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**DISPERSED EMPLOYMENT,
COMMUTING AND MIXED
LAND-USE IN MODERN CITIES**

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Commuting, Ricardian Rent and Housing Appreciation in Cities with Dispersed Employment and Mixed Land-Use

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I. Introduction

Since the 19th century, the dominant view of urban land use has been based on the Ricardian rent, monocentric city model. In this model, transportation frictions for commuting or commerce generate a rent “gradient” between a city center and urban periphery. This entire gradient increases as cities grow in population because travel distances expand. Implicitly, this model is behind the widespread popular belief that urban housing prices (and rents) grow over time significantly faster than inflation – increased location “scarcity” makes real estate assets a “good” investment.

This paper has two objectives. The first is to review a range of empirical evidence that suggests this vision of urban form is largely incorrect. Over time, in most cities, real estate rents and asset prices grow little more than inflation. In addition, recently released data on the spatial distribution of jobs (by place of work) suggest that firms and residences are remarkably well interspersed. The facts simply do not fit a “monocentric” model – nor do they support its comparative statics with respect to growth. The second objective is to develop a very simple model of urban spatial structure that does fit these facts. The model makes many simplifying assumptions, and has few embellishments, but it can generate city forms that are consistent with the data – and which do yield little or no growth in real estate values as cities expand in population over the long run. This model has five key features.

- Mixing or interspersed land use occurs if land is allocated to residences or commerce in proportion to their rent rather than to the strictly highest rent.
- The rent for each use, and thus the degree of mixing depends on the level of some economic agglomeration factor - as opposed to the ease of travel.
- Travel patterns are determined with mixed land use, and Solow type congestion is introduced. Congestion is highest with fully centralized employment and becomes negligible as commerce is fully dispersed and evenly mixed with residences.
- In a city with highly dispersed employment there turns out to be little or no Ricardian rent “gradient”, in contrast to a city with centralized employment.
- As a city with dispersed-employment increases in population and grows spatially, there is less, and in some cases no *increase* in Ricardian rent, again in sharp contrast to a city with centralized employment.

The paper is organized as follows. The next section reviews a range of empirical evidence about both the growth in real estate values, and the spatial distribution of jobs and people within cities. The following section provides an example of how dispersed employment and mixed land use can generate congested traffic flows between firms and residences. In the fourth section, these travel costs are used to generate equilibrium Ricardian rent and wage gradients for both firms and households. In section five, the model is closed, by incorporating an “idiosyncratic” auction function that allocates land use based on the relative magnitude of Ricardian rents from the two uses. The sixth through eighth sections of the paper show how the degree of employment dispersal varies with the level of urban agglomeration, the level of transportation infrastructure, and population growth. Section nine then compares how land rents change with population

growth in cities with fully dispersed employment, as opposed to traditional monocentric cities. The final section concludes with a long list of suggested future research.

II. Empirical Evidence.

The first evidence that real estate values might not consistently rise in real terms comes from the unique Eicholtz [1999] repeat-sale price index for housing in the city of Amsterdam: from 1628 through 1975. The deflated series in Figure 1 shows no obvious trend, and statistical tests confirm its stationarity¹. Thus, during almost 350 years, the greater Amsterdam area has grown in population and jobs by 1200% without having house prices rise faster than inflation.

Figures 2 and 3 show the longest standing US series on residential markets –apartment rents from the US Commerce Department’s CPI repeat sample survey. In Figure 2, the indices for five “older” cities are shown, while in Figure 3, the indices for three “newer” cities are displayed (in constant dollars). None seems to have an upward trend, and the series for the “newer” cities actually have significant downward trends. If the series are examined over just the post-war period (1946-1999), the same statistical conclusions hold². Over these 81 years, the cities in this sample had population growth (in their metropolitan areas) of between 110% (New York) to 2100% (Houston).

The final data examined is the much more recent OFHEO repeat sales price index for US single-family housing. Here again, Figure 4 shows deflated prices for five older cities, while Figure 5 does so for 5 newer areas: from 1975 to the present. In four of the five older city series, there does appear to be an upward trend in prices, but in the newer cities there is none. In fact, this is validated statistically, although the time series behavior of the two groups of cities is quite different³. It is important to see that the rent series in Figure 3 show a similar upward trend as do prices – if examined from 1975 – an artifact largely of the short time frame.

In recent years, there has been an unrelated, but growing empirical literature that suggests employment is far more dispersed than previously thought. Mills [1972] estimated employment as well as population density gradients, and claimed that jobs

¹ To test stationarity first requires identifying the underlying behavior of a series. To do this, the following model is estimated: $\Delta y_t = \alpha_0 + \alpha_1 \Delta y_{t-1} + \beta y_t + \delta t + \varepsilon_t$. An augmented “F” statistic is used to determine if β and δ are collectively significant. If so, then the “t” statistic of δ ascertains whether the series is stationary. If a random walk model cannot be rejected, then the “t” statistic on α_0 ascertains whether there is significant “drift”. For the Dutch series, the “F” test passes the 5% level using the augmented Dickey-Fuller criteria, but the trend has a “t” statistic of only .6.

² The same “F” statistic is generally less conclusive about whether the CPI data is a random walk or mean reversionary. The addition of a mean reverting trend adds explanatory power in all of the series, but often only at 10-20% significance. In older cities, the trend “t” statistics range from 1.6 to -1.4. In the new cities they range from -1.2 to -3.2. In all cases, the value of α_0 is not significant from zero

³ One must place limited credence in any time series tests with only 25 observations. Still, in all 5 older city series mean reversion is statistically significant, and in four of five the trends are significant at the 5% level. In the new cities, the random walk model cannot be rejected and there is no significant drift. If a mean reversion model is *imposed* the trends also are completely insignificant in all cases.

were indeed much more dispersed than a monocentric model assumes. More recent research continues to show that employment in most metropolitan areas of the world, is both dispersing and in some cases clustering into subcenters [Guiliano and Small, 1991; McMillen-Macdonald, 1998]. Finally, there is equally voluminous evidence of the spatial variation in wage rates, which is necessary to accommodate job dispersal [Eberts, 1981; Madden, 1985; Ihlanfeldt, 1992; McMillen-Singell, 1998; Timothy-Wheaton, 2001].

A more detailed analysis of job dispersal has recently become possible with the release of Commerce Department information on employment at place of work – by Zip Code. As reported in Glaeser-Kahn[2001, with zip code level data, it is possible to construct the cumulative spatial distribution of employment as well as population – from the “center” of each MSA outward. This is shown in Figures 6 and 7, for the two areas in the US that are held up as representing “older” as opposed to “newer” cities: the New York and Los Angeles CMSAs. While employment is slightly more concentrated than population in New York, the closeness of the two is unexpected – as is the remarkable similarity to Los Angeles – where the two distributions are virtually identical.

It is easy to construct a simple measure of how “concentrated” the spatial distributions are in Figures 6 and 7. One takes the integral of the distribution – up to (say) the 95% point and then divides by the distance at that point. Cities that are “fully” concentrated at single central point have a value of unity, while cities with an even distribution have a value of .5. A city with virtually all employment or population at its “edge” would have a value that approaches zero. In Figure 8, the value of this measure for both population and employment is shown for 120 MSAs (or CMSAs). Across these areas, the measure of population concentration varies from about .5 to .75 depending on how dense and spread out population is. What is remarkable is the extremely close correlation between the two measures ($R^2 = .71$). Cities in which population is spread out – jobs are as well – and vice versa. A true “monocentric” city would have an employment value of something like .8 and a population value of less than .5. There are no cities like this.

