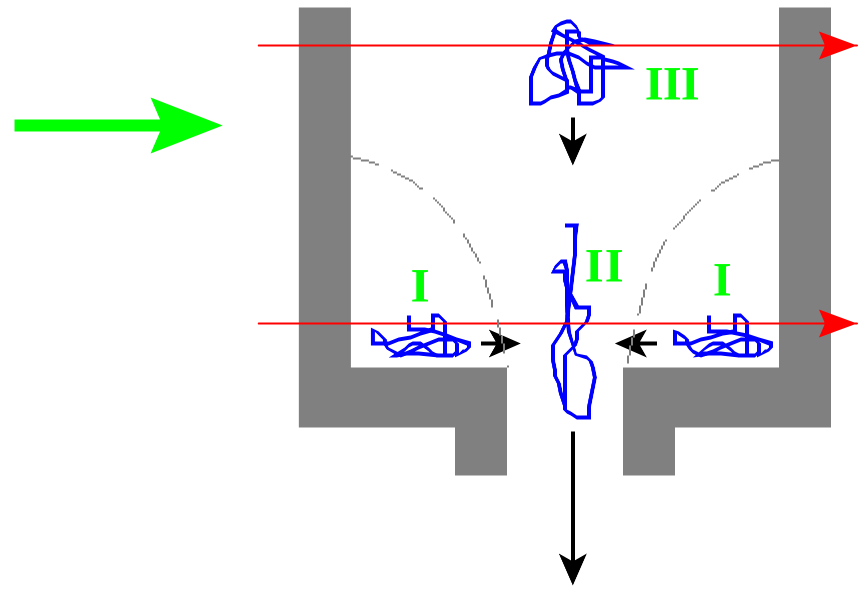
- Integrated measures dependent on optical train used.

$$I_1 = \sin\delta' \cos 2\chi' = \int \sin\delta \cos 2\chi \, dl$$

$$I_2 = [1 - \cos\delta'] \sin 2\chi' \cos 2\chi' = \int [1 - \cos\delta] \sin 2\chi \cos 2\chi \, dl$$

