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# A Re-evaluation of the Stokes Drift in the Polar Summer Mesosphere

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The Stokes drift argument put forth by Coy et al. (1986) to explain the monthly mean downward vertical velocities observed in the summer polar mesosphere by radars is critically re-examined. The size of the effect is quite sensitive to the choice of gravity wave wavenumber and phase speed. We reproduce the Coy et al. (1986) result for a monochromatic gravity wave and then generalize to a Garrett-Munk type spectrum. This allows us to easily incorporate experimentally determined wave field parameters in order to predict a Stokes drift magnitude. The Stokes drift we calculate for mesospheric spectra reported in the literature is less than 4 cm/s. This is nearly a factor of 10 smaller than the mean June and July downward vertical velocities we have verified from 4-year averaging of the Poker Flat, Alaska, radar data base. In this extended analysis we also find that the upward winter mean vertical velocity is smaller than previously reported but still in apparent conflict with the mesospheric winter circulation theory. We suggest, as an alternative to the Stokes drift idea, that the observed summer velocity might be the terminal velocity of charged aerosols. Such an explanation would have the advantage of relating the mean velocity observations to the large VHF radar cross section exhibited by the polar summer mesosphere. This possible connection is discussed further in a companion paper.

## INTRODUCTION

The polar summer mesosphere is at least 70°K colder than radiative equilibrium calculations predict. In fact, temperatures as low as 120°K have been reported [*von Zahn and Meyer*, 1989]. Only a mechanically forced circulation with upward mass flow near the summer pole can explain this low temperature. Such a circulation is provided, for example, by the zonal forcing due to breaking gravity waves [*Lindzen*, 1981; *Holton*, 1982, 1983; *Matsuno*, 1982]. In the summer, westward propagating gravity waves are filtered out by the westward stratospheric and mesospheric jet. Eastward propagating waves pass through this region and deposit their momentum in the upper mesosphere. This wave-momentum deposition represents an eastward force acting on the mean flow and causes a reduction in the magnitude of the westward jet. The Coriolis force approximately balances this eastward force due to waves, resulting in a meridional motion away from the pole. Continuity then requires an upward motion near the pole. This upward motion should be of the order of 1 cm/s to explain the observed low temperatures. For example, the model of *Holton* [1983], which includes the zonal drag due to breaking gravity waves, produces an upward motion in the upper summer mesosphere of about 1 cm/s.

In order to make statistically significant measurements of small mean vertical velocities in the presence of large fluctuations, long-term observations are necessary. The radar observations from Poker Flat, Alaska [*Ecklund and Balsley*, 1981] for 1979–1985 provide such a data set. Using a portion of this data, *Balsley and Riddle* [1984] reported that during the period of May 1980 through July 1981 a downward average velocity of 20–30 cm/s was detected in the summer months, while in the winter an upward mean velocity of 10–20 cm/s existed. The winter monthly means had larger error bars because the radar echoes are far more sporadic. However, the summer observations are of high quality and are clearly in contradiction to theory in both sign and magnitude. On the other hand, the zonal and meridional monthly means determined by the same radar agreed with the seasonal circulation theory, showing an equatorward motion of the order of 10 m/s in the summer and a weaker poleward motion in the winter. We emphasize that if one believes the meridional velocity measurements, then the polar vertical velocity measurements are extremely puzzling. The meridional mass flow away from the pole must be supplied by rising mass near the pole.

The Poker Flat radar observations have been corroborated in large part by the Saskatoon, Canada, MF radar for a year of observations in 1985–1986 [*Meek and Manson*, 1989], even though these researchers observed a change in the sign of the mean winter vertical velocity near 90 km. In addition, the Cornell University Portable Radar Interferometer (CUPRI) also observed a downward vertical velocity of 14 cm/s in the upper mesosphere during a 1988 summer campaign from northern Norway [*Hall*, 1991], although the CUPRI data set represents only about 5 days of data and so is barely significant statistically.

*Coy et al.* [1986] put forth an argument explaining the summer vertical velocity observations as due to the induced Eulerian velocity of second-order compressional gravity wave effects. In the presence of large wave oscillations, a mean residual Lagrangian motion $\overline{w}^L$ can differ from the Eulerian (fixed point) motion $\overline{w}$. If we assume that the backscatter is beam filling and that the radiation scatterers faithfully track the atmospheric motion, then a radar measures Eulerian motions of the atmosphere. The difference between averages made in the two reference frames is defined as the Stokes drift

$$\overline{w}^S \equiv \overline{w}^L - \overline{w}. \quad (1)$$



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The adiabatic cooling necessary to explain the summer temperature is a statement about expansion of fluid parcels, and its magnitude is given by the vertical velocity following the parcels. This is a Lagrangian velocity, and the temperature measurements dictate it should be about 1 cm/s upward. A Stokes drift magnitude much larger than 1 cm/s will imply a Eulerian velocity approximately equal and opposite to the Stokes drift. Now, slightly compressional waves have components of parcel motions in the direction of phase propagation, resulting in a nonzero Stokes drift. For a monochromatic gravity wave, *Coy et al.* [1986] express this Stokes drift in terms of the horizontal phase velocity and, choosing values for the wave parameters, obtain $\sim$ 10 cm/s upward Stokes drift (downward Eulerian). This value has the right sign, although its magnitude is smaller than the observed 20–30 cm/s.

In this paper we extend the analysis of the region by studying 4 years of Poker Flat data. The puzzling downward mean summer vertical velocity is observed in all 4 years. In order to interpret the results we start with the assumption that the radar Doppler measurements faithfully reflect the atmospheric fluid velocity (the assumption made by *Coy et al.* [1986]). We then attempt to explain the vertical velocity observations by employing the *Coy et al.* [1986] Stokes drift analysis. The magnitude of the Stokes drift effect is determined by making use of gravity wave spectral observations from various locations, including Poker Flat. As shall be seen, the resulting Stokes drift is less than 4 cm/s, too small by almost a factor of 10 to explain the summer mean vertical velocity. As part of a possible alternative explanation we question the assumption that radar scatterers faithfully follow the neutral atmospheric motion in the summer polar mesosphere. In the discussion section we suggest that the observed mean velocity is produced by charged aerosols falling under the influence of gravity. A companion paper [*Cho et al.*, this issue] explores other implications of charged aerosols for the summer mesosphere.

## DATA OVERVIEW

The *Balsley and Riddle* [1984] paper, which first brought attention to the unexpected values for the monthly mean vertical velocities, reported on slightly over 1 year of data. It seemed worthwhile to look at a longer length of data, given the importance of the result in understanding middle atmospheric circulation. This was easily done because hourly averages of the three wind velocity components exist now as part of the National Center for Atmospheric Research (NCAR) data base. As the experimental setup has been discussed extensively elsewhere [*Balsley et al.*, 1980], we describe it only very briefly here. The radar operated at Poker Flat, Alaska (65°N, 147°W), from 1979 to 1985. Three beams were used for transmitting and receiving: one vertical and two orthogonal beams 15° off vertical. From the three measured Doppler shifts, the wind vector may be constructed. The height resolution is approximately 2 km, and data were recorded every few minutes, although in the NCAR data base only hourly averages are reported. Summer mesospheric data are relatively continuous due to the strength of polar mesospheric summer echoes, while the winter mesospheric echoes are far more sporadic. Most of the echoes in the upper winter mesosphere are likely due to meteor trails.