What explains the remarkable degree of intermixed land use in US cities? In the following sections, a model is developed which brings together a number of previous ideas in the literature. Following White [1976, 1988], and Ogawa and Fujita [1980], the model motivates employers to seek proximity to residences in order to lower the commuting and hence wages that they pay workers. From Helseley-Sullivan [1991], and Anas-Kim [1996], the model introduces a spatial agglomeration factor for firms, which provides some rationale for centralization. Finally, Solow-type [1973] transportation congestion is introduced for the first time in such models. What makes the synthesis of these ideas analytically tractable is the introduction of “mixed” land use – firms and residences can “jointly” occupy land at the same site – depending on the relative magnitude of their land rents.

III. Commuting with Dispersed Employment and Mixed Land Use

Like a traditional monocentric city, the city modeled here will be circular and have a transportation technology that only allows inward radial commuting. The only distinguishing feature of land is thus its distance (t) from the geographic center of the city. Land that is not vacant is occupied either by household residences or by firms employing workers. At each location (t), the fraction of land used by firms is $F(t)$, and that by households is $1-F(t)$. The function $F(t)$ will be discussed later. For now, it is simply specified as a mapping from t into the zero-one interval.

A simplification in the current model will be the assumption that individuals consume a fixed amount of land space, both at their place of employment (q_f) as well as at the location of their residence (q_h). In most modern cities, workplace land consumption is far smaller than residential ($q_f < q_h$), but this is only an observation and is not necessary for the model at hand. Allowing density to be endogenous complicates the model, but probably does not change the qualitative conclusions derived here. In any case, it will be one of many suggested future extensions.

With fixed density at both residence and workplace, $e(t)$ and $h(t)$ are defined as the cumulative number of workers or households who live *up to the distance* t . Using the $F(-)$ function, these are:

$$e(t) = \int_0^t 2\pi x F(x) / q_f dx \quad (1)$$

$$h(t) = \int_0^t 2\pi x [1-F(x)] / q_h dx \quad (2)$$

The spatial distribution of employment and households will have an outer “edge” or limit. No jobs exist beyond the distance b_f and no households beyond the distance b_h . The following boundary conditions on $e(t)$ and $h(t)$ are obvious, where the equality between total jobs and households is made only for convenience.

$$e(b_f) = E = H = h(b_h) \quad (3)$$

$$e(0) = 0 = h(0)$$

The assumption that the *transportation technology allows only inward radial commuting* results in a very simple and easy characterization of traffic flows. The number of commuters passing each distance in the city is equal to the difference between the cumulative number of workers employed up to that point and the cumulative number of residents living up to that same distance. Since reverse commuting is not allowed, it also follows that workplaces must not be “more dispersed” than residences. Thus the demand (flow) of inward-only commuters $D(t)$, under this assumption is:

$$D(t) = e(t) - h(t) \geq 0 \quad (4)$$

$$D(b_h) = D(0) = 0$$

The result that jobs must be more centralized than residences, and that no reverse commuting is possible also insures the following condition:

$$b_f \leq b_h \quad (5)$$

These assumptions generate a resulting pattern of travel demand that has several interesting features. First off, it is clear that there exists a land use pattern (a particular function $F(t)$) for which there is *no commuting*. If at all distances, t , $e(t)=h(t)$, then travel demand is everywhere zero. Clearly this is the case if the fraction of land used by firms is always equal to their “share” of overall land demand:

$$F(t) = q_f / (q_f + q_h), \text{ implies } e(t) = h(t) \text{ for all } t, \text{ and } b_f = b_h. \quad (6)$$

A second observation is that when the land use pattern $F(t)$ does generate travel demand, then travel demand is at a (local) maximum or minimum at that distance where the aggregate land demand of each use is equal:

$$D(t) \text{ reaches a local maximum where } [1-F(t)] q_h = F(t) q_f \quad (7)$$

As will be shown later, maximum travel demand tends to occur at or around the edge of employment b_f , regardless of how dispersed that employment is. At this point in the city, the number of residents traveling inward is very high, since no one has yet to be “dropped off” at work.⁴

Finally, travel demand must be converted into travel costs. Here an amended version of the Solow [1973] congestion function is used. It’s assumed that the marginal cost of travel has a fixed component (β) as well as congestion which in turn is proportional to demand and inverse to capacity, $S(t)$.⁵

$$\partial C(t) / \partial t = C'(t) = D(t) / S(t) + \beta \quad (8)$$

Again for the sake of simplicity, this model assumes that transportation capacity can be provided without using up land and $S(t)$ reflects only road “capital”. This keeps the number of urban land uses to just two (employment, residence).

⁴ In conversation, William Vickery once observed that congestion was much worse around the edges of Manhattan than at its center.

⁵ None of the results here are based on congestion depending upon the ratio of demand to capacity. This is used only for convenience in the simulation results. Actual studies [Small, 1992] suggest that the ratio assumption is reasonable, but with a coefficient that is greater than one.

IV. Wage and Rent Gradients with Agglomeration and Dispersed Employment

With land consumption fixed, and assuming that travel “costs” incorporate a fixed value of time, household utility depends only on net income after receiving wages, paying for travel costs, and consuming land (paying rent). Household equilibrium thus requires that net income be constant across locations. Of course in this model, households make two location decisions: where to live and where to work. There thus must exist two “price” gradients to make households indifferent about each decision.

For households at a fixed place of residence, all alternative workplaces must yield them identical net income. Since rent is fixed by residence place, choice of workplace impacts net income through commuting. For indifference, a wage gradient $W(t)$ must exist and vary directly with the marginal cost of travel:

$$\partial W(t)/\partial t = W'(t) = -C'(t) \quad , \quad W(0) = W_0 \quad (9)$$

For households at a fixed place of employment, all alternative residential choices must be equally attractive. This requires a land rent gradient $R_h(t)$ that varies with the marginal cost of travel and everywhere is greater than the reservation rent for land (A), except at the edge of residential development:

$$\partial R_h(t)/\partial t = R'_h(t) = -C'(t)/q_h \quad , \quad R_h(b_h) = A \quad (10)$$

To identify the location preferences of firms, the model needs some additional cost of production – that varies with location – besides that of labor and land. The intent of this model is not to delve deeply into the issue of spatial agglomeration; this again is reserved for model extensions. Fujita [1980], and Anas, et.al [1998], for example, have modeled intra-urban agglomeration as the aggregation of distances between businesses. In a circular city, such as this, such a measure yields the greatest costs at the center and the least at the edge. The objective here, however, is only to examine the impact of such agglomeration and not its generation. Thus the current model will simply have an output/worker $Q(t)$ gradient, which declines with distance from the urban center. Greater agglomeration implies a steeper gradient. Making this gradient exogenous again provides great analytic simplicity.

