MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 6, No. 3, Summer 2004, pp. 225–236
ISSN 1523-4614 | EISSN 1526-5498 | 04 | 0603 | 0225

informs®
DOI 10.1287/msom.1040.0035

Controlling the Risk for an Agricultural Harvest

Stuart J. Allen
Penn State University, stuart99allen@yahoo.com

Edmund W. Schuster
MIT Auto-ID Labs, 23 Valencia Drive, Nashua, New Hampshire 03062, schuster@ed-w.info

Gathering the harvest represents a complex managerial problem for agricultural cooperatives involved in harvesting and processing operations: balancing the risk of overinvestment with the risk of underproduction. The rate to harvest crops and the corresponding capital investment are critical strategic decisions in situations where poor weather conditions present a risk of crop loss. In this article, we discuss a case study of the Concord grape harvest and develop a mathematical model to control harvest risk. The model involves differentiation of a joint probability distribution that represents risks associated with the length of the harvest season and the size of the crop. This approach is becoming popular as a means of dealing with complex problems involving operational and supply chain risk. Significant cost avoidance, in the millions of dollars, results from practical implementation of the Harvest Model. Using real data, we found that the Harvest Model provides lower-cost solutions in situations involving moderate variability in both the length of season and the crop size as compared to solutions based on imposed risk policies determined by management.

Key words: harvest risk; agriculture
History: Received: July 30, 2002; accepted: December 12, 2003. This paper was with the authors $7\frac{1}{2}$ months for 6 revisions.

1. Introduction

Gathering the harvest is a complex managerial problem in controlling risk. Since the dawn of agriculture about 12,000 years ago, farmers have dealt with several categories of risk in producing the food needed to sustain their lives and to build urban centers of industry and learning (Garraty and Gay 1972, p. 50).

For an agricultural cooperative in modern times, the harvest is the primary source of assets needed to run the business. Hence, the harvest represents the base raw material and most important link in the supply chain for these firms. Before each harvest, managers must make critical assessments concerning the capability of harvesting the entire crop at optimum maturity.

In this article, we analyze the risk associated with the harvesting of Concord grapes through the development of a mathematical model and the presentation of results from practice. Specifically, the model calculates the optimal rate for a processing plant to receive grapes. This is a common situation in agriculture where growers deliver crops to a central receiving area for further processing and storage prior to sale. Determining the optimum rate to harvest and process the crop is a challenging problem in balancing the risk of overinvestment in capital with the risk of not harvesting the entire crop.

2. Structure of the Concord Grape Industry

The Concord grape, a flavorful, aromatic purple variety of grape, is grown in the cooler regions of the United States. Major growing areas include western New York, northern Ohio, and northern Pennsylvania (all three near Lake Erie); western Michigan; and south-central Washington state. In 1869 Thomas B. Welch, a dentist from New Jersey, used pasteurization as a means to preserve Concord grape juice. By heating the grape juice before bottling he was able to stop fermentation, creating the first nonalcoholic fruit juice product. This event marked the beginning of the bottled and canned fruit juice industry in the United States.

During the 130-plus years since this genesis, the market for Concord grapes has grown into a billion-dollar industry with an annual harvest of about

400,000 tons. Though the Concord grape comprises only about 6% of the total grapes grown in the United States, it maintains a high profile because of intensive marketing; it remains a popular ingredient for juices, fruit jellies and jams, and concentrates. Not suitable for the fresh market because of its short shelf life, nearly all of the Concord grapes harvested are immediately converted into juice.

The leading player of the industry is Welch Foods, Inc., which is owned by the National Grape Cooperative Association. Welch Foods controls about 55% of the Concord grapes grown in the United States. The next-largest player controls less than 7%. Collectively, five business structures comprise the Concord grape industry.

(1) *Private packers* that purchase Concord grape juice as a major ingredient for their products, both branded and house brands.

(2) *Private processors* that contract with growers on an annual basis for delivery of grapes, then process the grapes into concentrate for sale as a raw ingredient.

(3) *Private processors with packaging capabilities*, primarily involved in production of house brands.

(4) *Cooperatives* owned by growers that process grapes into raw ingredients, sell to other companies, and return profits to the growers.

(5) *Vertically integrated cooperatives* involved in growing, processing, packaging, distributing, and marketing Concord grape products.

The industry is a mix of both vertically integrated and modular structures (Fine 1998). However, the long-term trend is toward supply chain integration similar to the transition taking place in the poultry industry (Kinsey 2001). Product life cycles are long, with few radical product innovations. Intense competition and lack of pricing power in the market combine to force a focus on cost control.

2.1. Operational Design, Organization, and Financial Implications

All businesses that convert Concord grapes into juice—cooperatives and private companies alike—follow similar steps during the harvest process. In the fall, usually beginning in late September, fruit growers deliver grapes picked by mechanical harvesters to processing plants located near the vineyards. The grapes are then converted into juice through a complex pressing process. This step takes place within eight hours after picking to avoid fermentation and deterioration of quality. Exceeding this time limit means the grapes must be diverted, at severe cost penalty, to other uses, such as wine making. The economics of refrigerated warehousing, as well as biological degradation at low temperatures, preclude storage of raw Concord grapes for later pressing into juice.

In each growing area, approximately 100 mechanical harvesters pick grapes on a continuous basis. Since the cost of an individual harvester is very high, approaching $200,000 per unit, growers usually pool resources through joint ownership or contracting. A single harvester picks an average of three separately owned vineyards. Coordination of the field operations becomes critical to ensure a steady feed of freshly picked grapes to the processing plant.