It is important to note that the vertical beam must be truly close to vertical. As *Balsley and Riddle* [1984] pointed out, tropopause measurements indicated large seasonal mean zonal and meridional winds, while mean vertical winds were zero within the limits of measurement. This precludes the possibility of horizontal winds having a projected component in the radar beam direction. It is possible for a truly vertical beam to still experience horizontal contamination of the vertical velocity if the scattering layers being advected through the radar volume are both aspect sensitive and tilted off horizontal. In fact, polar mesospheric summer echoing layers are often aspect sensitive [*Czechowsky et al.*, 1988], and *Palmer et al.* [1991] have observed a significant influence of this effect on vertical velocity determinations. However, if this aspect sensitivity effect is to be influential over long-term averages, the scattering layers would need to have a preferred tilt direction. *Hall* [1991], using the Poker Flat data base, has seen no evidence for such a preferred tilt direction.

In Figure 1 we show a composite year of monthly average zonal velocities averaged into four height bins. The data are taken from the two years 1981 and 1982. The error bars represent the rms fluctuations divided by the square root of the number of samples, the statistical uncertainty in knowledge of the mean assuming a normal distribution. The strong summer westward jet is readily apparent, as is a weaker winter eastward jet. As expected, the magnitude of the jet is reduced at the top of the mesosphere. Figure 2 shows the same plot for the meridional wind. Below 80 km

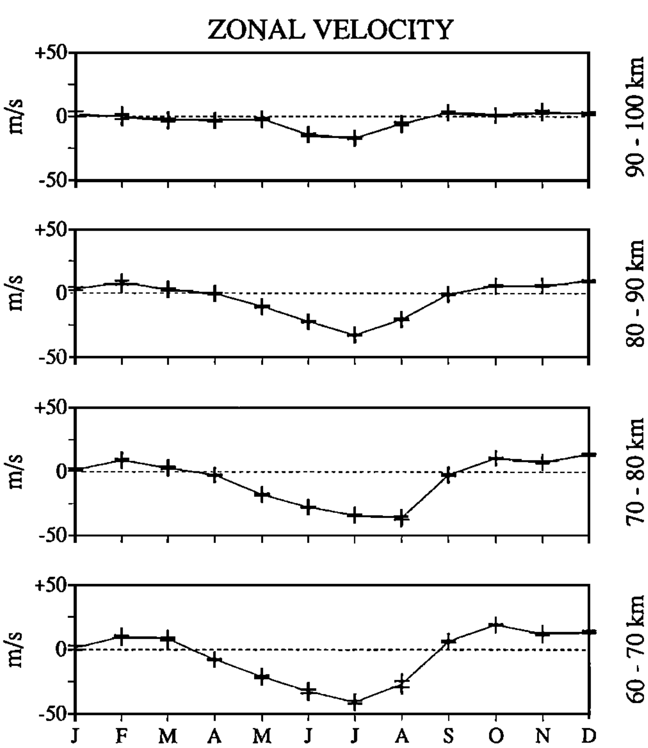


Fig. 1. Mean monthly values of the composite year formed from 1981 and 1982 data. Positive is to the east (eastward). Data have been averaged into four height bins, and the error bars are the rms fluctuation divided by the root of the number of samples. Notice the strong summer eastward jet and weaker westward jet in the lower two height bins. These jets are substantially closed by the top of the mesosphere.

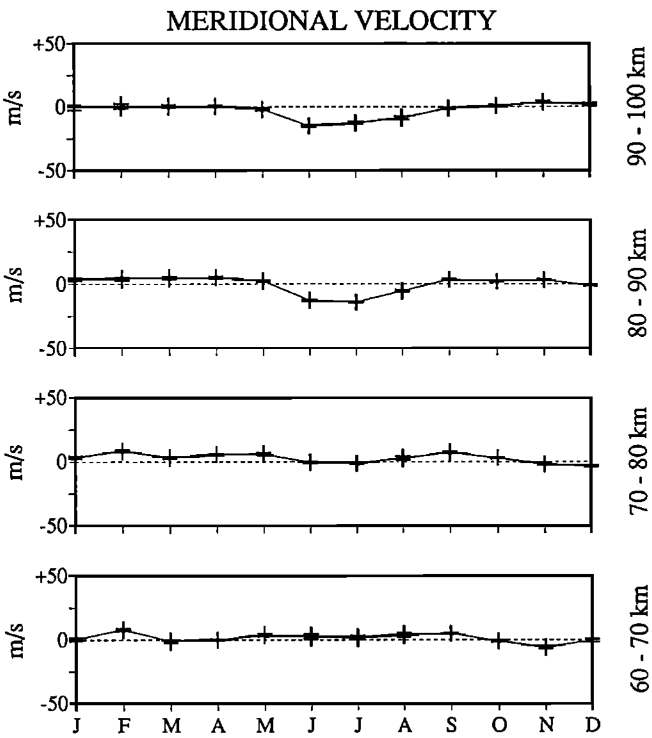


Fig. 2. The same as Figure 1 but showing the meridional wind. Positive is to the north (northward). The lower two height bins do not display well-structured seasonal dependence. In the upper mesosphere however, where the zonal jet is being reduced, there is motion toward the winter pole. This is particularly noticeable in the summer months.

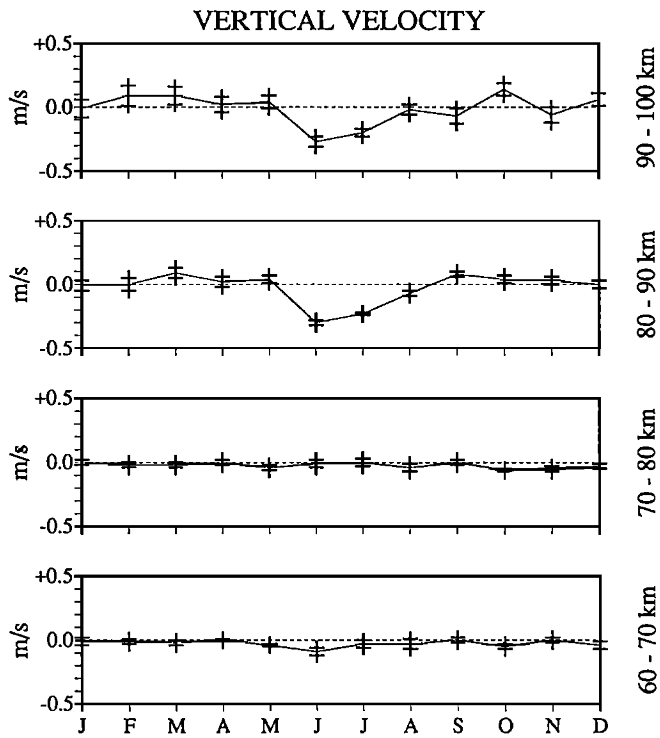


Fig. 3. A composite year of monthly average vertical velocities employing 4 years and 3 months of data. Note that the velocity scale is 100 times smaller than Figures 1 and 2. The summer months in the upper mesosphere show a clear downward motion 20–30 cm/s in magnitude. The winter months are near zero.

the seasonal variation is not well structured. However, above 80 km, where the zonal jet is being significantly reduced in magnitude by gravity wave momentum deposition, the summer meridional wind is 10–15 m/s equatorward, while the winter values are a few to 10 m/s poleward. Again, this is consistent with an upper mesospheric summer-to-winter circulation cell forced by the wave activity, and it is also consistent with the 1-year analysis of *Balsley and Riddle* [1984].