The assumption that worker productivity declines with distance ($Q'(t)<0$) simply provides some justification for firms to locate centrally. Assuming that firm technology does not allow substitution, firm profits equal output per worker minus wages and land costs/worker. For firms to be equally profitable at all locations it must be true that worker productivity, wages and firm land rents all exactly offset each other. This yields a firm land rent gradient $R_f(t)$ that obeys the following conditions:

$$\begin{aligned} \partial R_f(t)/\partial t = R'_f(t) &= [Q'(t) - W'(t)]/q_f & (11) \\ &= [Q'(t) + C'(t)]/q_f \quad , \quad \text{from (9)} \quad , \quad R_f(b_f) = A \end{aligned}$$

Thus if the decline in productivity is larger than marginal travel costs, firm rent gradients decrease with distance. From (4), this is most likely to occur at the very center ($t=0$) and at the residential edge ($t=b_h$). On the other hand, if marginal travel costs are larger than the decline in productivity, then firm rent gradients might increase with distance. Again from (7) this is most likely at or near b_f .

Thus, if equations (9) through (11) hold both firms and households can be in a locational equilibrium. The rent and wage gradients that insure this, however, depend crucially on congestion, which in turn, depend on the travel flows that result from the pattern of land use mixing.

V. Rent Gradients and “Mixed” Land Use

In the traditional theory of competitive land markets, land use at each location deterministically is based on which use offers the highest Ricardian rent. By definition this precludes land use mixing, except possibly in the case where the rent from two uses is identical. Even in the case where rents are “tied”, the exact fraction of land that is assigned to each use is indeterminant. As a result, traditional theory tends to create land use patterns in which there are exclusive zones, rings or areas for each use. Here we adopt a new convention of assuming that there are random or idiosyncratic effects that make $F(t)$ vary continuously over the 0-1 interval - depending on the relative magnitude of the rent levels for each use.⁶

There are a number of functions that one could use to effectively model “idiosyncratic land use competition”, including for example logistic choice. Here, two more simple forms are illustrated, again for analytic ease. The only requirement is that each map positive rents into the zero-one interval. The first, in (12) assumes that with equal Ricardian rents, the two uses get equal (50%) land use assignment. In (12’), it is assumed that with equal Ricardian rent, each use gets assigned a share that is proportional to their land use consumption levels (q_f, q_h).

$$\text{Equal rents imply equal shares: } F(t) = R_f(t) / [R_f(t) + R_h(t)] \quad (12)$$

$$\begin{aligned} &\text{Equal rents imply shares} \\ &\text{proportional to land consumption: } F(t) = q_f R_f(t) / [q_f R_f(t) + q_h R_h(t)] \quad (12') \end{aligned}$$

In addition, for any of these functions, it is necessary to assume that there is a reservation (e.g. agricultural) use to land that has a uniform rent of A , and that if $R_f(t) \leq A$ then $F(t) = 0$.

⁶ One might imagine that other location considerations, or truly random effects generate stochastic Ricardian Rents. In this case, each use would have a probabilistic rent distribution for any location and application of the highest use principle would yield probabilistically mixed land use.

With the specification of the $F(t)$ function, the model is “closed”. An equilibrium can be imagined with the following mappings. Equations (1)-(2) take the $F(t)$ function and determine $[e(t), h(t)]$. Then equations (4),(8),(9),(10),(11) take $[e(t),h(t)]$ and determine $[D, C, W, R_f, R_h]$. Finally, one of the equations (12) maps $[R_f, R_h]$ back into $F(t)$.

While it is quite straightforward to find reasonable equilibrium solutions, it may not always be the case that a fixed point exists, at least without further restrictions than those above. A particular troublesome problem can arise for seemingly reasonable representations of $C(t)$ and $Q(t)$. At points of maximum congestion it is possible that $C' > Q'$ and firm bid rents will slope up with greater distance. Since according to (7) this is likely to happen near to the edge of employment, it can become difficult to find a solution to b_f .

While it is difficult to prove that an equilibrium always exists with mixed land use, it is quite easy to find particular equilibria, and to demonstrate that these require certain conditions. As a first step, it is useful to examine the range of land use patterns that are possible with the equations from the previous sections. Two patterns are possible: mixed use (over a range), and fully dispersed employment. In the first, firms are more centralized than households, but there is a range of locations where both uses exist. In the latter, firms and households are perfectly intermingled. Each of these outcomes can be generated with particular combinations of urban productivity, transport capacity and population size.

VI. The Dispersal of Employment with Agglomeration.

When an idiosyncratic land use function is used, such as (12) or (12'), then some degree of land use mixing occurs almost by definition. The degree of employment dispersal, and hence land use mixing, however, will depend heavily on the level of agglomeration or on how rapidly productivity declines with distance.

Proposition 1. Lower Agglomeration generates greater employment dispersal in mixed use cities.

[By contradiction]. In a city that has a mixed land use pattern, $b_f < b_h$, and $F(t) < 1$, for $t < b_f$. With less agglomeration, $Q'(t)$ decreases at all locations. If greater employment centralization were to result, then b_f would have to be smaller, and $F(t)$ higher over the interval $0-b_f$. From (1)-(8), however, this would cause C' to be greater over this same interval. From (10) and (11) lower Q' and higher C' when combined with a smaller b_f will reduce commercial rents $R_f(t)$, and increase residential rents $R_h(t)$ over this interval. With either (12) or (12') this will reduce rather than increase $F(t)$ over $t=0-b_f$. Through the reverse argument, the converse holds as well - greater agglomeration cannot lead to employment dispersal.

At the extreme case, fully dispersed employment requires that agglomeration must either be non-existent, or else be such as to exactly offset exogenous congestion costs.

Proposition 2. A fully dispersed employment equilibrium is possible with (12) and (12').

Full employment dispersal requires that $F(t) = q_f / (q_f + q_h)$ and $e(t) = h(t)$ over all t . Hence $b_f = b_h$, $D(t) = 0$, and $C'(t) = \beta$ (no congestion, only exogenous travel costs).

Example #1: If $\beta = \lambda q_h^2$, and $Q' = -\lambda [q_f^2 + q_h^2]$, where λ is any positive scalar, then $W'(t) = -\lambda q_h^2$, $R'_h = -\lambda q_h$, and $R'_f = -\lambda q_f$. Furthermore, it is necessary for $A = 0$, so that rent levels become proportional to their slopes. Then using (12), if $F(t) = R_f(t) / [R_f(t) + R_h(t)]$, it is also true that $F(t) = q_f / (q_f + q_h)$.

Example #2: If $\beta = 0$, and $Q' = 0$, then, $W'(t) = 0$, $R'_h = 0$, and $R'_f = 0$. Furthermore, it is necessary for $A > 0$. Thus both rent gradients are flat and equal to the positive value: A . In this case, and using (12'), if $F(t) = q_f R_f(t) / [q_f R_f(t) + q_h R_h(t)]$, then $F(t) = q_f / (q_f + q_h)$.

In the first example, the assumptions that $C' = \lambda q_h^2$ and $Q' = -\lambda [q_f^2 + q_h^2]$ may seem somewhat contrived. In actuality, the value of λ can vary between cities of different size and with different transportation systems. The critical assumption is that for any given city, the ratio C'/Q' is equal to the residential share of squared land consumptions across the two uses. This case is interesting in that even with no commuting and congestion, if there exists exogenous travel costs, then some rent and wage gradient based upon these is “necessary” to insure that employment remains dispersed.