Axisymmetric FIB Upstream of 4:1:4 Contraction-Expansion



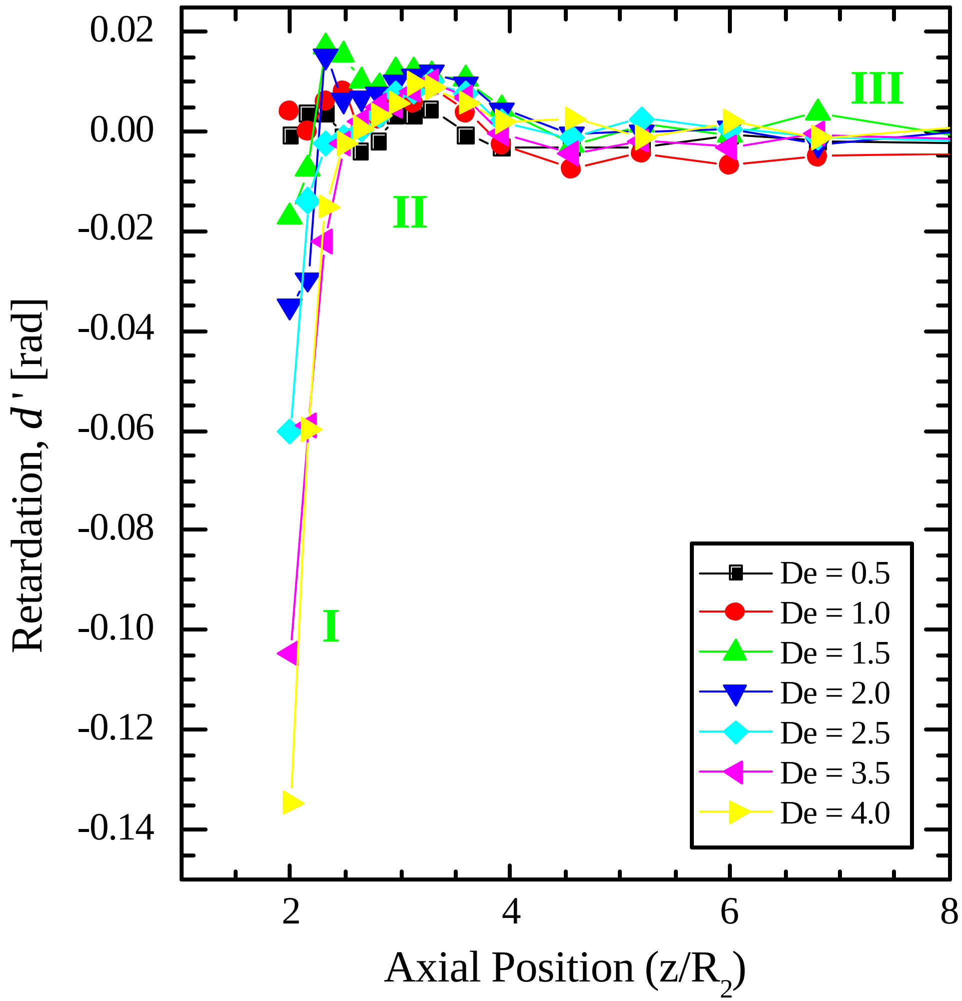


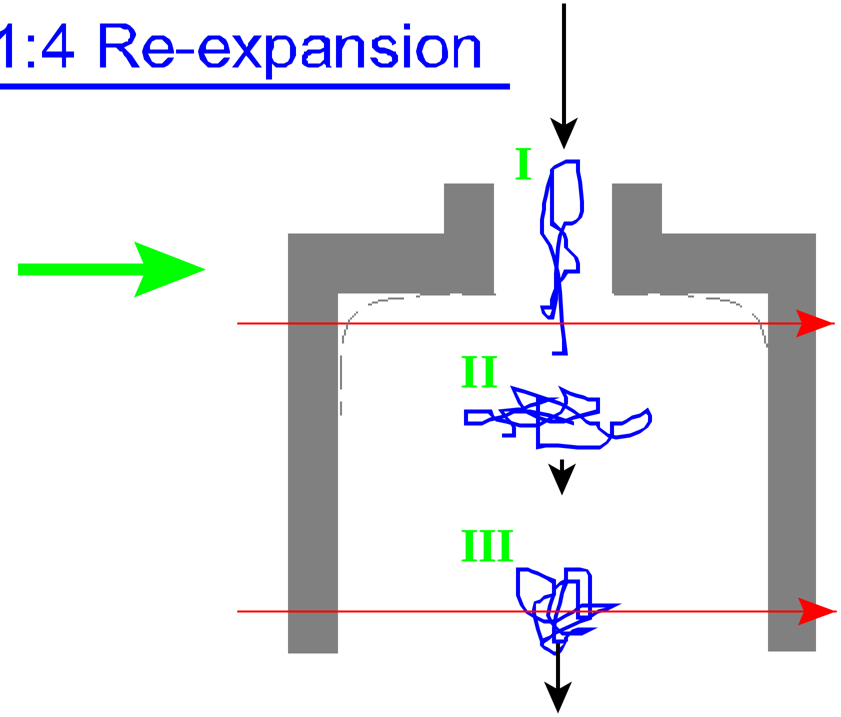
- Axial compression ($dN > 0$) near contraction ($z = 0$) due to shear flow along contraction plane.

- At large *De*, strong axial elongation ($dN < 0$) results from extensional flow along centerline.

- Growth in size and strength of axial elongation region coincides with corner vortex growth.