In Figure 3 we form a composite year of monthly mean vertical velocities incorporating 4 years and 3 months of data from September 1979 to December 1983. Note that the velocity axis has been rescaled by a factor of 100 from that of Figures 1 and 2. The multiyear averaging has not diminished the strong downward summer motion first reported for a single year by *Balsley and Riddle* [1984]. We emphasize again that this vertical velocity observation is in contradiction to the mesospheric circulation theory in both sign and magnitude. The simultaneous observations of downward and equatorward flow near the pole are not consistent with mass continuity.

The upward winter vertical velocity values are smaller than those originally reported by *Balsley and Riddle* [1984]. The average nonsummer (months excluding May, June, July, and August) vertical velocity for the 4.25-year period is 2 $\pm$ 1 cm/s upward. Because of the larger magnitude of the downward summer mean vertical velocity, in this paper we concentrate on this season. In the next section we look closely at the Stokes drift mechanism to see if it can explain the observed downward monthly mean values in the summer. However, although smaller in magnitude the nonsummer vertical velocity measurements are still in conflict with the mesospheric circulation theory, and they certainly cannot be explained by the Stokes drift mechanism. A gravity wave Stokes drift effect cannot induce an upward Eulerian mean flow unless the gravity waves have upward phase propagation, which is not observed. Moreover, even if we considered the nonsummer mean vertical velocity to be approximately zero, the Stokes drift mechanism still apparently fails in this season. It would need to turn off or greatly diminish in magnitude, but *Meek and Manson* [1989] see no evidence for greatly reduced gravity wave activity in the winter mesosphere compared to summer. However, we note that *Hall* [1991] has suggested that the sparse radar scattering in the winter mesosphere may cause radars not to be sensitive to the Eulerian mean flow induced by a Stokes drift.

## THEORETICAL DEVELOPMENT

Our goal is to derive an expression for the gravity wave-induced Stokes drift in terms of gravity wave spectral parameters. Experimentally determined values of these parameters can then be employed to see whether or not the resulting Stokes drift magnitude is large enough to explain the puzzling vertical velocity observations. Before writing down equations, however, it is useful to briefly outline qualitatively the physics involved, as it is somewhat counterintuitive. *Hall* [1991] contains a more thorough discussion.

Any finite amplitude wave that has a component of fluid parcel motion parallel to the phase propagation will produce a Stokes drift. The fluid parcels will spend slightly more time moving with the phase propagation then against and will therefore have a net drift in this direction. Now, external boundary conditions may dictate that the total mass flow is zero. There must then be an induced mean flow that cancels the Stokes drift.

In the polar summer mesosphere the total vertical mass flow rate is determined by continuity from the horizontal flow away from the pole. Normalized by the density, this mass flow rate is the small value of 1 cm/s upward. On the other hand, there is significant gravity wave activity present in the region, and these waves may have vertical Stokes drifts substantially larger in magnitude than 1 cm/s. A mean vertical velocity must therefore be set up to approximately cancel the gravity wave Stokes drift, so that the difference is the actual mass flow rate. It is counterintuitive but nevertheless true that although the mean atmospheric fluid velocity at a point in space may be downward, the mean mass flow rate through the point can be zero or even upward. This ability of the fluid, as will be demonstrated in equations, is due to a correlation between the vertical velocity and density fluctuations.

Gravity waves have predominantly downward phase propagation, and one might at first think that this would be the direction of the Stokes drift. In fact, however, downward phase propagating gravity waves have upward Stokes drifts. Because of compressibility, the parcel trajectories are tilted off the phase fronts in such a way that parcels actually are moving upward, in the direction of the group velocity, when they have velocity components in the direction of downward phase propagation. The geometry is illustrated in Figure 2 of *Coy et al.* [1986]. This upward Stokes drift, if much larger in magnitude than 1 cm/s, implies there must be induced an approximately equal and opposite downward Eulerian mean flow. Our aim is to determine whether this induced mean flow has the correct magnitude to explain the radar observations.

The existence of horizontal gradients in the wave amplitude can also cause gravity waves to have vertical Stokes drifts. This type of drift does not require parcel motions along the phase propagation direction and therefore is not related to compressibility. It is unlikely, however, that averaged over a season, horizontal amplitude gradients will be large enough to produce a Stokes drift capable of explaining the observed vertical velocities.

We now turn to equations. In the presence of wave fluctuations the total mass flux is given by

$$\overline{\rho w} = \overline{\rho}\,\overline{w} + \overline{\rho' w'}, \qquad (2)$$

where $\rho$ is the atmospheric density, $w$ the vertical velocity, primes indicate wave fluctuating quantities, and overbars indicate time averaging. The density normalized mass flux rate required to keep the summer mesopause the observed 60°–80°K below radiative equilibrium is approximately 1 cm/s. On the other hand, the observed magnitude of vertical velocity is much larger, so that if the Stokes drift is responsible for the observed vertical velocities, we may say that

$$\overline{w} \approx -\frac{\overline{\rho' w'}}{\overline{\rho}}. \qquad (3)$$

The term $\overline{\rho' w'}/\overline{\rho}$ is the vertical Stokes drift $\overline{w}^S$ of wave motion in the absence of horizontal wave envelope gradients. For a derivation of this expression as the difference between Lagrangian and Eulerian velocity averages, see *Coy et al.* [1986].

The most straightforward way to obtain $\overline{\rho' w'}$ for an acoustic gravity wave is to write down two-dimensional linearized equations of motion on an $f$ plane, assume plane wave solutions, derive a $\rho'$ $w'$ polarization relation, and perform the averaging of the product. The momentum, energy, and continuity equations for an atmosphere of uniform Brunt-Väisälä frequency are

$$u'_t - fv' = -\phi'_x \qquad (4)$$

$$v'_t + fu' = 0 \qquad (5)$$

$$w'_t + gr' = -(\phi'_z - \phi'/H) \qquad (6)$$

$$r'_t - (N^2/g)w' = \phi'_t/c^2 \qquad (7)$$

$$r'_t + u'_x + w'_z - w'/H = 0 \qquad (8)$$

Here $u'$ and $v'$ are zonal and meridional velocity fluctuations, $\phi' \equiv p'/\overline{\rho}$ for the pressure $p'$, $r' \equiv \rho'/\overline{\rho}$, $g$ is the gravitational acceleration, $N$ the Brunt-Väisälä frequency, $H$ the density scale height, $f$ the local inertial frequency, $c$ the sound speed, and subscripts refer to differentiation.

Notice that this set is non-Boussinesq. In a Boussinesq atmosphere we would neglect the pressure term in equation (7) and the time derivative in equation (8). However, equation (7) would then imply that $w'$ and $r'$ (or $\rho'$) are out of phase, giving $\overline{\rho' w'} = 0$. It is the compressional effects that produce a nonzero Stokes drift in the absence of wave envelope gradients.