Example #2 provides a result that quite consistent with many more recent empirical studies (e.g. Waddell [1993]), which show that in newer or more dispersed cities, there is little or no rent gradient.

Figures 9-10 illustrate a mixed land use city that is extremely close to looking like a traditional centralized monocentric city. There are 2 million households (and workers), firm land consumption is .0001 square miles per worker and household land consumption is .0005 square miles per worker. Exogenous marginal transport costs are set to $\beta = 100$ and transportation capacity is set so as to be constant across distance. The business zone extends to ring 96 and the city edge is at ring 195 (rings are tenths of miles). Congestion is zero at the very center and edge, and reaches a maximum at the edge of the business zone (Figure 9). This edge is determined by where the firm rent gradient equals the reservation rent for land (Figure 10), and firm rents are based on a spatial decline in worker productivity (Q' of 1200) that is about twice the maximum marginal travel cost. This is sufficient so that firm rent gradients are much steeper than those of households (Figure 10). As a result that portion of the city with mixed use (to ring 96) has on average

almost 80% of its land used by firms. In this simulation, aggregate travel expenditures are 5.4 billion, and central residential rents reach 105 million.⁷

Figures 11-12 illustrate a similar mixed land use city, but one where the decline in productivity is a quarter as much – 300 as opposed to 1200 per mile. The employment border moves out (from 96 to 155) and the fraction of land inside this border that is devoted to commercial use decreases to an average of 40%. Central residential rents drop, here to 78 million, because greater land use mixing is lowering the necessity to commute and thereby the marginal cost of travel. In the aggregate, travel expenditure is down sharply - to 2.5 billion – less than half of their value in the more centralized city (Figures 9-10).

Finally in Figures 13-14, a city with fully dispersed employment is illustrated. This is simply accomplished by setting the level of agglomeration at that specified in Example # 1 of Proposition 2. There is no commuting and hence no congestion, so marginal travel costs everywhere are equal to 100. At each location, firms occupy 17% of land and residences 83%. Even with fixed travel costs, aggregate travel expenditures are equal to zero since no one has to travel! In the fully dispersed city, the absence of congestion generates a huge difference in land rents – central residential rents now are only 37 million.⁸

These three simulations show how the dispersal of employment can greatly reduce commuting costs and in turn residential land rents. Commercial land rents also are greatly reduced, from 700 million down to only 7 million, but some of this is due directly to the reductions in agglomeration that generate the increasingly more dispersed employment patterns.

VII. The Dispersal of Employment and Transportation Capacity.

While changes in agglomeration alter the slope of a firm's rent gradient, changes in transportation capacity alter the rent gradients for both commercial and residential uses, by impacting congestion. As expanded capacity improves travel flows, both firms and residences move further “apart” to realize other location advantages.

Proposition 3. Expanded Transport capacity generates employment centralization.

⁷ In the top figure of each pair, population and employment depict the normalized variables: $h(t)/H$, $e(t)/E$. Travel costs are: $C' = 25D(t)/S(t) + \beta$. Transport capacity is set to be uniform over t : $S(t) = 125600$. Transport costs are in yearly \$/mile, and rents are in \$ square mile. The annual opportunity rent of land at the urban edge is: $A = \$1000000$. per square mile. Finally, in various simulations worker productivity is assumed to decline linearly with distance in the range: $Q' = 100$ to 1200. These simulations use (12') for the assignment of land use.

⁸ The mixed use city simulations in Figures 9-12 use the same $F(t)$ function as in the fully dispersed simulation results of Figures 13-14, that is (12'). Only the values of agglomeration and exogenous travel costs are changed, so that partial mixing occurs rather than fully dispersed employment.

[By contradiction]. In a mixed land use pattern, $b_f < b_h$, and $F(t) < 1$, for $t < b_f$. With a uniform expansion of $S(t)$, and no change in land use, $C'(t)$ will decrease everywhere. From (10)-(11) this causes R'_f to increase at all locations, while R'_h is less at all locations. Given that b_h is fixed (from (1) and (2)), $R_h(t)$ will be lower at all locations. Suppose b_f were to increase, then $R_f(t)$ would be higher at all locations. From either (12) or (12'), these rent changes would cause $F(t)$ to be greater at the same time that b_f increases – violating (1). To meet conditions (1), (2), b_f must shrink as $F(t)$ increases – the definition of employment centralization.

Figures 15-16 illustrate a simulated city with mixed employment that is in all respects identical to that in Figure 11-12, except that the value of transport capacity $S(t)$ has been doubled at each location. The employment border moves in (from 155 to 128) and the fraction of land inside this border that is devoted to commercial use increases. Central commercial rents increase while residential rents decrease.

The land use changes in Figures 15-16 provide an interesting example of “Braess’ paradox”: adding transportation capacity may actually increase aggregate or average travel expenditures - in the absence of congestion-based user pricing [Small, 1992].⁹ In the base case simulation of Figure 11-12, marginal travel costs per mile are higher, but job dispersal keeps the aggregate miles of travel low. The combination of the two yields aggregate travel expenditure of 2.5 billion. By adding capacity, the city in Figures 15-16 has lower marginal travel costs, but the more centralized land use pattern leads to longer trips. The product of longer but speedier travel leads to greater travel expenditure - about 10% higher, at 2.7 billion.

VIII. The Dispersal of Employment with Population growth.

Increasing population will deteriorate travel conditions, acting just in reverse to the expansion of transport capacity. Firms and household should seek to mitigate this situation by moving closer together. Because residential density in this model is both exogenous and invariant across locations, it is useful to consider two types of urban growth.

Definition 1: Internal metropolitan growth.

With internal growth, E and H increase through comparable decreases in q_h and q_f (increased density), while b_h and b_f remain fixed.

Definition 2: External metropolitan growth.

With external growth, E and H increase through expansion of b_h and b_f , (to b_{1h} and b_{1f}) while q_h and q_f remain fixed.

⁹ If each driver’s actions are based on minimizing total system costs, as they would be with congestion pricing, then the envelope theorem applies and the general equilibrium impact on total system costs of adding capacity equals the partial impact – always beneficial. In the real world, where efficient congestion pricing is absent, there are many documented examples of the paradox.

Proposition 4. Internal growth generates greater employment dispersal.

[By contradiction]. Consider starting from a city whose land use pattern is neither fully dispersed nor fully centralized. In this case $b_f < b_h$, and $F(t) < 1$, for $t < b_f$. If E and H expand internally by a factor $\gamma (> 1)$, because q_h and q_f decrease by $1/\gamma$, then at each t , $e(t)$, $h(t)$ and the difference $D(t)$ grow by $1/\gamma$. Without any expansion of travel capacity, $C'(t)$ increases everywhere. This causes R'_f to decrease, while R'_h is greater at all locations (in absolute values). Given that b_h is fixed, $R_h(t)$ will be greater at all locations. Suppose b_f were to decrease, then $R_f(t)$ would be less at all locations. From either (12) or (12'), these rent changes would cause $F(t)$ to be less at all locations up to b_f . To meet conditions (1), b_f must expand as $F(t)$ is lower. An expansion of b_f and decrease in $F(t)$ for $t < b_f$ is the definition of greater employment dispersal.

