Optimizing Product Line Design

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Agenda

1. Product line design exercise

2. Research on design optimization methods

Timbuk2 Has A Problem

Timbuk2's Problem

1. Price: $70 - $100
2. Size: small or large
3. Color: black or red
4. Sloan logo
5. Handle
6. PDA holder
7. Cell phone holder
8. Mesh pocket
9. Sleeve closure
10. Reinforcing "boot"

Exercise 1

1. Which five bags will maximize Timbuk2's aggregate profit?
2. The market: MBA students
3. We will provide you with:
   - The menu of product features
   - The cost of each product feature
   - The features of the 3 competing bags
4. Groups of 4

Exercise 1

> Change the prices and features in the yellow box.
> Try to design bags that will maximize profits.
> Be sure to consider the three competing Coop bags.

Set price from $70 to $100 in $5 increments.
Enter "1" to add a feature, "0" or "Delete" to remove it.

Feature	Firm's five laptop bags					Feature Cost
	1	2	3	4	5	
Price	$70	$70	$70	$70	$70	
Large Size						$3.50
Red Color						$0.00
Sloan Logo						$2.00
Handle						$3.50
PDA Holder						$3.00
Cell Phone Holder						$3.00
Mesh Pocket						$2.00
Velcro Flap						$3.50
Boot						$4.50
Bag Profitability	$35.00	$35.00	$35.00	$35.00	$35.00	

Feature	Coop's Three Bags		
	A	B	C
Price	$70	$85	$100
Large Size			1
Red Color			1
Sloan Logo			1
Handle			1
PDA Holder		1	1
Cell Phone Holder		1	1
Mesh Pocket		1	1
Velcro Flap		1	1
Boot		1	1

Really 2 Problems

1. **Measurement**: which product features do customers prefer? (*customer behavior*)

2. **Optimization**: how should firms act? (*firm behavior*)

Solution Methodology

1. **Measurement:** use Conjoint Analysis to measure customer preferences

2. **Optimization**: use Discrete Optimization Methods to determine how firms should act

Timbuk2 Data

Timbuk2 Data

- 2001 study involving MIT Sloan students
- 324 students participated (92% response)
- 16 paired comparison questions

http://conjoint.mit.edu/newdemo/FastPace/html/page01.htm

Results For 1 Student

Features	Regression Coefficients
High Price	-0.7
Large Size	82.0
Color Red	-47.0
With Logo	12.2
With Handle	3.2
PDA Holder	-16.1
Phone Holder	32.5
Mesh Pocket	-43.9
Full Closure	61.3
With Boot	94.2

Different for each student

Exercise 2

1. Which five bags will maximize Timbuk2's Aggregate profit
2. Use the same groups and focus on the same market
3. Additional information:
   - Importance weights for the 10 features for each of the 324 students
   - A spreadsheet calculating the profit from any combination of five bags

Exercise 2

> Change the prices and features in the yellow box.

> Try to maximize profits.

Profits: **$11,576**

Set price from $70 to $100 in $5 increments.

Enter "1" to add a feature, "0" or "Delete" to remove it.

Feature	Firm's five laptop bags					Feature Cost
	1	2	3	4	5	
Price	$95	$80	$100	$75	$100	
Large Size	1		1		1	$3.50
Red Color	1					$0.00
Sloan Logo	1		1			$2.00
Handle	1		1	1	1	$3.50
PDA Holder	1	1	1			$3.00
Cell Phone Holder	1	1	1			$3.00
Mesh Pocket	1	1	1			$2.00
Velcro Flap	1	1	1			$3.50
Boot	1	1	1			$4.50
Bag Profitability	$35.00	$29.00	$40.00	$36.50	$58.00	
# Purchased	53	44	133	57	18	**305**
Profits Generated	$1,855	$1,276	$5,320	$2,081	$1,044	**$11,576**

Feature	Coop's Three Bags		
	A	B	C
Price	$70	$85	$100
Large Size			1
Red Color			1
Sloan Logo			1
Handle			1
PDA Holder		1	1
Cell Phone Holder		1	1
Mesh Pocket		1	1
Velcro Flap		1	1
Boot		1	1
# Purchased	15	1	3