We now substitute plane wave solutions and perform the necessary algebra to obtain

$$\overline{w}_S = \frac{1}{2} C \cos\Phi\, |w'|^2, \qquad (9)$$

where

$$C \equiv \frac{\omega\gamma m H}{g[(1-\frac{\gamma}{2})^2 + \gamma^2 m^2 H^2]} \times$$

$$\sqrt{(1-\frac{N^2}{\omega^2})^2 + \frac{[(1-\frac{\gamma}{2})(1-\frac{\gamma}{2}\frac{N^2}{\omega^2}) + \gamma^2 m^2 H^2 \frac{N^2}{\omega^2}]^2}{\gamma^2 m^2 H^2}} \qquad (10)$$

and

$$\Phi = \arctan[\frac{(1-\frac{\gamma}{2})(1-\frac{\gamma}{2}\frac{N^2}{\omega^2}) + \gamma^2 m^2 H^2 \frac{N^2}{\omega^2}}{\gamma m H(1-\frac{N^2}{\omega^2})}]. \qquad (11)$$

The wave intrinsic frequency is $\omega$, $m$ is the vertical wavenumber, and $\gamma$ is the ratio of specific heat at constant pressure to constant volume. Before we evaluate expression (9) by substituting (10) and (11), it is worthwhile to examine some limiting cases. First consider the magnitude factor, equation (10). In the limit, $N^2/\omega^2 \gg 1$, there are two regimes in $m$ dependence. For the Boussinesq limit, $m^2H^2 \gg 1$,

$$C \sim \frac{N^2}{g}\frac{1}{\omega}. \qquad (12)$$

while in the opposite limit, $m^2H^2 \ll 1$,

$$C \sim \frac{\gamma N^2}{2g(1-\frac{\gamma}{2})}\frac{1}{\omega} \qquad (13)$$

The difference in $C$ evaluated at the two wavenumber extremes is only the numerical factor $\gamma/2(1-\frac{\gamma}{2}) \approx 2.3$. More importantly, we see that in either wavenumber regime, in this low frequency limit the density-velocity correlation factor $C$ increases with decreasing frequency. This makes physical sense. For a given magnitude of vertical velocity fluctuation a longer period will result in more vertical parcel displacement relative to the density scale height. This produces greater compressional effects, and it is these effects that are responsible for the density-velocity correlation.

Now consider the phase. In the low-frequency limit, equation (11) becomes

$$\Phi \sim \arctan[(1 - \frac{\gamma}{2})\frac{1}{2mH} - \gamma mH]. \quad (14)$$

Again, this has interesting long and short vertical wavelength limits. For $mH \gg 1$,

$$\Phi \sim \arctan(-\gamma mH) \sim -\frac{\pi}{2}, \quad (15)$$

showing that in a Boussinesq atmosphere, density and vertical velocity are out of phase. This is the familiar scenario of pure internal gravity waves where density fluctuations are created solely by adiabatic expansion or compression of vertical parcel displacement. The parcel motion is in a plane perpendicular to the phase velocity, so that there is no net mass flux. On the other hand, for $mH \ll 1$,

$$\Phi \sim \arctan[(1 - \frac{\gamma}{2})\frac{1}{2mH}] \sim \frac{\pi}{2}. \quad (16)$$

Here, the background density has changed by so many scale heights in one vertical wavelength that the motion is close to that of a surface wave at a density discontinuity. The propagation of such waves is trapped in the horizontal at the discontinuity, and the lack of vertical phase propagation means there can be no vertical Stokes drift. This limit is unlikely to be physically realized in the Earth's atmosphere, because for most frequencies, it would require horizontal wavelengths larger than the $f$ plane approximation allows. Nevertheless, it is apparent that there is an intermediate regime where density and vertical velocity fluctuations are far from out of phase. In fact, the phase goes through zero at

$$m = \sqrt{\frac{1}{2\gamma}(1 - \frac{\gamma}{2})}\,\frac{1}{H}. \quad (17)$$

This is the wavenumber for which the wave is most strongly coupled to the basic state. For a density scale height of 6 km the corresponding vertical wavelength is about 114 km.

If, as argued by *Coy et al.* [1986], the wave amplitude reaches saturation and is therefore limited to the intrinsic phase speed $\omega/m$, we can evaluate equation (9) for the Stokes drift. In Figure 4 we plot the Stokes drift at this limiting amplitude versus the horizontal intrinsic phase speed $\omega/k$ ($k$ the horizontal wavenumber). Here, $m$ has been expressed in terms of $k$ by employing the dispersion relation

$$m^2 = k^2\frac{N^2 - \omega^2}{\omega^2 - f^2} - \frac{1}{4H^2} + \frac{\omega^2}{c^2}. \quad (18)$$

Figure 4 is identical to the family of curves from *Coy et al.* [1986]. The value of the Stokes drift is quite sensitive to the choice of phase speed and wavenumber. *Coy et al.* [1986] chose monochromatic wave values of 40 m/s and $2\pi/50$ km, respectively, to obtain an approximate 10 cm/s Stokes drift. While this value is the same order as the reported downward velocities, it is significantly smaller than the observed 30 cm/s downward mean value in June. Furthermore, we argue next, using a different measure of the wave properties, that 10 cm/s is more likely an upper bound for the Stokes drift, rather than a typical value.

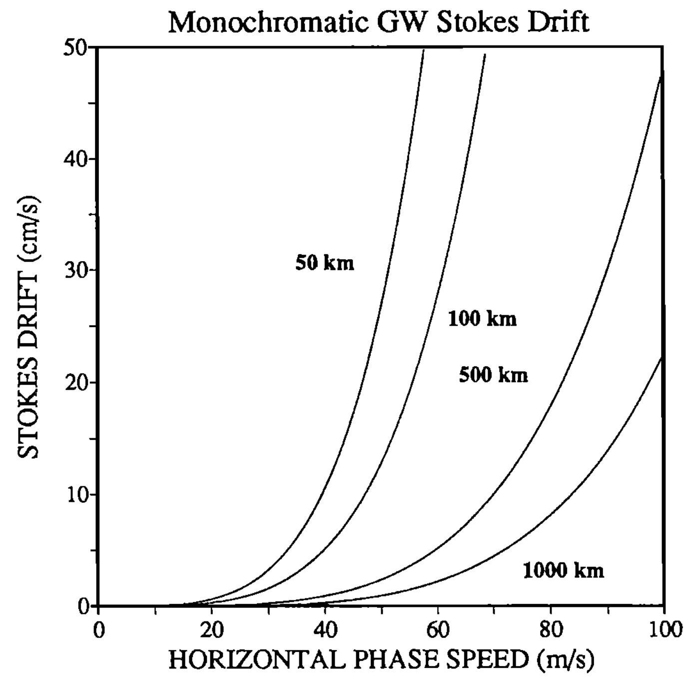


Fig. 4. The Stokes drift of a gravity wave versus its intrinsic horizontal phase speed for four different horizontal wavelengths. The wave amplitude has been taken at saturation as the phase speed. This is identical to the result of *Coy et al.* [1986].

Because the long-term average effect of atmospheric gravity waves cannot be modeled monochromatically, it seems reasonable to obtain the Stokes drift for a spectrum of waves. *Waltersheid and Hocking* [1991] have performed numerical calculations of Stokes drifts for specific gravity wave spectra and arrived at results similar to those we will produce. Spectrally integrating the Stokes drift has the advantage of allowing us to employ observational determinations of spectral parameters in our evaluation. In order to do this we replace the monochromatic wave magnitude with the two-dimensional vertical velocity fluctuation spectra $E(\omega, m)$. From equation (9) the expression for the Stokes drift now becomes

$$\overline{w}^S = \frac{1}{2}\int_0^\infty dm \int_f^N d\omega\, C(\omega, m)\cos\Phi(m)E(\omega, m). \quad (19)$$

The frequency integration is performed from the inertial to the Brunt-Väisälä frequency. The wavenumber integration is insensitive to the limits as long as the lower limit is significantly smaller than the spectral bandwidth introduced below, and the upper limit is much larger than $H^{-1}$.