Figures 17-18 illustrate a simulated city with a mixed land use pattern that has half the population, and also half the density levels, of the city in Figures 11-12. The increase in population and density (moving from figure 17-18 back to 11-12) causes the employment boundary to increase from 126 to 155, and the employment distribution to flatten. Congestion is everywhere greater and this leads commercial rents at the center to be 60% higher, while residential rents are 135% greater.

It might be hypothesized that external growth has a similar impact on urban form, that is, it encourages employment dispersal by increasing congestion. The difficulty with demonstrating this is that in order to meet conditions (1)-(2), external growth by definition must expand the employment border b_f , and change $F(t)$.

IX. The impact of Population growth on Rents and when Employment is dispersed as opposed to centralized

A major hypothesis of this paper is that population growth has quite different impacts on cities when employment is dispersed as opposed to more concentrated. Consider a city whose employment is concentrated due to high agglomeration, such as displayed in Figures 9-10. With increased population through internal growth (greater density), it is clear from Proposition 4 that greater congestion will cause household rent gradients to become steeper with distance (R'_h increases). This same increase in congestion leads the rents for firms to become less steeper (R'_f decreases). The edge of employment, however, also expands outward. Thus while household rent levels increase everywhere, the change in firm rent levels cannot be determined unambiguously. When the distribution of employment is at the other extreme – fully dispersed – the impact of population growth on rents is much easier to assess.

Proposition 5: Impact of Internal growth on rent with fully dispersed employment.

Using the dispersed example #1, if E and H expand internally by a factor $\gamma (> 1)$, because q_h and q_f decrease by $1/\gamma$, then the value of β , and hence C' is lower at all locations. In this case, both wage and rent gradients are flatter while the degree of mixing $F(t)$ remains unchanged (*Proposition 2*). Thus internal or vertical growth would actually reduce land rents, while aggregate travel expenditure remains unchanged (at zero).

A perhaps more realistic assumption is that as q_h and q_f decrease by $1/\gamma$ the value of λ increases by $(1/\gamma)^2$, thus leaving the value of β and C' unchanged. In this case, the rent gradients of both commercial and residential uses increase by γ , as would both rent levels.

In the case of dispersed example #2, proportional decreases in q_h and q_f have *no* impact on wages or rents. Wages do not vary by location and rent levels remain constant throughout the city and equal to the opportunity value of urban land, A .

In Table 1, a comparison is made between land rents in a city of 1 and 2 million inhabitants – across the two extreme types of land use. The expansion of population occurs through a doubling of density and so the overall boundary of the city remains unchanged. With centralized employment (the high agglomeration of Figure 10), commercial rents double and residential rents increase 200%. With fully dispersed employment, using example #1, commercial rents also double, but residential rents increase by only 100%. This occurs because the value of β is not allowed to change. With fully dispersed employment using the example #2, all rents remain unchanged at the value of $A=1$ million.

Table 1: Impact of Internal growth on Land Rents*

	1 Million		2 Million	
	Commercial	Residential	Commercial	Residential
Centralized	415	35	826	104
Dispersed #1	3.7	18.6	7.4	37.2
Dispersed #2	1	1	1	1

*all rent figures in millions per square mile.

When cities experience external growth, both the edge of employment and population expand outward. The impact of such expansion on congestion and the marginal cost of travel become difficult to determine. As such, the changes in rent are complicated as well. Intuitively it is difficult to imagine that rents do not increase, but magnitudes can only be determined numerically. In the case of fully dispersed employment, however, the impact is easily ascertained.

Proposition 6: Impact of External growth with fully dispersed employment.

In a dispersed city, using example #1, if E and H increase, while q_h and q_f remain fixed, then $F(t)$ remains unchanged (by (6)), while $b_h (= b_f)$ expands. The slopes of both rent gradients remain unchanged (since congestion is zero and marginal travel costs equal β). Denoting the new value of the border as $b_{1h} (= b_{1f})$ central commercial rent levels become $R_{1c}(0) = b_{1h}\lambda q_f$ as opposed to $R_c(0) = b_h \lambda q_f$, and similarly for residential rents. Thus the central rent level for both uses increase, but only linearly by the growth in the border. Aggregate travel expenditure remains unchanged (at zero).

In a dispersed city using example #2, wages do not vary across locations, and rent gradients remain flat at the opportunity cost of land, A, even as the border expands. Thus there is no change in rents with external growth.

Table 2 compares land rents in a city of 1 and 2 million inhabitants – again across the two types of cities. Here, the expansion of population occurs through a doubling of developed area, while densities remain unchanged. With centralized employment, commercial rents increase 50% and residential rents increase 100%. With fully dispersed employment, using example #1, commercial rents also increase 50%, but residential rents increase by only 50%. With fully dispersed employment using the example #2, all rents remain unchanged at the value of A=1 million.

Table2:Impact of External growth on Land Rents*

	1 Million		2 Million	
	Commercial	Residential	Commercial	Residential
Centralized	548	52	826	104
Dispersed #1	5.1	25.0	7.4	37.2
Dispersed #2	1	1	1	1

*all rent figures in millions per square mile

Clearly, these simulations and *Propositions 5-6* show that impact of growth on urban rents will definitely vary by the degree of employment dispersal. When commercial land use is highly centralized, growth generates both longer commuting distances, and because of this, greater congestion and hence higher marginal travel costs as well. On the other hand, with a fully dispersed land use pattern, growth has much less impact. There is no increase in congestion or marginal travel cost, and travel distances remain unchanged (at zero). The comparative impact of these on land rents depends to some degree on which model one chooses, but in all cases population growth has less impact on rent when employment is dispersed.

X. Future extensions

The example of “Braess’ paradox” in adding transport capacity suggests a significant difference exists between equilibrium and optimal land use in cities with dispersed employment. As long as productivity is exogenous, the primary market failure is due only to the presence of congestion. In this case, correct road pricing may be all that is needed to obtain the “optimal” degree of land use mixing. Such pricing would have two effects in the current (mixed use) model. First, it would tend to increase the steepness of the residential rent gradient. With the mixing function of the model, this encourages households to live closer to the center. At the same time, it would increase the wage gradient and decrease the steepness of the commercial rent gradient – encouraging firms to live farther from the city center. These two changes, of course, tend to generate *greater*

land use mixing or employment dispersal. This suggests that optimum employment locations should be more dispersed than those in a competitive equilibrium.

The paper also has only begun to analyze cities where land use mixing can occur. The list of extensions is quite long. First off, more realistic 2-way commuting might be examined, since clearly all travel modes have this feature. Furthermore, the marginal cost of traveling the “opposite” direction is very low. This would seem to further encourage job dispersal. Next, density or land consumption could be made endogenous, and this gives an additional dimension of adjustment to job dispersal decisions. Private land consumption decisions by households are not efficient, and the residential case has been well studied [Wheaton, 1998], but the density decision of firms should impact congestion as well.

Lastly, the modeling of joint work-residence location decisions might begin to incorporate the existence of the labor-market frictions, that now are so widely modeled in macroeconomics. The only significant work here is the recent paper by Rouwendal [1998] who examines worker “commuting” sheds when job choice is repeatedly made in the presence of uncertainty. Does worker turnover, and job switching alter the results of the model presented here?