2.2. Harvest Scheduling and Capacity

Prior to the harvest, planners assign a constant rate of picking to each harvester operator based on acreage, estimated yields, the estimated length of the harvest, and the standard pressing rate for the receiving plant. This practice establishes fairness: Each vineyard owner faces the same risk of losing crop because of poor weather, such as a frost. A hard frost, where temperatures dip below 28°F, weakens stem tissue, causing the grapes to fall from the vines. This is called *shelling.* Every grower wants to complete the harvest before a hard frost causes shelling in the vineyards.

The total daily schedule for all harvesters equals 80% of the standard plant pressing rate. The reserve of 20% allows planners flexibility during harvest operations to accommodate individual growers who might experience crop maturity problems, mechanical breakdowns, or an abnormally high risk of crop loss due to frost. The reserve is completely used as part of day-to-day harvest operations so that the rate of picking always matches the rate of pressing at the receiving plant. This implies that the rate of picking and pressing is dictated at the same time that capacity investment decisions are made (i.e., once per year, projecting five years into the future).

In the midst of harvest operations, it is very difficult to change the rates of picking and pressing grapes because of the large number of harvester operators and the complexity of receiving plant operations. When day-to-day pressing or picking rates vary by even small amounts a classic system dynamics response occurs, producing substantial loss of efficiency that extends many days past the original disruption. Extensive preharvest efforts focus on testing and maintenance of plant equipment to ensure reliability in achieving the standard pressing rate per day.

Upon receipt from the vineyards, the grapes are heated and pressed into unsettled juice that is stored in large refrigerated tanks. Unsettled juice is the intermediate step in processing. Welch Foods alone has over 55 million gallons of tank storage capacity. Throughout the remainder of the year, the unsettled juice is further processed and used for bottling and as an ingredient in other food products. The manufacturing structure takes on a "V" shape typical in the process industries where a small number of raw materials are used to produce a large number of end items (Umble 1992). Final processing of the entire harvest takes close to one year.

2.3. Financial Implications Investment in Capital Assets

Investment in capital assets to process Concord grapes into juice is substantial. For cooperatives this investment in pressing capacity, that remains idle for 10 months per year, is a drain on resources. Despite tax advantages, cooperatives involved in processing raw fruits and vegetables typically have a low return on capital compared with the rest of the food industry. In this case, the proper sizing of harvest processing capacity becomes an important strategic decision with significant financial impacts.

3. Problem Definition

Consider the typical situation in harvesting Concord grapes, or other crops, where a planner faces a task with an uncertain start date, an uncertain end date, and whose size cannot be known in advance with certainty. As an example, the maturation dates of Concord grapes depend on random factors such as weather and soil conditions as well as cultivation practices. Random frost dates may signal an abrupt end to the harvest season and total crop size is not known in advance of the harvest process.

In such cases, the planner must decide what degree of effort to devote to the harvest in terms of the pressing rate and the corresponding capital investment in equipment. These are decisions made in advance of harvest as part of rolling five-year plans because of the long lead times for capital equipment. A balance must be made between the costs of the resources committed and the costs of a partially completed task (grapes remaining in the vineyard). With whatever methods employed, the derivation of the optimal harvest rate is critical for the grower in controlling risk and in determining the proper capital investment.

3.1. The Elements of Uncertainty

With uncertainty in both the size of future grape crops and the time window available for future harvests, it is impossible to calculate the optimal rate of pressing using direct methods. Simply dividing the historical average crop size by an estimate of the available days of harvest will result in a solution that ignores the inherent risk of agriculture.

The essence of this problem can be defined by three variables:

$H =$ crop size in tons, a *random variable*;
$L =$ length of harvest season in days, a *random variable*;
$R =$ pressing rate in tons per day, the *decision variable*.

From these variables two sets of expressions can be defined, one set each for the *amount of crop loss* (ACL) and the *excess harvest rate* (EHR).

$$\text{ACL} = \begin{cases} H - RL & \text{if } H > RL \\ 0 & \text{if } H \leq RL \end{cases} \tag{1}$$

$$\text{EHR} = \begin{cases} 0 & \text{if } H \geq RL \\ RL - H & \text{if } H < RL. \end{cases} \tag{2}$$

These two sets of expressions have natural interpretations. If H is greater than what can be harvested at R, then there will be loss in the vineyards, as represented in Equation (1). This often occurs in practice when a frost cuts short the harvest season. However, if R is too high, the opportunity is lost for harvesting the crop with less effort, as represented in Equation (2). This also occurs in practice when there is overinvestment in pressing equipment. Because both of the above equations contain two random variables, we will have to form the expected values of each by integrating over the joint density function, which we denote by $g(H, L)$.

3.2. Important Model Assumptions and Justifications

Allen and Schuster (2000, pp. 32–33) report that for one of the Concord grape growing regions a correlation coefficient of 0.14 exists between H and L based on 22 years of data at an observed significance of 53%. The results of this analysis offer support that H and L are statistically independent. This makes sense because H is dependent on many variables such as precipitation, soil conditions, temperatures, and available sunlight, which are uncorrelated to the time window of good weather available to gather the harvest. Then the joint density function can be formed as the product of two marginal distributions.

$$g(H, L) = f(H) \times f(L). \tag{3}$$

Another important assumption is the shape of the probability distributions used to estimate H and L. Extensive historical information from state agricultural experiment stations, spanning more than 40 years of history, allows calculation of the mean and standard deviation in each case. Histograms of this data show that both H and L are mound shaped and reasonably symmetrical. An analysis of data from Allen and Schuster (2000, pp. 32–33), using a chi-square goodness-of-fit test, shows insufficient evidence ($p = 0.53$ and $p = 0.66$) to reject the null hypothesis that H and L are normally distributed. In both cases, one degree of freedom exists. The modified histograms of H and L contain four observations in one and two intervals, respectively. This is outside the minimum guideline of five observations for the chi-square goodness-of-fit test. It is also important to note that in similar situations researchers use symmetrical distributions (Jones et al. 2001). These observations lead us to assume L and H are normally distributed with mean and standard deviation μ_L, σ_L and μ_H, σ_H respectively.