Exercise 1 Profit Maximization Results

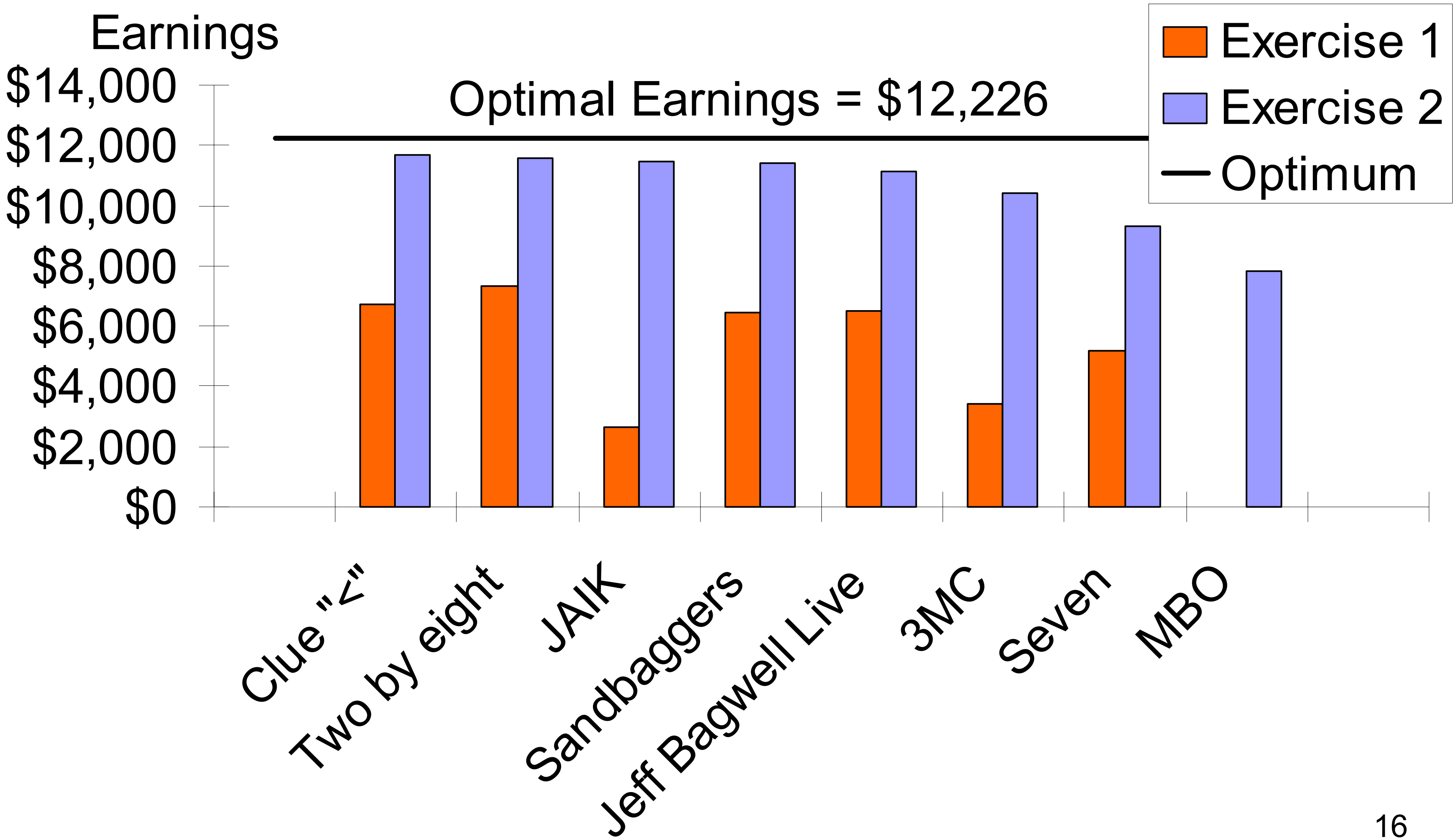
Team	Profits
Two by eight	$7,338
Clue "<"	$6,721
Jeff Bagwell Live	$6,500
Sandbaggers	$6,468
Seven	$5,176
3MC	$3,419
JAIK	$2,643
Average	**$5,466**