The empirical implications of the model here are very pointed. Job dispersal per se should, if the model is correct, lead both to lower marginal commuting costs and travel distances. The only research on this question is a 12 year old paper by Richardson and Kumar [1989] that uses aggregate data and quite rough approximations for measures of job dispersal. Their results clearly support the model. The US press, however, daily speaks of horrible congestion and commuting patterns in modern “dispersed” cities. The empirical question clearly needs to be examined anew, and this time with much more disaggregated data. If commuting has not improved with job dispersal, then the model is clearly missing some critical ingredients.

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FIGURE 1. Amsterdam Single Family House Price Index: 1628-1972 (constant guilder)

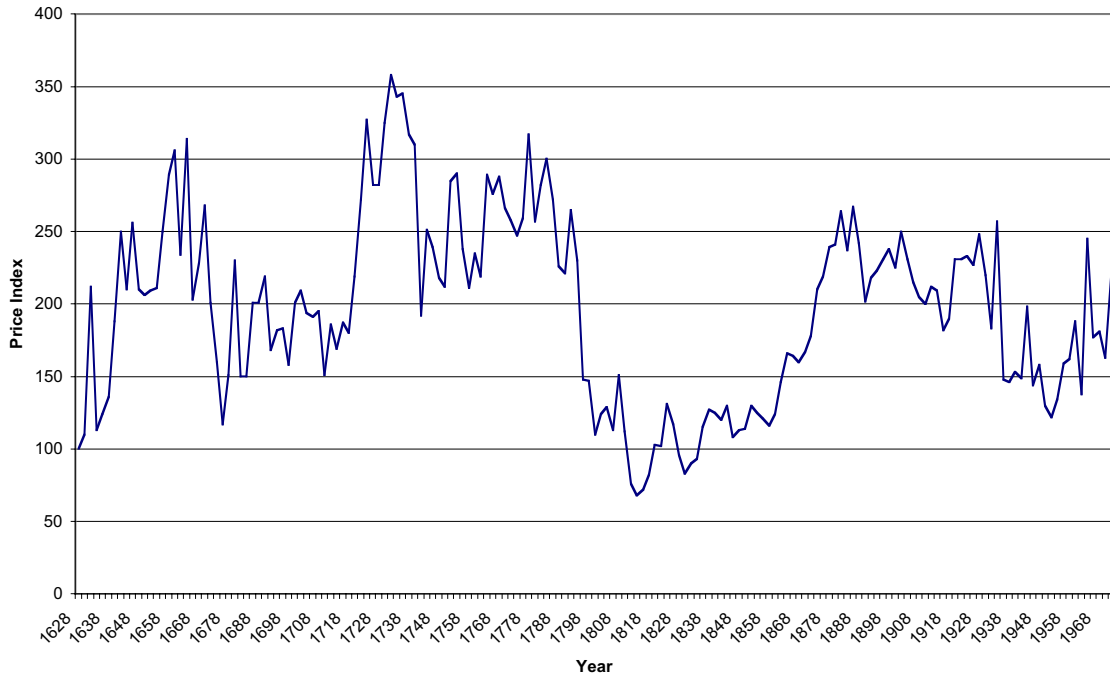


FIGURE 2. CPI Apartment Rent Indices for Selected "traditional" Cities: 1918-1999 (constant \$)

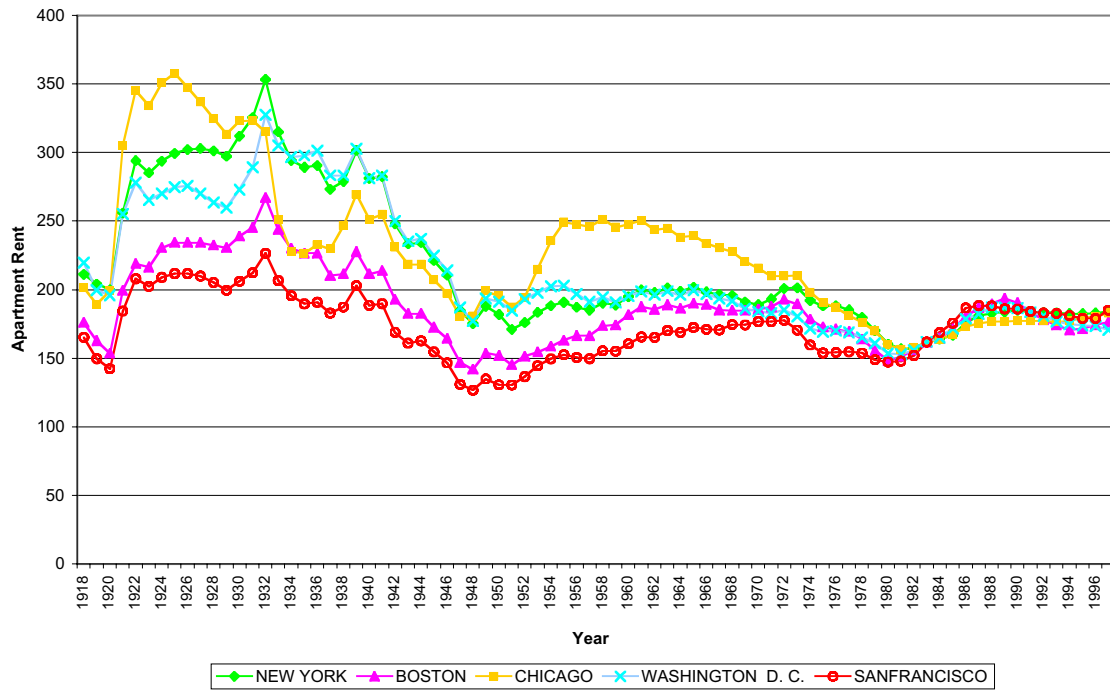


FIGURE 3. CPI Apartment Rent Indices for Selected "new" Cities: 1918-1999 (constant \$)

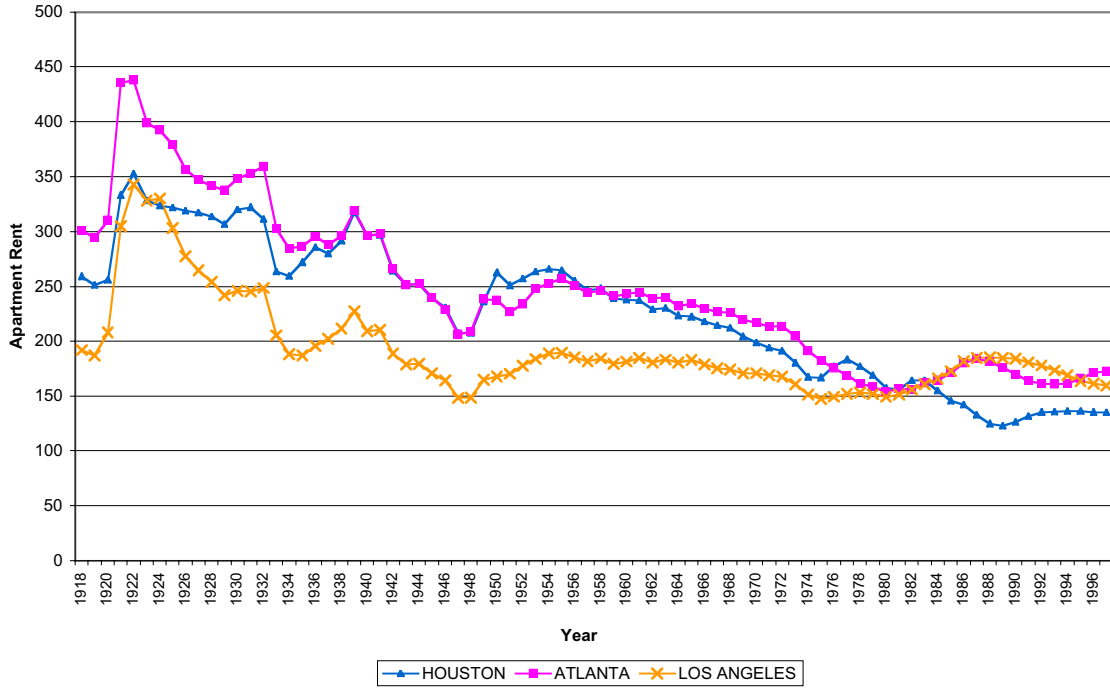


FIGURE 4. Repeat Sale House Price Indices for Selected "Traditional" Cities: 1975-1999 (constant \$)