In relating the gravity wave Stokes drift to the vertical velocity fluctuation spectrum, we have made the assumption that all vertical velocity fluctuations in the frequency range $f$ to $N$ are due to gravity waves. A number of studies have shown consistency between middle atmospheric observations of frequency and wavenumber energy spectra and model energy spectra of gravity waves [*Scheffler and Liu*, 1985; *Smith et al.*, 1985, 1987; *Muraoka et al.*, 1990]. On the other hand, *Gage et al.* [1986] and *Gage* [1979] have argued an imortant role for two-dimensional turbulence in horizontal velocity fluctuations. Such a role would influence our subsequent development. If two-dimensional turbulence is important, energy spectral parameters are not solely indicative of the atmospheric wave field, and we cannot employ these observationally determined parameters in a simple way to obtain the vertical velocity spectrum needed to evaluate (19). In such a scenario the assumption of gravity wave dominance of the energy spectrum will lead to an overestimate of the total vertical velocity variance and, therefore, an overestimate of the Stokes drift. However, because our conclusion will be that the gravity wave Stokes drift is too small in magnitude to explain the seasonal mean vertical velocity measurements in the polar summer mesosphere, if our calculated Stokes drift magnitude is an overestimate, our conclusion will only be strengthened.

The next step is to relate the vertical velocity spectrum to a wave energy spectrum. Note that for gravity waves the vertical velocity represents a small part of the total wave kinetic energy; except for very near the Brunt-Väisälä frequency the horizontal velocity dominates. In addition, there is potential energy associated with the motion against mean atmospheric stratification. *Scheffler and Liu* [1985] have derived a relationship between energy spectra and velocity component spectra for arbitrary velocity components in a non-Boussinesq atmosphere. Their expression, evaluated for the vertical velocity, is

$$E(\omega, m) = E_0 B(\omega) A(m) \frac{Q(m,\omega)}{F(\omega)}, \tag{20}$$

where

$$Q(m,\omega) = \frac{(\omega^2 - f^2)(c^2m^2 + \omega_d^2)}{(N^2 - f^2)(c^2m^2 + \omega_d^2) + (N^2 - \omega^2)^2} \tag{21}$$

and

$$F(\omega) = 1 + \sqrt{\frac{\omega^2 - f^2}{N^2 - \omega^2}}. \tag{22}$$

Here $\omega_d^2 = c^2/4H^2 - N^2$. The wave energy spectrum, assumed separable in frequency and wavenumber, is $E_0 B(\omega) A(m)$ where $B(\omega)$ and $A(m)$ have the Garrett-Munk [*Garrett and Munk*, 1975] form

$$B(\omega) = \frac{p-1}{f^{1-p} - N^{1-p}} \omega^{-p} \tag{23}$$

and

$$A(m) = \frac{t-1}{(m_* + m)^t} m_*^{t-1} \tag{24}$$

The wave field energy density $E_0$ and the wavenumber bandwidth $m_*$ are parameters to be fit to observed spectra. The frequency and wavenumber power laws $p$ and $t$ may also be considered as fitting parameters to observed spectra, although there is theoretical expectation for the values $p = 5/3$ and $t = 3$. This is discussed in a bit more detail below. Our procedure is to use values for $E_0$, $m_*$, $p$ and $t$ determined from various mesospheric observations to estimate the Stokes drift by numerical integration of equation (19).

Figures 5 and 6 display the spectrally integrated Stokes drift versus $m_*$ for various values of $E_0$. In Figure 5 the frequency power law index is $p = 2$, while in Figure 6 it is $p = 5/3$. We have not shown results of varying the wavenumber power law $t$ because the integration is insensitive to $t$ in the area of $t \approx 3$ suggested by experiment [*Muraoka et al.*, 1990]. The curves have peak values of $m_*$ at $\lambda_z \approx 21$ km. That there should exist a maximum as a function of $m_*$ can be understood by referring to Figures 7, 8, 9, and 10. Figure 7 is a surface plot of $C \cos \Phi$ versus

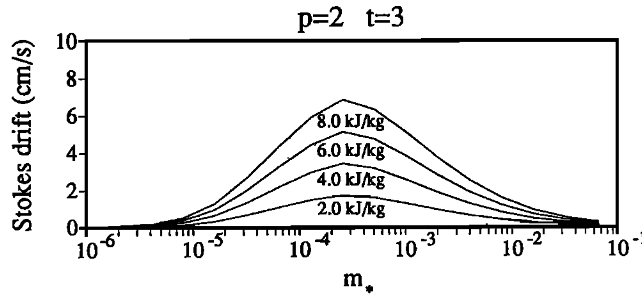


Fig. 5. The Stokes drift for a spectrum of downward phase propagating gravity waves versus the bandwidth of the wavenumber energy spectrum (see text for discussion). Curves for four values of wave field energy density are plotted. Here, the spectral power laws are $t = 3$ in wavenumber and $p = 2$ in frequency.

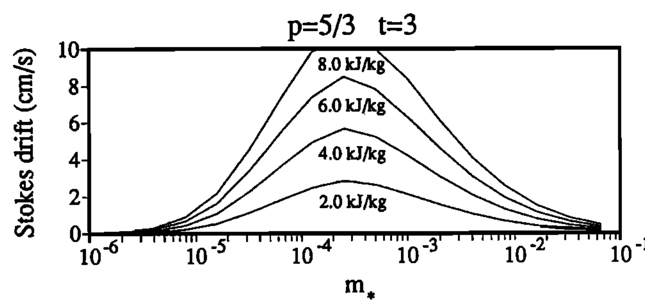


Fig. 6. Identical to Figure 5, except the frequency power law is $p = 5/3$. This change in power law roughly doubles the peak Stokes drift value.

log ($\omega$) and log ($\lambda_z$), while in Figures 8, 9, and 10, vertical velocity spectra for three values of $m_*$ are plotted on the same axes. For large $m_*$, most of the vertical velocity energy is tied up in wavelengths where the density and velocity fluctuations are out of phase. On the other hand, as $m_*$ decreases, increasingly more of the vertical velocity energy is tied up in high frequencies for all wavelengths. The density-velocity correlation magnitude goes inversely with the wave frequency. Therefore there should exist an intermediate $m_*$ that causes the product of the correlation magnitude and vertical velocity spectrum to maximize.