3.3. The Objective Function

The Harvest Model minimizes a total cost function that contains two costs.

- *Underage cost* (C_H, *dollars per ton*). This reflects the cost of harvesting too slowly, therefore foregoing sales of grapes left in the field at the end of the harvest season.
- *Overage cost* (C_R, *dollars per ton*). This reflects the cost of harvesting too quickly, therefore committing an excessive amount of resources to the harvesting effort.

The formation of the Harvest Model involves applying costs to ACL and EHR. Because the Harvest Model is probabilistic in nature, expected values are needed for both ACL and EHR. Once expressions exist for each, costs can be applied to form a total cost function. The minimum of this function will provide the optimal harvest rate at minimum cost.

The remainder of this study will focus on the matter of harvesting Concord grapes with an uncertain length of harvest season and an uncertain crop size. Section 4 provides an overview of the literature relating to the Harvest Model. Section 5 gives specifics about the derivation of the expected values and the minimization of the total cost function. In §6 detailed numerical results of the Harvest Model are presented based on real data, followed by §7, which provides managerial insights.

4. A Review of Cognate Research

The analysis of harvest risk draws upon three general areas of previous research, including the analysis of harvesting operations, agricultural decision making involving risk, and the newsvendor model.

4.1. Analysis of Harvest Operations

Thornthwaite (1953) addresses the problem of harvesting peas at the peak of maturity. Subsequent analysis by Kreiner (1994) shows that Thornthwaite's method was still in use more than 40 years later and has reduced harvest and processing costs, and lost product. Though Thornthwaite's work involves annual crops (peas), and does not deal with the probability of losing crop as a result of frost, it remains a classic in the application of operations research. To our knowledge this is the first reference on the scientific study of harvest operations.

Porteus (1993a, 1993b) develops a two-part case study of the National Cranberry Cooperative. In this case, he examines complex trade-offs in capital investment and capacity for the processing of cranberries during a harvest season. Although the harvest and processing of cranberries share common elements with Concord grapes, the author did not address the risk of crop loss from a frost, or from overmaturation.

In aggregate, these four references provide important background information on harvesting operations plus specifics about the uniqueness of this area of management.

4.2. Agricultural Decision Making Involving Risk

Maatman et al. (2002) note that "various methods of risk reduction exist, including those aimed at *prevention* of risk (e.g., irrigation), *dispersion* of risk (by diversification of risky activities such as the cultivation of different varieties of crops), *control* of risk (e.g., by sequential decision making) and '*insurance*' against risk" (p. 400, italics in original).

Much of research in agricultural decision making focuses on the dispersion of risk through improved individual farm planning. Mathematical programming is often the tool used for modeling farmers' strategies under uncertain conditions (see Anderson et al. 1977, Hazell and Norton 1986). These models are static in nature, assuming that probabilities of occurrence are known in advance and that all decisions are made at one time.

This class of models is similar in spirit to what is presented in this article. However, the Harvest Model does not focus on the dispersion of economic risk and does not employ traditional mathematical programming. Rather, we concentrate on control of risk for a pooled harvest situation (a cooperative) through optimization based on a generalization of the newsvendor problem.

Jones et al. (2001) study the control of risk for production of hybrid seed corn from the perspective of both random yields and demands. A follow-up study reports results in practice (Jones et al. 2003). The authors' work is similar in style to the Harvest Model. However, the authors do not address uncertain start and end times for a harvest, choosing instead to focus on a two-stage production model that involves two different regions (North America and South America) with staggered growing seasons. In this regard, their work differs from the Harvest Model, and will no doubt lead to further work in controlling agricultural risk.

Previous research involving Concord grapes and agricultural cooperatives deals with decision making in raw materials management (Schuster and Allen 1998, Schuster et al. 2000) and production planning (Schuster and Finch 1990, Allen and Schuster 1994, Allen et al. 1997, D'Itri et al. 1998). Though these models yield significant financial savings in practice, none considers the risk associated with harvesting Concord grapes.

In a recent article, Allen and Schuster (2000) address harvest risk for Concord grapes through a working model with the successful outcome of a closed form solution. Using this approach, senior management specifies risk levels as policy. An example would be "Harvest 100% of the crop, 85% of the time." The policy level becomes an important input. Despite offering a large improvement over previous methods that involved heuristics, this model does not include an explicit treatment of the cost of lost crop or seasonal operating costs.

4.3. The Newsvendor Model

The newsvendor approach is effective in a number of applications, including the defense industry (Masters 1987), and is the underpinning of our harvest model. Nahmias (1997, pp. 272–280) provides a comprehensive review of the newsvendor model including derivation, historical context, and several important extensions.

The process of harvesting grapes shares characteristics with the management of style goods such as fashion items in the clothing industry. As is true of style goods, grapes are perishable. Concord grapes have a limited time for peak maturity combined with a short time window for harvesting during the fall. In this regard, the harvest of Concord grapes shares similarities to seasonal sales of clothing where there are substantial costs of obsolescence and limited opportunities to store for future sale.