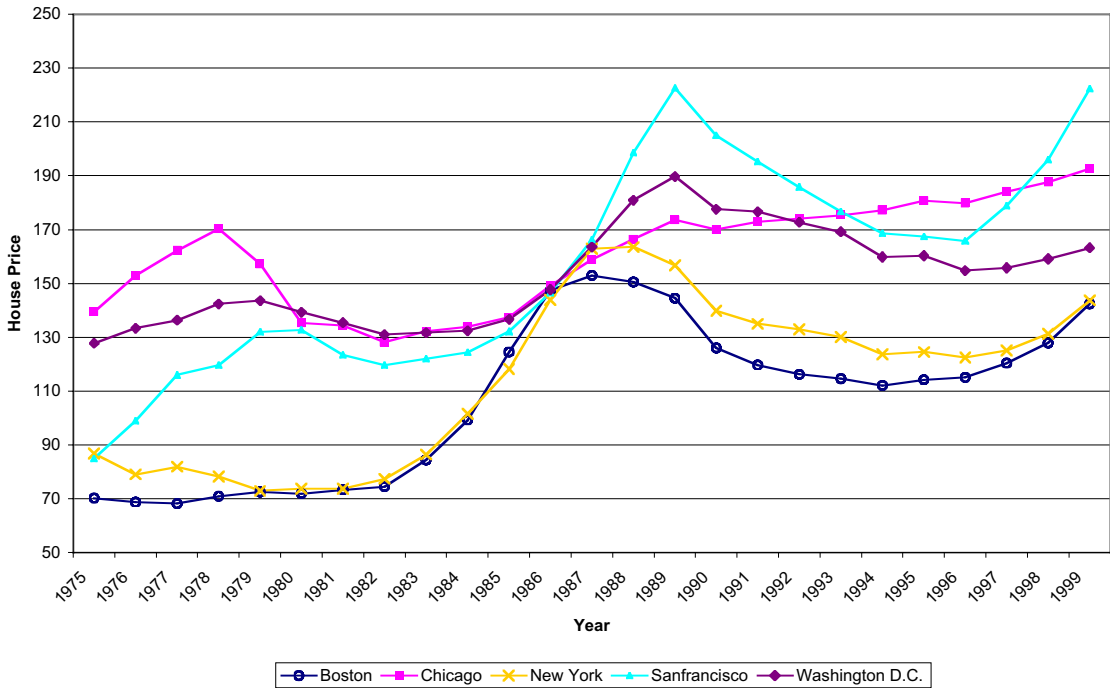


FIGURE 5. Repeat Sale House Price Indices for Selected "new" Cities: 1975-1999 (constant \$)

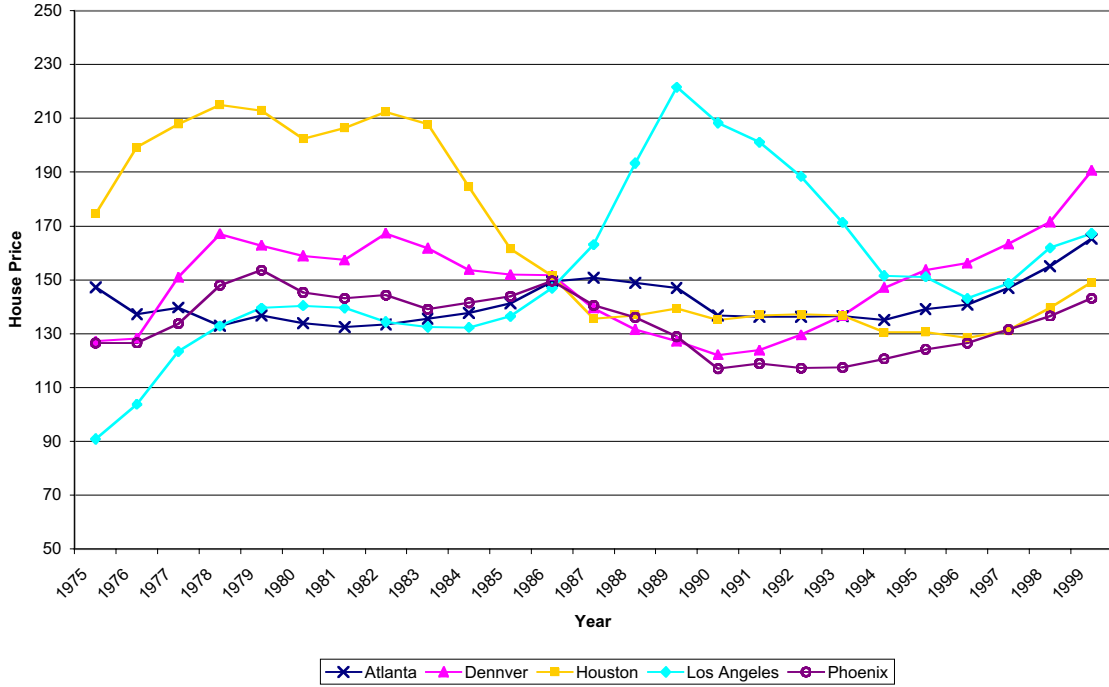
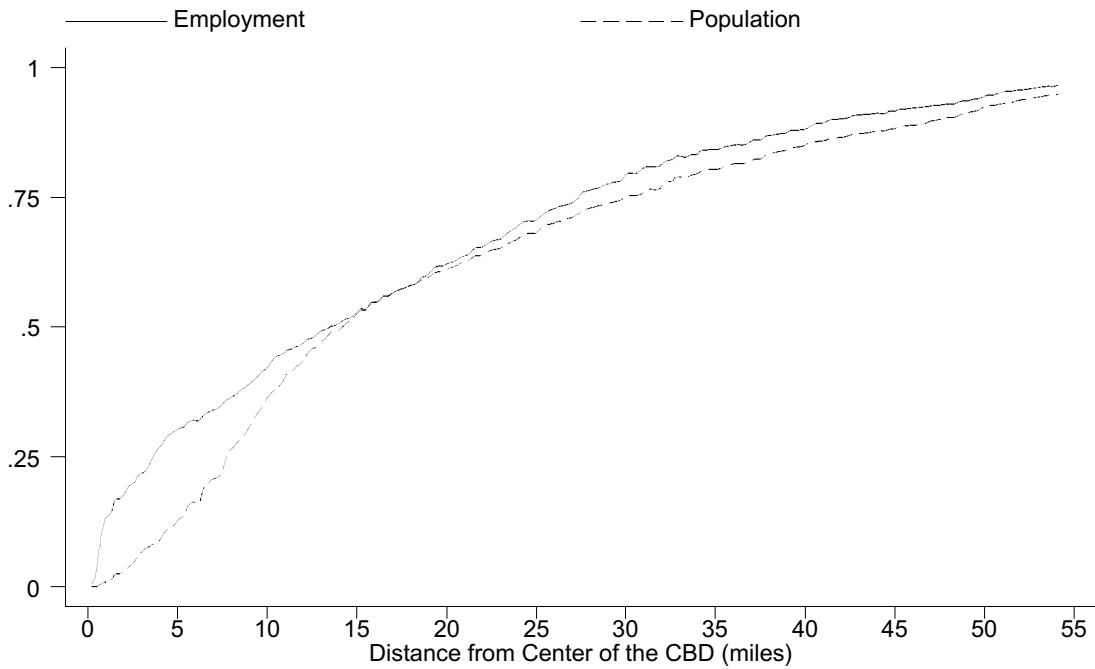