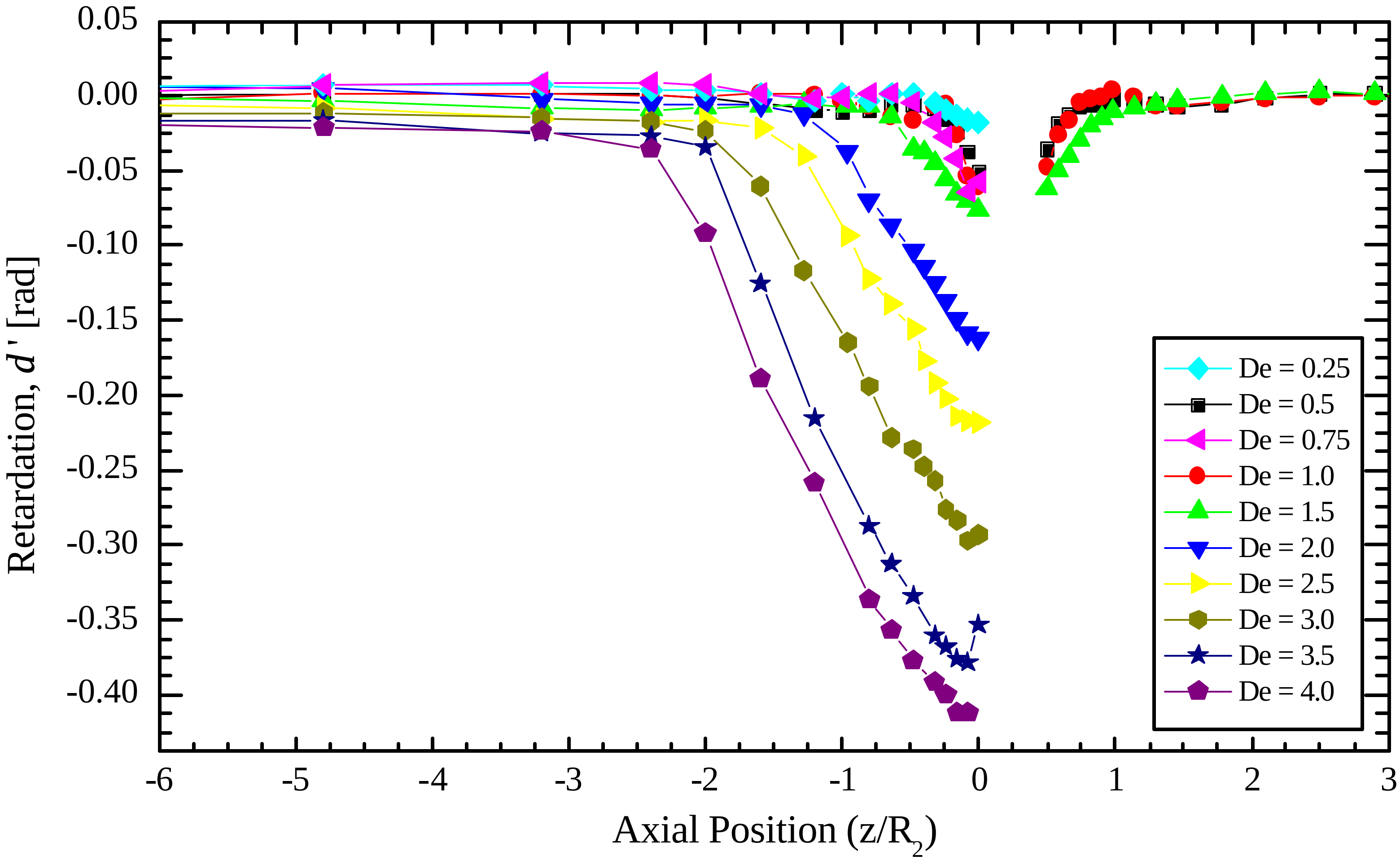
Axisymmetric FIB Downstream of 4:1:4 Re-expansion





- Region of axial compression (z=-3) resulting from deceleration of flow out of contraction.
- Magnitude and amplitude of compression doesn't change with Deborah number.
- At large Deborah numbers, strong axial elongation near re-expansion plane resulting from extensional flow upstream and strong shear in the throat.

Axisymmetric FIB in a 2:1:2 Contraction-Expansion



- Again, strong extension is observed upstream corresponding to vortex growth.
- Very little compression is observed from the shearing flow at the contraction plane.
  - No other distinct qualitative differences are observed between different contraction ratios.

Normal Stress Ratio from FIB Measurements

C Can now form a normal stress ratio from axisymmetric FIB measurements.

$$\aleph = \frac{N_1 / \eta_0 \dot{\gamma}}{(\tau_{zz} - \tau_{rr}) / \eta_0 \dot{\varepsilon}} = \frac{S_R(\dot{\gamma})}{T_R(\dot{\varepsilon})}$$

C To calculate normal stress ratio from FIB measurements we:

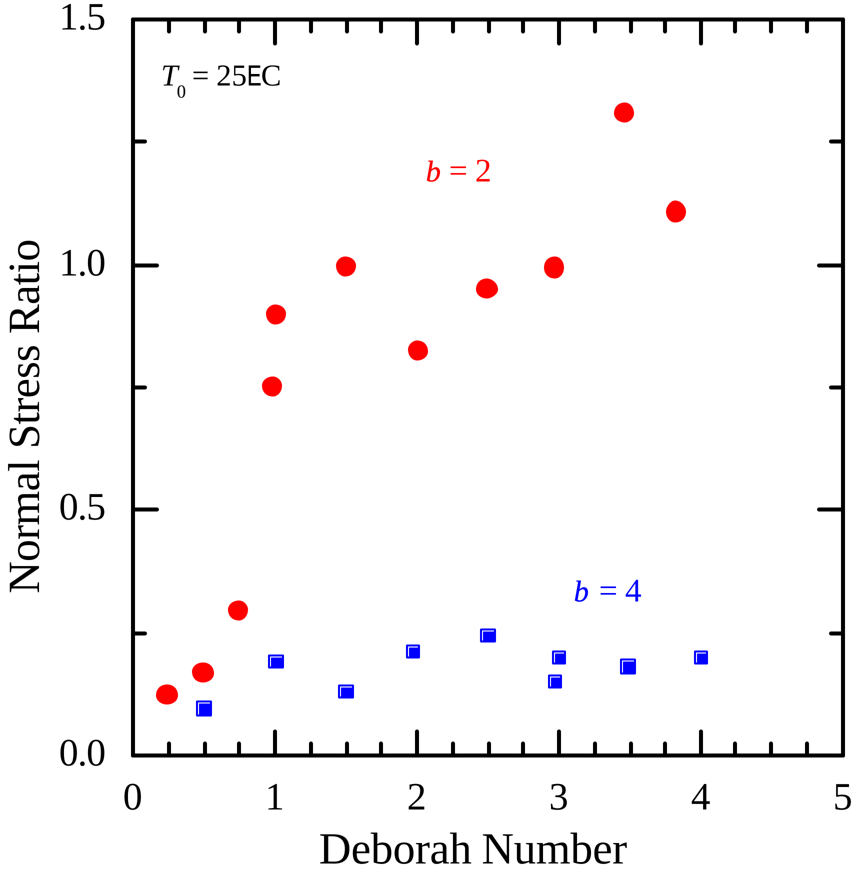
6 Use simple shear rheology to determine N_1 at the contraction wall.