Exercise 2 Profit Maximization Results

Team	Profits
Clue "<"	$11,672
Two by eight	$11,586
JAIK	$11,443
Sandbaggers	$11,409
Jeff Bagwell Live	$11,126
3MC	$10,444
Seven	$9,316
MBO	$7,833
Average	**$10,604**

Optimal Profits: $12,226

Chart of Profit Maximization Results



Optimal Solution

Feature	Firm's 5 Laptop Bags				
	T1	T2	T3	T4	T5
Price	$80	$95	$95	$100	$100
Large Size		■		■	
Red Color		■			
Sloan Logo		■		■	■
Handle		■	■	■	■
PDA Holder	■	■			■
Cell Holder	■	■			■
Mesh Pocket	■	■		■	■
Velcro Flap	■	■	■	■	
Reinforcing Boot	■	■	■	■	

Enumeration won't work

- There are 7 x 2^9 = 3,584 possible laptop bag types.

- The number of possible combinations of five different bags is:

$$\frac{3{,}584 \times 3{,}583 \times 3{,}582 \times 3{,}581 \times 3{,}580}{5 \times 4 \times 3 \times 2 \times 1} \approx 4.9 \times 10^{15}$$

- Enumerating and evaluating the profit value of each combination is not a workable solution strategy.

Sample Optimization Problem

Project	Net Present Value ($million) (at 18% per year)	First-Year Investment Cost ($million)	LINUX Transportable	Managers Required
A	$17	$5	1	3
B	8	5	1	3
C	11	4	1	1
D	14	2	0	3
E	18	1	0	1

- 5 potential projects
- First-year budget: $11 million
- At least 2 projects must be LINUX transportable
- Elite software project managers available: 9

Solution

Which projects should the firm undertake?

Answer: A, C, E

What is the Net Present Value of the projects?

Answer: $46 Million

Sample Optimization Problem

Project	Net Present Value ($million) (at 18% per year)	First-Year Investment Cost ($million)	LINUX Transportable	Managers Required
A	$17	$5	1	3
B	8	5	1	3
C	11	4	1	1
D	14	2	0	3
E	18	1	0	1
F	18	1	1	2
G	16	3	1	3
H	18	6	0	2
I	9	4	1	3
J	20	9	0	0
K	9	1	1	1
L	19	8	0	3
M	13	1	0	3
N	16	4	0	3
O	11	4	1	1
P	9	6	1	3
Q	10	8	1	1
R	17	1	0	1
S	13	4	1	2
T	6	8	1	1

- 20 potential projects
- First-year budget: $85 million.
- At least 6 projects must be LINUX transportable
- Elite software project managers available: 28

Solution???

Which projects should the firm undertake?

What is the Net Present Value of the projects?

Possible Solutions

- Tracy’s feasible plan has NPV = $167 million
- Mark’s feasible plan has NPV = $164 million
- Tom’s feasible plan has NPV = $175 million
- Laura’s feasible plan has NPV = $188 million
- Should we go with Laura’s plan? How good is it really?

Decision Variables

XA = 1 if we undertake project A, 0 if we do not

…

…

XT = 1 if we undertake project T, 0 if we do not

Objective Function and Constraints

Maximize	NPV =	17 XA + 8 XB + … + 6 XT
s.t.	(budget:)	5 XA + 5 XB + … + 8 XT ≤ 85
	(LINUX:)	XA + XB + … + XT ≥ 6
	(managers:)	3 XA + 3 XB + … + 1 XT ≤ 28

XA, XB, … , XT are binary (0 or 1) variables

Solution

Decision Variable	Optimal Value
XA	1
XB	0
XC	1
XD	1
XE	1
XF	1
XG	1
XH	1
XI	0
XJ	1
XK	1
XL	1
XM	0
XN	1
XO	1
XP	0
XQ	1
XR	1
XS	1
XT	1

Solution

Which projects should the firm undertake?

Answer: A, C, D, E, F, G, H, J, K, L, N, O, Q, R, S, T

What is the Net Present Value of the projects?

Answer: $233 million

What is the optimal resource utilization?

First-year budget: $69 million < $85 million

LINUX: 9 > 6

Managers: 28 = 28

Guarantee of Optimality

- Tracy's feasible plan has NPV = $167 million
- Mark's feasible plan has NPV = $164 million
- Tom's feasible plan has NPV = $175 million
- Laura's feasible plan has NPV = $188 million

- Optimal NPV = $233 million

Without a *guarantee* of optimality, we cannot know if a proposed plan is very good or not.