An important body of literature addresses the style goods problem. Fisher and Raman (1996) develop a response-based production strategy to deal with the uncertain demand characteristic of style goods. Other authors improve style goods forecasts by using a Bayesian approach (Eppen and Iyer 1997, Murry and Silver 1966). However, with the harvest of Concord grapes, little opportunity exists to change the harvest rate during seasonal operations, rendering Bayesian methods less effective.

As a concluding comment, the distinguishing aspect of the Harvest Model is that it contains two random variables (H and L) as compared to the traditional newsvendor model that contains only a single random variable (demand). In this way, the harvest risk model shares similarities with the work of Song et al. (2000) that considers two random variables.

5. Mathematical Development

We now turn our attention to expressions for the expected ACL and EHR.

5.1. The Expected ACL

Since the ACL is discontinuous at $H = RL$, the expected value is given by

$$\mathrm{E(ACL)} = \int_{L=-\infty}^{L=\infty} \left\{ \int_{H=RL}^{H=\infty} (H - RL) f(H)\, dH \right\} f(L)\, dL. \tag{4}$$

The inner integral is the familiar normal loss function. We define the standard normal variate for H as

$$z_H = \frac{H - \mu_H}{\sigma_H}, \tag{5}$$

and

$$Q = \frac{RL - \mu_H}{\sigma_H}. \tag{6}$$

Then the expected ACL becomes

$$\mathrm{E(ACL)} = \sigma_H \int_{-\infty}^{\infty} G(Q) f(L)\, dL \tag{7}$$

where

$$G(Q) = \Phi(Q) - Q[1 - \Psi(Q)] \tag{8}$$

and $\Phi(\cdot)$ is the standard normal density function while $\Psi(\cdot)$ is the cumulative of the standard normal density function. The form of the expected ACL can be simplified by introducing the nondimensional harvest rate, r. In a risk-free environment, the risk-free harvest rate becomes $R_0 = \mu_H/\mu_L$. In this situation, R is known because both H and L are known in advance. Of course in reality uncertain crop size and uncertain harvest start and end times exist, so R is different from R_0. The definition for the nondimensional harvest rate becomes $r = R/R_0$. Next, we define the standard normal variate for L as

$$z_L = \frac{L - \mu_L}{\sigma_L}. \tag{9}$$

Finally, we define the two coefficients of variation

$$k_H = \sigma_H/\mu_H, \quad k_L = \sigma_L/\mu_L.$$

Now Q can be expressed entirely in terms of the two coefficients of variation, the standard variate, z_L, and the dimensionless harvest rate, r:

$$Q = \frac{r(1 + k_L z_L) - 1}{k_H}. \tag{10}$$

Then the expected ACL becomes

$$\mathrm{E(ACL)} = \mu_H k_H \int_{-\infty}^{\infty} G(Q)\Phi(z_L)\, dz_L. \tag{11}$$

Silver and Smith (1981) provide the tools to carry out the integration of this expression and subsequent results. First note that Q in Equation (10) can be expressed in the form $Q = az_L + b$, where

$$a = r(k_L/k_H); \qquad b = (r-1)/k_H. \tag{12}$$

In the interest of notational conciseness, we define another recurring parameter as

$$c = b/\sqrt{1 + a^2}. \tag{13}$$

Using Equations (12) and (13), the integrated form of Equation (11) becomes

$$\mathrm{E(ACL)} = \mu_H k_H \sqrt{1 + a^2}\, G(c). \tag{14}$$

In Equation (14) and all subsequent results, $G(\cdot)$ is defined by Equation (8) and we emphasize that $\Phi(\cdot)$ is the standard normal density function (i.e., zero mean and unit variance) and $\Psi(\cdot)$ is the cumulative of the standard normal density function.

5.2. Expected EHR

Similar to ACL, this function is also discontinuous at $H = RL$ so

$$\mathrm{E(EHR)} = \int_{L=-\infty}^{L=\infty} \int_{H=-\infty}^{H=RL} (RL - H) f(H) f(L)\, dH\, dL.$$

Now add and subtract the complement to the inner integral and rearrange to get

$$\mathrm{E(EHR)} = \int_{L=-\infty}^{L=\infty} \left\{ \sigma_H G(Q) + \int_{H=-\infty}^{H=\infty} (RL - H) f(H)\, dH \right\} \cdot f(L)\, dL,$$

where Q is as previously defined in Equation (10). The inner integral simplifies to $[RL-\mu_H]$ and on subsequent integration over L this becomes $[R\mu_L-\mu_H]$. This should be in nondimensional form as well, so using earlier definitions, the result is

$$\mathrm{E(EHR)} = \mu_H k_H \int_{-\infty}^{\infty} G(Q)\Phi(z_L)\,dz_L + \mu_H(r-1). \quad (15)$$

The integrated form of Equation (15) becomes

$$\mathrm{E(EHR)} = \mu_H k_H \sqrt{1+a^2}G(c) + \mu_H(r-1). \quad (16)$$

5.3. Expected Total Cost and Optimal Harvest Rate

The integral form of the total cost (TCP) per ton of harvest can now be expressed as an expected value using Equations (11) and (15):

$$\begin{aligned}\mathrm{E(TCF)}/\mu_H &= C_H k_H \int_{-\infty}^{\infty} G(Q)\Phi(z_L)\,dz_L \\ &\quad + C_R k_H \int_{-\infty}^{\infty} G(Q)\Phi(z_L)\,dz_L \\ &\quad + C_R(r-1). \end{aligned} \quad (17)$$