6 Assume all extensional stress is held by a single uniformly stretched filament of width $2R_2$.

$$(\tau_{zz} - \tau_{rr}) = \delta\lambda / 4\pi C R_2$$

C Normal stress ratio strongly dependent on contraction ratio.

C Final test would be to compare with FIB measurements of PIB/PB Boger fluids.



Conclusions

- We have coupled global flow field measurements of the effect of viscoelasticity with local conformation measurements for a model polymer solution in a prototypical complex flow.

- These measurements generate a comprehensive data set for the validation of constitutive equations and numerical methods.
  - Pressure drop measurements
  - Axisymmetric flow induced birefringence measurements
  - Velocity measurements (PIV and LDV)
  - Vortex growth measurements
  - Characterization of an elastic flow instability

- We have rationalized the dependence of elastic lip and corner vortices on contraction ratio and test fluid rheology with a new dimensionless group, the normal stress ratio.

$$\aleph = \frac{N_1/\eta_0\dot{\gamma}}{(\tau_{zz}-\tau_{rr})/\eta_0\dot{\varepsilon}} = \frac{S_R(\dot{\gamma})}{T_R(\varepsilon)}$$

Flow Induced Birefringence (FIB)

- C Can use FIB to determine microscopic anistropy in polymer chain conformation

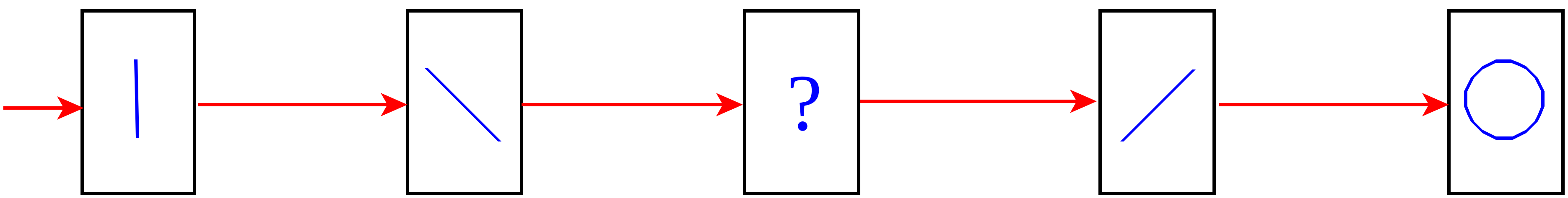
birefringence → stress-optical coefficient → chain end-to-end vector

$$\Delta n' \sin 2\chi = 2C\, n_m \mathrm{k_B} T ?_{12} \qquad \mathbf{A} = \langle \mathbf{QQ} \rangle$$

$$\Delta n' \cos 2\chi = C\, n_m \mathrm{k_B} T \left(?_{11} - ?_{22} \right) \qquad \delta = 2\pi \Delta n' d / \lambda$$

- C Need a technique to measure retardance (*) and extinction angle (P) simultaneously

*** Polarization Modulated Flow Birefringence ***



Polarizer @ 0E

Photoelastic Modulator @ 45E
* = A$_c$sin Tt

Flow Cell
P = ?
* = ?

Analyzer @ -45E

Photodetector measures $I(t)$

$$I(t) = I_{dc}[1 + 2J_1(A_c) M_{34} \sin \omega t + 2J_2(A_c) M_{32} \cos 2\omega t]$$

$$M_{34} = \sin\delta \cos 2\chi$$

$$M_{32} = [1 - \cos\delta] \sin 2\chi \cos 2\chi$$

→

$$R_\omega = \frac{I_\omega}{2J_1(A_c) I_{dc}} = M_{34},$$

$$R_{2\omega} = \frac{I_{2\omega}}{2J_2(A_c) I_{dc}} = M_{32},$$