Product Line Design Optimization Model

- **Decision Variables**
- **Objective Function**
- **Constraints**

Decision Variables

- Bag variables:
  - List all bag types: $7 \times 2^9 = 3{,}584$ different types
  - $Y_j = 1$ if the firm produces bag j , 0 otherwise
  - j = 1, ..., 3,584

- Student purchase variables

  $P_{ij} = 1$ if student i purchases bag j , 0 otherwise

  i = 1, …, 324, and j = 1, …, 3,584

- Total of 1,161,216 decision variables

Constraints

- The firm will produce exactly 5 laptop bags:

$$Y_1 + Y_2 + Y_3 + Y_4 + \ldots + Y_{3,584} = 5$$

- Other constraints that enforce presumed consumer behavior:
  - Each student will purchases exactly one laptop bag from those offered by the firm and those offered by the Coop
  - Each student will purchase the laptop bag that maximizes his/her individual utility

- Total of 3,483,973 constraints in model

Objective Function

- Maximize Profit $= \sum$ (Profit to firm generated by each student's utility-maximizing decision)

- The firm's profit generated by a given student depends on which bag the student purchases, and on the cost of each feature of that bag

Traditional Solution Methods…

- The binary optimization model includes over 1 million decision variables and over 3 million constraints
- We ran the model in OPL Studio (custom software for optimization)
- Software ran out of memory and crashed
- Running in Excel would be even more hopeless

…Traditional Solution Methods

- We tried a half-dozen other integer optimization formulations of the problem. None could be solved by existing methods and software.

A Sophisticated Solution Method

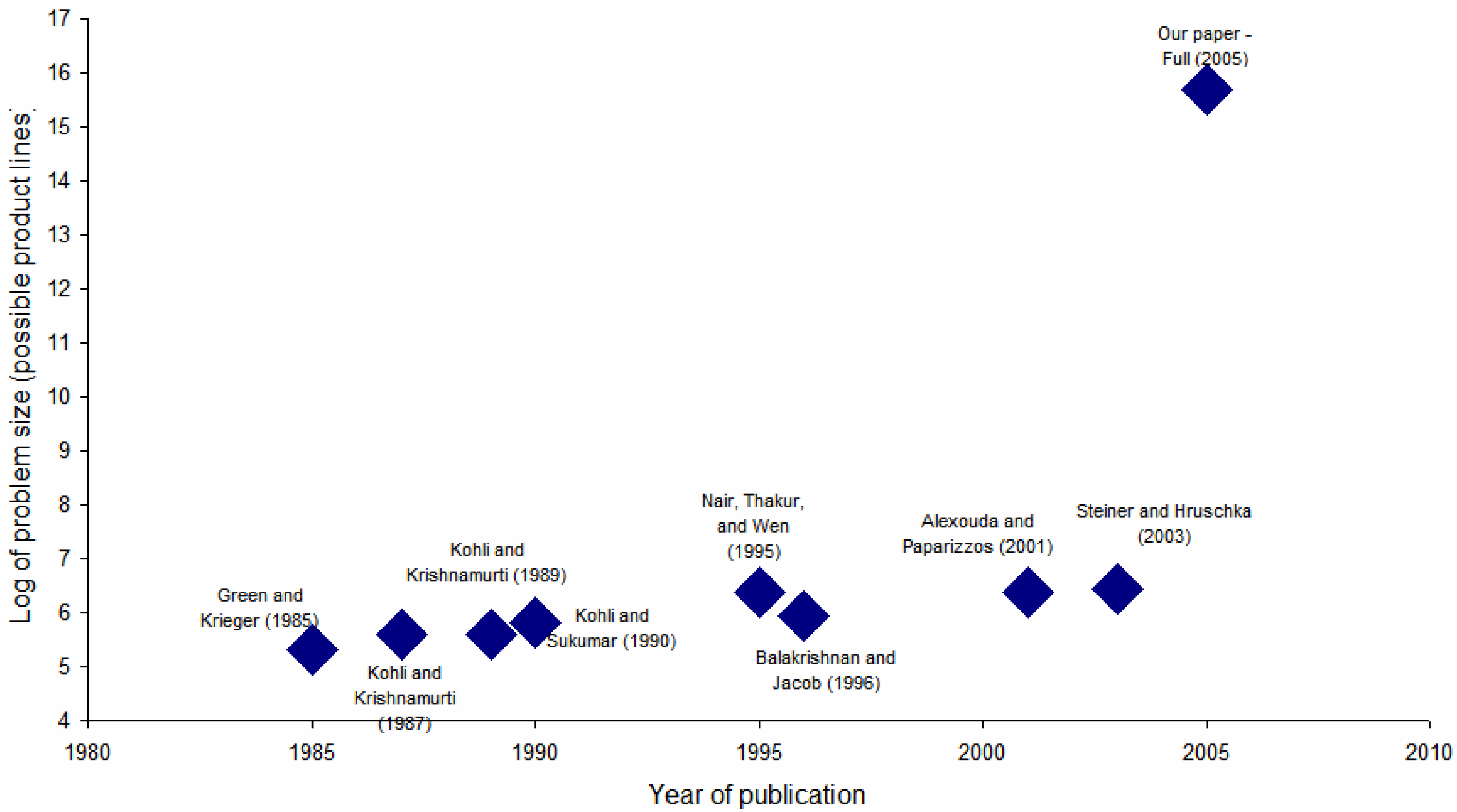
- Use “Lagrangian Relaxation” to reduce the number of constraints and to help to produce an optimality guarantee