The integrated form of Equation (17) is

$$\begin{aligned}\mathrm{E(TCF)}/\mu_H &= (C_R+C_H)k_H\sqrt{1+a^2}G(c) \\ &\quad + C_R(r-1). \end{aligned} \quad (18)$$

Next, an expression is obtained for the optimal dimensionless harvest rate. Taking the first derivative of Equation (17) with respect to r, setting the result to zero, and rearranging terms yields an implicit expression for the optimal dimensionless harvest rate (r^*):

$$\int_{-\infty}^{\infty} [1-\Psi(Q)](1+k_L z_L)\Phi(z_L)\,dz_L = C_R/(C_R+C_H). \quad (19)$$

Again using Silver's results, the integrated form of Equation (19) yields

$$\Psi(c) + k_L\big(a/\sqrt{1+a^2}\big)\Phi(c) = 1 - C_R/(C_R+C_H), \quad (20)$$

where a and c are functions of the decision variable r and the parameters k_H and k_L through Equation (12).

It deserves emphasis that r^* depends only on the nondimensional ratios k_H, k_L; and the cost ratio $C_R/(C_R+C_H)$. Furthermore, the second derivative of Equation (17) with respect to r is positive for all possible z values so that Equation (20) will yield the minimum r.

5.4. Expected Percentage Crop Recovery

Beyond r^*, the Harvest Model also permits calculation of the expected amount of crop recovery. As mentioned previously, the introduction of risk into planning means that the harvest will be less than 100%.

We define the expected percentage crop recovery by

$$\begin{aligned}\mathrm{E(PCR)} = [&\text{expected amount of crop harvested} \\ &/\text{mean harvest size}] \times 100.\end{aligned}$$

The expected proportion of ACL is $\mathrm{E(ACL)}/\mu_H$. Then

$$\mathrm{E(PCR)} = [1-\mathrm{E(ACL)}/\mu_H]\times 100. \quad (21)$$

5.5. An Alternative Approach to Harvest Risk Management

Some decision makers prefer a policy approach (as opposed to a cost-based method) for dealing with risk management. In an earlier study, Allen and Schuster (2000) examine a policy approach. The objective here is to explore the connections between that study and the cost-based analysis in the present study. In this alternative, instead of specifying cost penalties management specifies a policy for the probability of harvesting the entire crop. It is important to emphasize that this is not at all the same as specifying the proportion of crop harvested as was addressed in the previous section.

The probability of harvesting the entire crop is found from

$$\begin{aligned}&\text{Prob(complete crop harvest)} \\ &\quad \equiv \mathrm{P(CCH)} = \mathrm{P}(H\le RL) = 1-\mathrm{P}(H>RL)\end{aligned}$$

$$\mathrm{P(CCH)} = 1 - \int_{L=-\infty}^{L=\infty}\left\{\int_{H=RL}^{H=\infty} f(H)\,dH\right\} f(L)\,dL.$$

The inner integral is simply $[1-F(RL)]$, so again using the definition of Q in Equation (10) employed in our earlier work, we obtain

$$\mathrm{P(CCH)} = 1 - \int_{-\infty}^{\infty}[1-\Psi(Q)]\Phi(z_L)\,dz_L.$$

We will illustrate the use of this policy-based risk management method for the case of a specified $\mathrm{P(CCH)} = 0.85$ (85%) in §6.2. Integration of the above yields the following implicit equation for the dimensionless harvest rate, r:

$$\Psi(c) = \mathrm{P(CCH)} = 0.85. \quad (22)$$

6. Numerical Results

This section draws from real data on the harvest of Concord grapes documented by Allen and Schuster (2000) as a basis for examples and analysis. Their work deals specifically with the decision-making process involving the harvest of Concord grapes for Welch Foods. The authors were directly involved in planning and managing harvest operations for a period of 10 years.

6.1. Data

The model requires two sets of historical data: H (tons) and L (days). Both H and L vary from year to year. In each growing area, the state agricultural experiment station keeps detailed records by year on yields and crop size. Historical records also show the harvest start date (when the crop is at the correct level of maturity) and the first 28°F day (hard frost) that signifies the end of the harvest. The value of L for each harvest can be calculated for a growing region by counting the days between the start and end dates. Similar to the crop size, L varies from year to year.

Besides H and L, the Harvest Model requires two costs, C_H and C_R. The calculation of C_H is straightforward; if a grower does not harvest a ton of grapes, then the relative value of the loss equals the open market price. This is a conservative approach to valuation. The examples presented assume $C_H =$ *$250 per ton of grapes*. This is a long-term estimate of the open market price. The market price of grapes will change a considerable amount on a year-to-year basis depending on industry crop size, carryover from the previous year, consumer demand, and the pricing of alternative raw ingredients.

The calculation of C_R is a more complex matter. Harvesting at an excessive rate means that the entire harvest will take place well in advance of a hard frost. If this happens, then R is too high and the long-term investment in capital is excessive. In this situation C_R relates to the amortized cost of the unneeded capacity. It is unique to each type of business organization.

Estimating C_R is difficult because it involves capital equipment that is added in steps. There is no question that the cost is nonlinear over large ranges of R. However, for small changes of R the cost becomes linear. An estimate can be calculated on a cost-per-ton basis by analyzing small increases of R, e.g., an increase of 10% or less and the corresponding capital required to support the higher pressing rate. This was the procedure used at Welch Foods.