## DISCUSSION

Refer again to Figure 4. *Coy et al.* [1986] consider a saturated wave with 50 km horizontal wavelength and 40 m/s horizontal phase velocity to obtain an approximate upward 10 cm/s Stokes drift. However, to get the observed ~ 30 cm/s value in June, we would need a phase speed of about 50 m/s. What magnitude vertical velocity fluctuation does this imply? Taken together, $\omega/k \approx 50$ m/s and $k \approx 2\pi/50$ km imply the intrinsic frequency $\omega \approx 0.006$ s$^{-1}$, which is roughly a third of the Brunt-Väisälä frequency (considering a 6-min Brunt-Väisälä period). The magnitude of these quantities indicates we may use the approximate dispersion relation

$$m^2 \approx k^2 \left(\frac{N^2 - \omega^2}{\omega^2}\right). \tag{25}$$

This gives a vertical wavelength $\lambda_z \approx 6$ km. We will now use the saturation condition and the polarization relation from the heat equation to set limits on the fluctuating vertical ve-

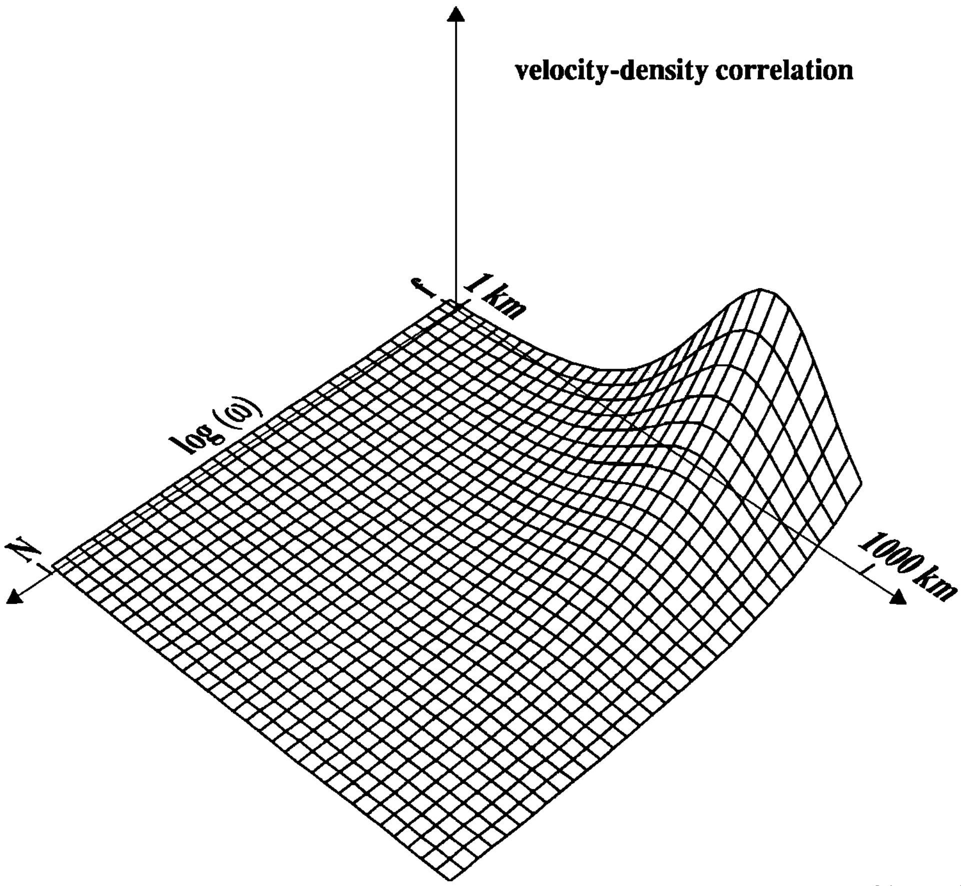


Fig. 7. A surface plot of the magnitude of the fluctuating density-vertical velocity correlation. It is this correlation, zero for incompressible motion, that is responsible for the Stokes drift under consideration here. The correlation is plotted versus log scales of frequency and vertical wavelength. The vertical axis is linear. See text for discussion of location of peak.

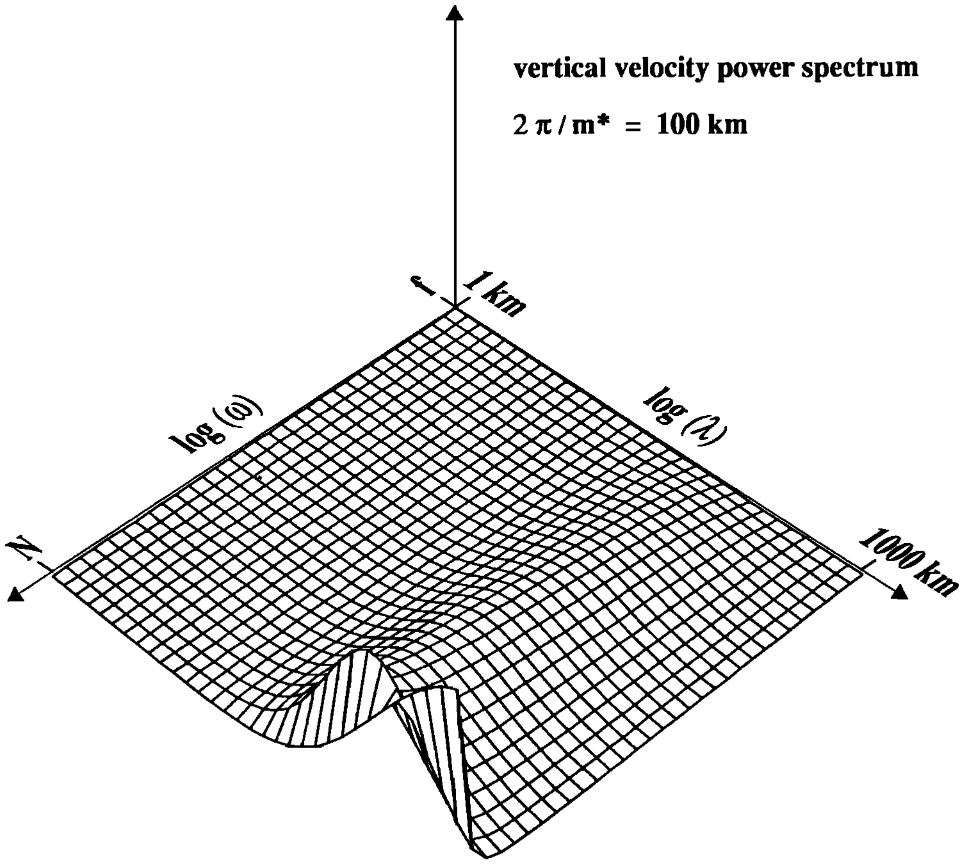


Fig. 8. Surface plot of the vertical velocity fluctuation spectrum versus log scales of frequency and vertical wavelength. Here, $m_* = 2\pi/100$ km. The vertical axis is linear and normalized to one.