- Use “Branch and Bound” to avoid having to do exhaustive enumeration

Comparison With Previous Research

- Academic researchers have been working on the optimal product line design problem for over twenty years.
- Previous research has relied on complete enumeration to guarantee optimality.
- Using Lagrangian relaxation with branch and bound, we have solved the problem presented in this exercise.

Size of Problems Solved in Previous Research



Our Sophisticated Method Is Impractical

- Our sophisticated method is extremely complex. Most firms would not have the ability to implement it.
- The method takes about 7 days to run.
- However, by providing a *guaranteed* optimal solution, the sophisticated method can benchmark more practical methods.

Practical Methods

- Coordinate ascent seeks local improvement by changing individual product features. The algorithm terminates when no further local improvement is possible.

- Simulated annealing is similar to coordinate ascent, except that it sometimes accepts negative changes. This enables the algorithm to escape from a local optimum and continue searching for a better solution.

- The product-swapping heuristic starts with a random solution and seeks local improvement by swapping new products into the solution. The algorithm terminates when no local improvement is possible.

- Genetic algorithms start with a population of random solutions and seek better solutions with a process that mimics natural selection.

Comparison of Methods

	Lagrangian Relaxation	Coordinate Ascent	Simulated Annealing	Product-swapping Heuristic	Genetic Algorithm
Trial 1	12,226	12,021	12,226	12,219	12,226
Trial 2		11,971	12,226	12,219	12,226
Trial 3		11,850	12,226	12,219	12,226
Trial 4		11,760	12,226	12,219	12,226
Trial 5		11,640	12,226	12,219	12,056
Trial 6		10,848	12,226	12,219	12,056
Trial 7		10,780	12,226	12,219	12,056
Trial 8		10,519	12,226	12,219	12,041
Trial 9		10,481	12,226	12,219	12,025
Trial 10		10,297	12,226	12,210	11,913
Avg. earnings	**12,226**	**11,217**	**12,226**	**12,218**	**12,105**
% of optimum	100.0%	91.7%	100.0%	99.9%	99.0%
Avg. run time	7 days	0.2 sec	128.7 sec	14.1 sec	16.5 sec

Comparison of Methods

- Relatively simple methods such as product-swapping, simulated annealing, and genetic algorithms are very effective even though they do not produce guarantees

- In general, the longer a method takes to run, the better it performs. The outlier in this trend is product swapping, which achieves near-optimal earnings in about 14 seconds.

- The most successful methods continue to perform well even when there is moderately large error in part-worth estimates.

Some Limitations

- Part-worth and conjoint analysis might not model consumer behavior with sufficient accuracy
- Cost assessments might not be sufficiently accurate
- The premise of the optimal product line design model does not consider competitive response to new product introductions

Lessons

- Measurement is necessary but not sufficient
- The same is true for optimization
- Measurement precedes optimization
- Formal optimization models may look unwieldy, but can be very effective

Questions???