For example, at one grape processing plant located in Pennsylvania, increasing R by 180 tons per day requires a capital investment of $1.5 million. During the course of an average harvest, this increased rate means that 5,400 extra tons of grapes can be pressed before a hard frost. Assuming a 10 year life span, the estimated amortized cost of this extra capacity is $150,000 per year. If part or all of this extra capacity is not needed, then C_R becomes the cost of the incremental capacity divided by the additional tons harvested at the higher rate ($150,000 per 5,400 tons), or about *$30 per ton of grapes*. This represents the estimated cost of incremental capacity occurring around the point in time when a hard frost might occur.

6.2. Results

We now provide numerical results from the Harvest Model for the following practical problems:

- calculation of r^* under different scenarios,
- determination of the percent recovery for a specific r^*, and
- finding r under conditions of an imposed harvest policy.

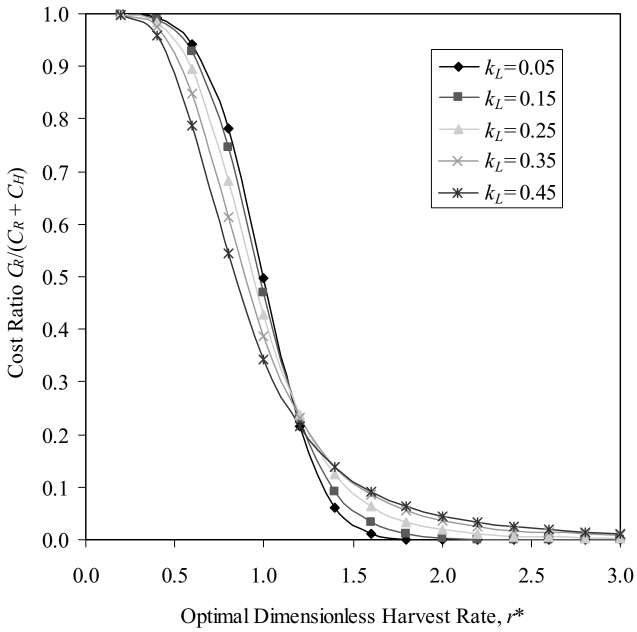
Using the historical data for H and L along with C_H and C_R as inputs, the Harvest Model calculates r^* providing the opportunity to analyze a number of harvest risk scenarios.

Table 1 gives results for r^* of various combinations of k_H and k_L for a cost ratio, $C_R/(C_R + C_H)$, of 0.10 using Equation (20). These were obtained by using the Goal Seek utility in Excel with a tolerance of 0.0001 for all calculations of r. The data show that r^* increases with increases in variability for H and L. This is a consistent result. The greater the risk, the greater the r^*.

Table 1 **Optimal r for $C_R/(C_R + C_H) = 0.1$**

		k_H				
		0.05	0.1	0.15	0.2	0.25
k_L	0.25	1.3350	1.3564	1.3828	1.4195	1.4620
	0.30	1.3880	1.4019	1.4291	1.4618	1.5004
	0.35	1.4306	1.4449	1.4662	1.4954	1.5305
	0.40	1.4586	1.4732	1.4879	1.5167	1.5489
	0.45	1.4713	1.4860	1.5008	1.5239	1.5532

Figure 1 Cost Ratio vs. r^* for $k_H = 0.25$



Additional descriptive results can be observed by evaluating the left-hand side of Equation (20) over ranges of r^* for representative values of k_H and k_L. The resulting values correspond to the cost ratios at optimality. The set of graphs in Figure 1 are for a constant k_H of 0.25 and a range of k_L values. An estimate of r^* can be made directly from the graph without invoking a search engine by specifying the cost ratio and the coefficient of variation for the length of harvest season. This provides planners with a visualization of the relationship between the cost ratio and r^*.

Computations of expected percentage of crop recovery using Equations (14) and (21) for the r^* of Table 1 (for $C_R/(C_R + C_H) = 0.10$), appear in Table 2 below.

Increasing variability in crop size and length of season means lower crop recovery. This reflects the trade-off between the cost of unharvested crop and the cost of an EHR. Table 2 is an important tool to explain why 100% recovery of a crop is not optimal given the trade-off between crop loss and a consistent early completion of harvest operations.

The total cost per ton for each value of r^* can be calculated using Equation (18). These results appear in Table 3. As is the case with r^*, the data show that cost per ton increases with increases in variability for H and L.

Finally, Goal Seek can be used to find r for specified values of k_H and k_L under an imposed policy for crop recovery, Equation (22). Decision makers often use this approach to demonstrate how r changes for policy alternatives (suboptimal values of r) approaching a complete harvest. Since a policy alternative might not be optimal, it is important to understand the cost difference between the optimal solution and the chosen policy. In some cases, choosing an r that is not optimal is justified based on the timing of capacity additions in relation to legacy equipment, an established consensus by growers who are risk adverse, and the uncertainty surrounding cost data or a shift in historical trends. By substituting the values of r into Equation (18) total costs can be computed for each imposed policy alternative. These results appear in Tables 4 and 5. The imposed policy for this example is "Harvest 100% of the crop 85% of the time."