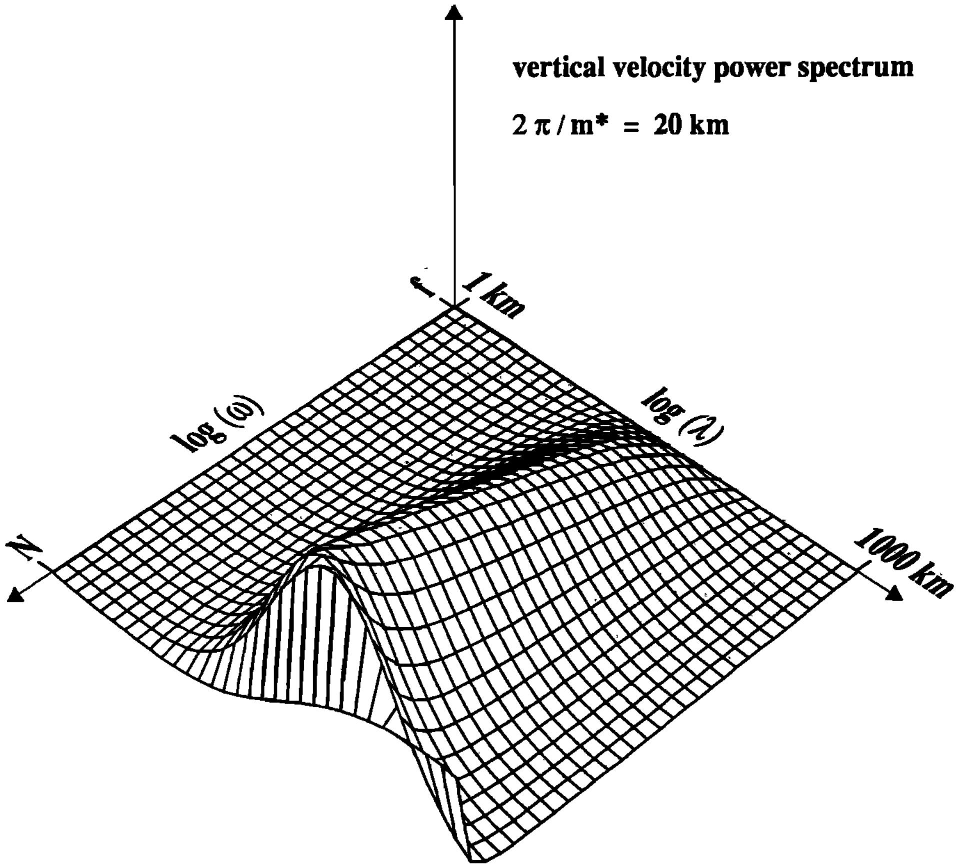


Fig. 9. The same as Figure 8 but with $m_* = 2\pi/20$ km.

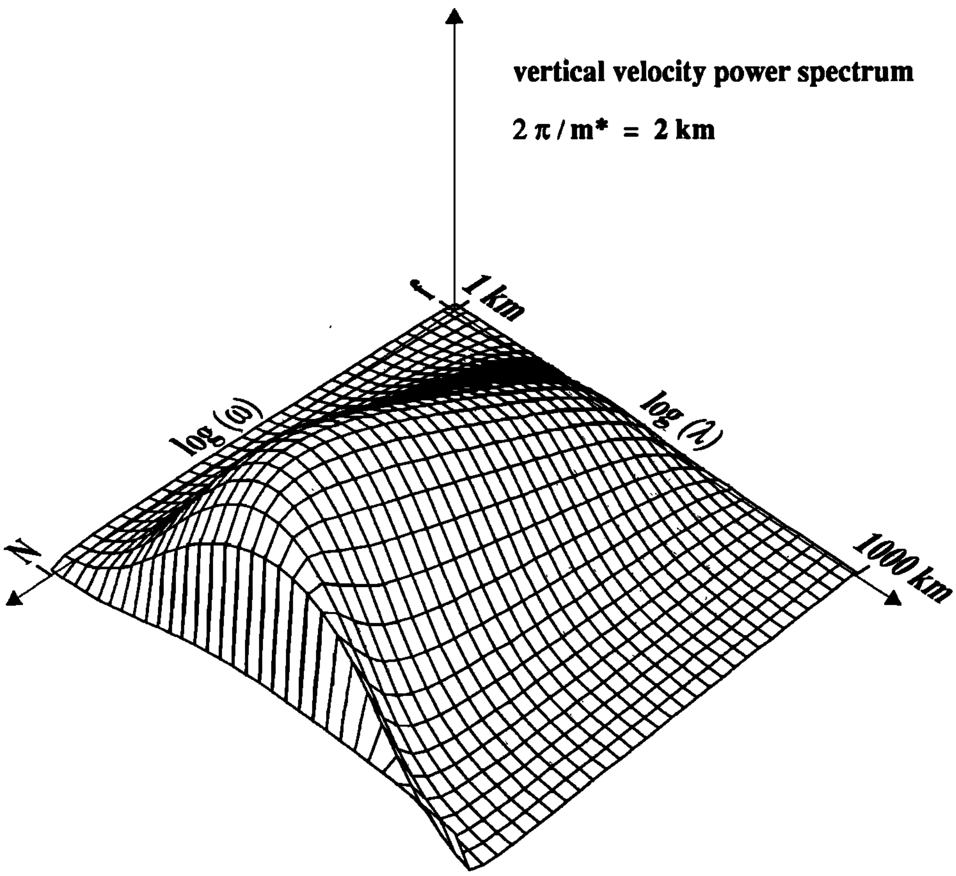


Fig. 10. The same as Figure 8 but with $m_* = 2\pi/2$ km.

locity amplitude. If we substitute plane wave solutions into equation (7), take the $z$ derivative, and divide by $\omega N^2/g$, we obtain

$$\frac{r'_z}{N^2/g} = \frac{w'}{\omega/m} + \frac{g}{c^2N^2}\phi'_z. \qquad (26)$$

Upon expressing $r'$ in terms of the background density $\overline{\rho}$ and instantaneous density $\tilde{\rho}$, equation (26) becomes

$$\frac{(\overline{\rho}_z/\overline{\rho}) - (\tilde{\rho}_z/\overline{\rho})}{(\overline{\rho}_z/\overline{\rho}) + (g/c^2)} = \frac{w'}{\omega/m} + \frac{g}{c^2N^2}\phi'_z. \qquad (27)$$

The motion becomes convectively unstable when the wave density fluctuations become large enough so that

$$\frac{\tilde{\rho}_z}{\overline{\rho}} = -\frac{g}{c^2}. \qquad (28)$$

Neglecting the small pressure term in equation (27), condition (28) implies $w' \approx \omega/m$. Using the values $\lambda_z \approx 6$ km and $\omega \approx 0.006$ s$^{-1}$, we arrive at a vertical velocity magnitude of $w' \approx 6$ m/s.

Vertical velocity fluctuation magnitudes as large as 6 m/s may well exist at times, but they seem implausibly large over long-term averaging. For example, a typical profile of rms vertical velocities in the polar summer mesosphere from *Czechowsky et al.* [1989] shows a range from 1.1 to 1.7 m/s. Upon multiplication by $\sqrt{2}$ we obtain the range 1.6 to 2.4 m/s for the equivalent monochromatic amplitude. In a more relevant example, *Fritts and Yuan* [1989] averaged 8 days of summer mesospheric data from a 1986 Poker Flat campaign to obtain a vertical velocity variance of 6 m$^2$/s$^2$ in the region from 82 km to 89 km, which gives an equivalent monochromatic amplitude of 3.5 m/s. In the same study, *Fritts and Yuan* [1989] report a mean vertical velocity of 30 cm/s downward, consistent with our 4-year average. These reported vertical velocity variances are too small to produce the Stokes drift necessary to explain the mean vertical velocity. Now, 3.5 m/s is not much different than 6 m/s, particularly in light of uncertainties in wave parameters. Our main point is that the 3.5 m/s equivalent monochromatic amplitude actually comes from a spectrum of wavelengths and frequencies. Most of the wavelengths and frequencies have values offering smaller contributions to the Stokes drift than 50 km and 0.006 s$^{-1}$. Here, we merely note that even if the wave variations were concentrated near ideal wavelengths and frequencies, they still fall a bit short of the magnitude needed for a 30 cm/s Stokes drift. For a more quantitative comparison of theory to experiment we consider the results for an observed spectrum of waves. By performing a Stokes drift calculation as a function of observed wave spectral parameters, we by design allow for realistic wave amplitudes in various frequency and wavelength regimes.