Table 2 E(PCR) in %

		k_H				
		0.05	0.1	0.15	0.2	0.25
k_L	0.25	97.16	97.08	96.95	96.79	96.61
	0.30	95.96	95.91	95.80	95.65	95.48
	0.35	94.52	94.46	94.35	94.21	94.04
	0.40	92.75	92.71	92.56	92.44	92.27
	0.45	90.67	90.63	90.49	90.35	90.18

Table 3 Total Cost per Harvest Ton at Optimal r

		k_H				
		0.05	0.1	0.15	0.2	0.25
k_L	0.25	17.31	18.04	19.19	20.66	22.37
	0.30	22.04	22.68	23.69	25.01	26.59
	0.35	27.28	27.85	28.76	29.97	31.44
	0.40	33.00	33.52	34.36	35.48	36.86
	0.45	39.13	39.62	40.41	41.47	42.78

Table 4 Policy r for 85%

		k_H				
		0.05	0.1	0.15	0.2	0.25
k_L	0.25	1.3547	1.3682	1.3927	1.4225	1.4573
	0.30	1.4570	1.4684	1.4883	1.5144	1.5458
	0.35	1.5729	1.5842	1.6001	1.6249	1.6533
	0.40	1.7094	1.7265	1.7371	1.7545	1.7837
	0.45	1.8716	1.8903	1.9002	1.9192	1.9384

Table 5 **Total Cost in US$ per Harvest Ton for Policy** r

		k_H				
		0.05	0.1	0.15	0.2	0.25
k_L	0.25	17.33	18.05	19.19	20.66	22.38
	0.30	22.26	22.86	23.84	25.13	26.67
	0.35	27.98	28.52	29.36	30.51	31.91
	0.40	34.72	35.28	35.98	36.96	38.27
	0.45	42.75	43.32	43.94	44.87	45.96

Table 6 shows the cost penalties for the 85% policy as compared to r^* for each case of k_L and k_H. The cost penalty for the 85% policy can be computed using the following formula:

$$\text{percent cost penalty} = (\text{policy cost–optimal cost}) \cdot 100/\text{optimal cost}.$$

At the extreme ($k_L = 0.45$), the maximum cost penalty for the 85% policy is about 9.2%. For lower values of k_L, the penalty is less than 3%. It is interesting to note that in all cases for a given k_L the cost penalty decreases as k_H increases. The greater variability in crop size has the impact of narrowing the cost difference between r, at 85% policy, and r^*. Since in all cases the penalties are modest, we conclude that the 85% policy gives an r that is close to optimum in terms of cost (for the set of parameters examined).

The Harvest Model also provides results based on various policy alternatives that are close to those obtained in the study by Allen and Schuster (2000) with some small differences in the second decimal place for the lower values of k_H. The more accurate values are probably those of the prior study that did not rely on Goal Seek. This validates the Harvest Model. The above results are not optimum from the point of view of minimizing costs but rather are the result of an imposed policy. This is an effective way of communicating harvest risk to growers investing in receiving plants through agricultural cooperatives.

Table 6 **Cost Penalty for Policy** r **in %**

		k_H				
		0.05	0.1	0.15	0.2	0.25
k_L	0.25	0.14	0.07	0.03	0.00	0.00
	0.30	0.96	0.81	0.63	0.46	0.31
	0.35	2.56	2.39	2.07	1.81	1.50
	0.40	5.22	5.25	4.72	4.17	3.82
	0.45	9.23	9.34	8.74	8.19	7.43

The Harvest Model gives consistent answers to a number of important questions involving risk. Based upon considerable use in practice, the Harvest Model is suitable as a general solution for problems encountered in agriculture where a sudden event, such as a fall frost, concludes the harvest season.

7. Managerial Insights

The true test of any model is performance in practice. The Harvest Model consistently provides reasonable results in addition to new insights. As an example, the traditional belief that the Concord harvest season should last 30 days to ensure complete harvest of the crop proves inaccurate when analyzed using the Harvest Model. Regional climatic variation plays a significant role in the probability of a frost. A large body of water, such as Lake Erie, acts as a heat sink in the fall, thereby extending the probable length of the harvest season for growers in close proximity. Analysis using the Harvest Model shows that extending seasonal operations beyond 30 days in parts of the Lake Erie region can be accomplished with low risk of crop loss. This translates into a slower harvest rate and reduced capital investment for cooperatives in this growing area.

However, similar analysis for growers in Michigan, who are located a considerable distance from any large body of water, shows that seasonal operations should be less than 30 days because of significant frost risk. In this growing region a small probability exists that a frost will occur before the beginning of the harvest. Given this case, the harvest rate must increase, which requires more capital investment.

The authors have experience implementing results of the Harvest Model, using normal probability distributions to express harvest size and length of season, for growing areas near Lakes Erie and Michigan. In total, capital avoidance equaled $2 million through establishing the proper harvest rate by region. This is one example of the benefits of controlling harvest risk, and of the adoption of precision agriculture in practice.

Beyond savings in capital, the Harvest Model provides other organizational benefits. In the case of Welch Foods, the Harvest Model is an important tool for corporate planning with results of the model being presented at meetings of the board of directors.

Welch Foods is owned by the National Grape Cooperative and is an example of vertical integration in agriculture. The company's span of control ranges from raw grapes to end-product marketing. A critical measure of effectiveness is the capability of receiving the grower's grapes with little risk of loss. By using the Harvest Model, management can communicate to growers—the owners of the cooperative—exact levels of risk associated with harvesting the crop. Shifting dialogue to a fact-based discussion offers great advantages in reaching a consensus concerning the proper investment in capital for pressing operations.

In addition, the Harvest Model provides a structure to evaluate the impact on picking and pressing operations from additions of vineyards to a cooperative. As product lines expand and sales grow, the need for Concord grapes also increases, causing additional burden in harvesting. The decision to add incremental acreage to the cooperative is influenced by the capability to harvest the crop at acceptable levels of risk and capital investment. The Harvest Model provides an effective structure to evaluate the change in risk when additional vineyards are planted. Since risk levels vary by growing region, the proper decision can be made concerning the correct area to plant additional grapes. This is a safeguard against expansion in high-risk growing regions where the probability of a killer frost is high. Over the long run, this type of prudent, risk-based decision making will result in an increased level of reliability for raw material supply.

8. Conclusion

We live in a world filled with quantifiable risk, yet there are few mathematical frameworks available to guide practitioners who must deal with agricultural harvest risk. Based on an extensive literature search, we conclude that harvest risk is an undeveloped area within operations management. Controlling risk by the application of rational models presents many opportunities to increase profits.

In this article, we develop a robust mathematical model to optimize the risk of an agricultural harvest. We test the model with representative data from the harvest of Concord grapes and find it gives acceptable solutions. During the course of our work we also find the model applies to other risk-bearing tasks in agriculture, such as the harvest of oranges, and in industries such as project management and style goods planning. We plan further research in this area.

Acknowledgments

The authors wish to thank the editor, senior editor, and two anonymous reviewers for the constructive comments provided during the review process.

References

Allen, S. J., E. W. Schuster. 1994. Practical production scheduling with capacity constraints and dynamic demand: Family planning and disaggregation. *Production Inventory Management J.* **35**(4) 15–21.

Allen, S. J., E. W. Schuster. 2000. Managing the risk for the grape harvest at Welch's. *Production Inventory Management J.* **41**(3) 31–36.

Allen, S. J., J. Martin, E. W. Schuster. 1997. A simple method for the multi-item, single-level, capacitated scheduling problem with set-up times and costs. *Production Inventory Management J.* **38**(4) 39–47.

Anderson, J. R., J. L. Dillon, B. Hardaker. 1977. *Agricultural Decision Analysis*. Iowa State University Press, Ames, IA.

D'Itri, M. P., S. J. Allen, E. W. Schuster. 1998. Capacitated scheduling of multiple products on a single processor with sequence dependencies. *Production Inventory Management J.* **40**(5) 27–33.

Eppen, G. D., A. V. Iyer. 1997. Improved fashion buying with Bayesian updates. *Oper. Res.* **45**(6) 805–819.

Fine, C. H. 1998. *Clockspeed*. Perseus Books, Reading, MA.

Fisher, M., A. Raman. 1996. Reducing the cost of demand uncertainty through accurate response to early sales. *Oper. Res.* **44**(1) 87–99.

Garraty, J. A., P. Gay. 1972. *Columbia History of the World*. Harper & Row Publishers, New York.

Hazell, P. B. R., R. D. Norton. 1986. *Mathematical Programming for Economic Analysis*. MacMillan Publishing Co., New York.

Jones, P. C., G. Kegler, T. L. Lowe, R. D. Traub. 2003. Managing the seed corn supply chain at Syngenta. *Interfaces* **33**(1) 80–90.

Jones, P. C., T. L. Lowe, R. D. Traub, G. Kegler. 2001. Matching supply and demand: The value of a second chance in producing hybrid seed corn. *Manufacturing Service Oper. Management* **3**(2) 122–137.

Kinsey, J. D. 2001. The new food economy: Consumers, farms, pharms and science. *Amer. J. Agricultural Econom.* **83**(5) 1113–1130.

Kreiner, H. W. 1994. Operations research in agriculture: Thornthwaite's classic revisited. *Oper. Res.* **42**(6) 987–997.

Maatman, A., C. Schweigman, A. Ruijs, M. H. van Der Vlerk. 2002. Modeling farmers' response to uncertain rainfall in Burkina Faso: A stochastic programming approach. *Oper. Res.* **50**(3) 399–414.

Masters, J. M. 1987. Analysis of the life-of–type buy decision. *J. Bus. Logist.* **8**(2) 40–56.

Murry, G. R., E. A. Silver. 1966. A Bayesian analysis of the style goods inventory problem. *Management Sci.* **12** 785–797.

Nahmias, S. 1997. *Production and Operations Analysis*. Irwin, Chicago, IL.

Porteus, E. L. 1993a. Case analysis: Analysis of the National Cranberry Cooperative-1. Tactical options. *Interfaces* **23**(4) 21–39.

Porteus, E. L. 1993b. Case analysis of the National Cranberry Cooperative-2. Environmental changes and implementation. *Interfaces* **23**(6) 81–92.

Schuster, E. W., S. J. Allen. 1998. Raw material management at Welch's. *Interfaces* **28**(5) 13–24.

Schuster, E. W., B. J. Finch. 1990. A deterministic spreadsheet simulation model for production scheduling in a lumpy demand environment. *Production Inventory Management J.* **31**(1) 39–42.

Schuster, E. W., S. J. Allen, M. P. D'Itri. 2000. Capacitated materials requirements planning and its application in the process industries. *J. Bus. Logist.* **21**(1) 169–189.

Silver, E. A., D. J. Smith. 1981. Setting individual item production rates under significant lead time conditions. *INFOR* **19**(1) 1–19.

Song, J. S., C. Yano, P. Lerssrisuriya. 2000. Contract assembly: Dealing with combined supply lead time and demand quantity uncertainty. *Manufacturing Service Oper. Management* **2**(3) 287–296.

Thornthwaite, C. W. 1953. Operations research in agriculture. *JORSA* **1**(1) 33–38.

Umble, M. M. 1992. Analyzing manufacturing problems using V-A-T analysis. *Production Inventory Management J.* **33**(2) 35–60.