Figure 5 is the result of the calculations using the spectral power laws corresponding to $p = 2$ and $t = 3$, values that have been approximately implicated in a recent report from the MU radar in Japan [*Muraoka et al.*, 1990]. In the figure the Stokes drift is plotted versus $m_*$ for several values of $E_0$. *Muraoka et al.* [1990] obtain $m_* \approx 10^{-5}$ m$^{-1}$ and $E_0 \approx 7 \times 10^3$ J/kg in their analysis of the spectra. These values give a Stokes drift less than 1 cm/s. Now, there are compelling theoretical reasons for a -3 vertical wavenumber power law. It arises from a convectively saturated wave field description [*Smith et al.*, 1987] as well as the Lumley buoyancy range turbulence theory [*Lumley*, 1964]. Moreover, equation (19) proves to be insensitive to the value of $t$ in the vicinity of 3. There is more controversy over the value of $p$. Saturated gravity wave theory assumes a frequency power law of -5/3, a result observationally determined by *Carter and Balsley* [1982]. The situation is clouded by the fact that if $p = 2$, rather than 5/3, the shape of the observed vertical velocity spectra is accurately predicted. Proponents of a 5/3 law believe that this result is due to the effects of Doppler shifting. In Figure 6 we reproduce Figure 5, but with $p = 5/3$. Changing the value of $p$ has a significant effect on the Stokes drift calculation, roughly doubling the peak value. In fact, the values $m_* \approx 2.1 \times 10^{-4}$ and $3.4 \times 10^{-4}$ m$^{-1}$, determined by *Smith et al.* [1985] for summer Poker Flat spectra during two different years, are near the peak of the Stokes drift curves. However, the energy densities determined from the same observations are $2.6 \times 10^3$ and $1.3 \times 10^3$ J/kg, respectively, still resulting in Stokes drift values less than 4 cm/s.

We conclude that the Stokes drift mechanism does not have the right magnitude to explain the size of the observed downward summer mean velocity. The wave energy density would need to be roughly 10 times the observationally deduced value to get the required 30 cm/s. Moreover, we emphasize again that even if such a large energy density were realized, the effect would need to turn off or greatly diminish in the nonsummer months to be consistent with the mean vertical velocity observation. *Meek and Manson* [1989] see no evidence for significantly reduced gravity wave activity in the winter and therefore no reduction in the Stokes drift, although it may be that the winter scattering mechanism results in velocity measurements insensitive to the Stokes drift [*Hall*, 1991].

Aside from the downward mean vertical velocity and low temperature, another well-documented unusual feature of the polar summer mesosphere is its extremely large VHF radar cross section compared to other seasons and latitudes. Could these phenomena be related? A connection would have the desirable property that only summer downward velocities need be explained.

The intensity of VHF radar scatter is likely due to the ability of the plasma to structure at shorter scales than the neutral gas [*Kelley et al.*, 1987; *Kelley and Ulwick*, 1988]. In order for this structuring to occur the ion gas must have a diffusivity smaller than the neutral gas viscosity. The ratio of the viscosity to the passive scalar diffusivity is the Schmidt number $Sc$, and *Kelley et al.* [1990] have reported values as large as 100. *Kelley et al.* [1987] suggested that low ion diffusivity could be produced by the large water cluster ions that are known to exist in the polar summer mesopause region. However, such clusters cannot become large enough to produce Schmidt numbers much larger than 10 [*Hall*, 1990]. One possible solution is the presence of multiply charged aerosols. These aerosols, in addition to enhancing $Sc$, might also have mean vertical velocities quite different than those of the neutrals and may well have terminal velocities on the order of the summer observations. In our companion paper, *Cho et al.* [this issue] show in detail the effect of the multiply charged aerosols on the plasma diffusivity and hence the radar cross section. They also show that the aerosol size required to produce $Sc \sim 100$ is consistent with a terminal velocity of 30 cm/s.

One still needs to argue that the radar observes the terminal velocity $w_T$ of the aerosol-laden plasma as the mean

Doppler velocity. A 50-MHz radar is sensitive to a 3-m structure in the electron density. As discussed in detail by *Cho et al.* [this issue], it is our hypothesis that a substantial fraction of the plasma density is provided by positively charged aerosols. Roughly speaking, in order to keep the plasma quasi-neutral an equivalent fraction of the electron gas must mimic any structuring of the aerosol density.

Now, the very presence of intense 50-MHz radar echoes in the polar summer mesosphere demonstrates the existence of strong 3-m structuring. For as long as a 3-m structure exists, it will fall with the aerosols, as these aerosols comprise the plasma responsible for scatter. The structure will also be influenced by many time scale motions in the neutral atmosphere, but over a season these motions should average to zero (we neglect the 1 cm/s upward neutral atmospheric motion because it is much smaller than the 30 cm/s downward aerosol fall speed). The enhancement of the Schmidt number in the polar summer mesosphere, aside from providing a 3-m structure in the passive scalar plasma, also eases the determination of a small velocity such as 30 cm/s. The lifetime of the plasma structure against diffusion is proportional to $k^2 D$ where $k$ is the radar wavenumber, so that the lifetime is enhanced proportional to the Schmidt number through its $D$ dependence. This insures that the scatterers have sufficiently long lifetimes that they can contribute to the radar-determined vertical velocities.

It would be useful to perform a simulation of a passive scalar being mixed by a given neutral gas forcing, while the scalar particles fall at a terminal velocity. This would provide a check of our assertion that structures in the passive scalar density move at the terminal velocity. We hope to perform such a simulation in the future.

## CONCLUSION

We have averaged over 4 years of hourly velocity measurements from the Poker Flat radar data base. The strong (20–30 cm/s) negative vertical velocities in the summer, first detected for a year of averaging by *Balsley and Riddle* [1984], are confirmed for the longer averaging. We find that the 4-year average nonsummer months exhibit an upward velocity of magnitude $2 \pm 1$ cm/s, somewhat smaller than values reported by *Balsley and Riddle* [1984] for one winter. Because of its larger magnitude, in this paper we have concentrated on the summer vertical velocity measurements. However, the smaller winter monthly mean values are also in contradiction to the current theory of the mesospheric seasonal circulation and remain a puzzle.

In this paper we show that the Stokes drift mechanism of *Coy et al.* [1986], offered as an explanation for the unexpected summer observations, seems not to have the right magnitude when Poker Flat estimates of wave field spectral parameters are incorporated in the calculation. Reported energy densities of $\sim 2 \times 10^3$ J/kg and wavenumber spectral bandwidths of $\sim 2 \times 10^{-4}$ m$^{-1}$ produce an induced downward Eulerian velocity of no more than 4 cm/s. As an alternative explanation we propose that the radar is detecting the terminal velocity of charged aerosols. These aerosols can also play an important role in the radar scattering mechanism, as is explored in depth in a companion paper. Such a connection between the polar mesospheric summer echo scattering mechanism and the observed mean vertical velocity has the attractive feature of turning off the downward drift in the winter when the temperature is too high for the necessary ion chemistry.

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