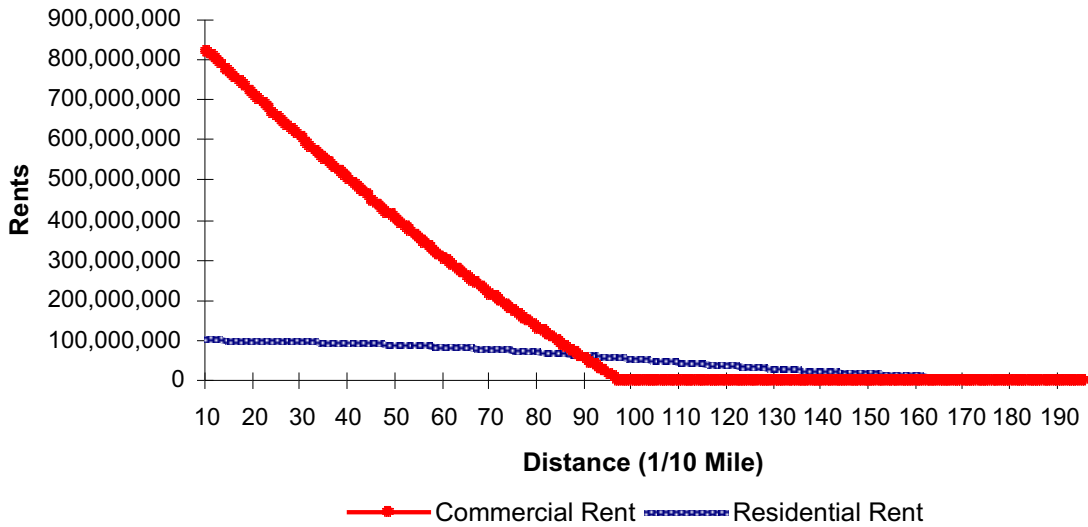
Figure 6: New York Spatial Distributions



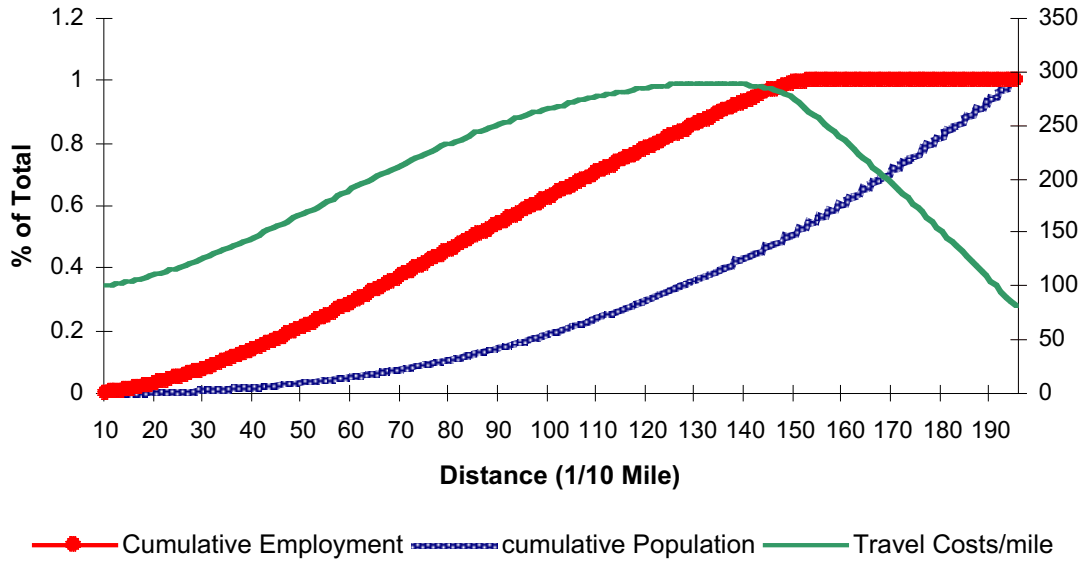
**Figure 9: Land Use and Travel Costs,
2 million inhabitants, mixed use city, high agglomeration**



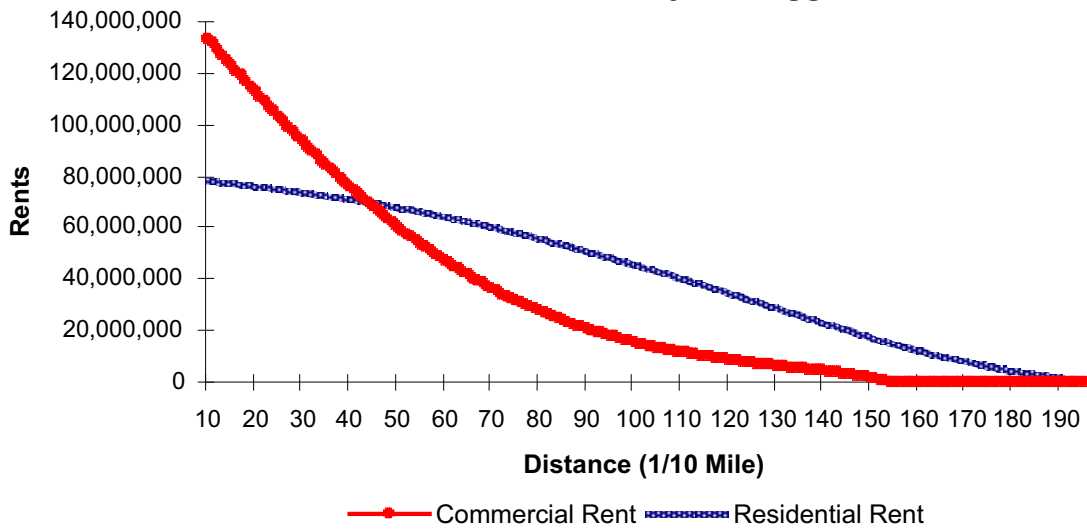
**Figure 10: Land Rents,
2 million inhabitants, mixed use city, high agglomeration**



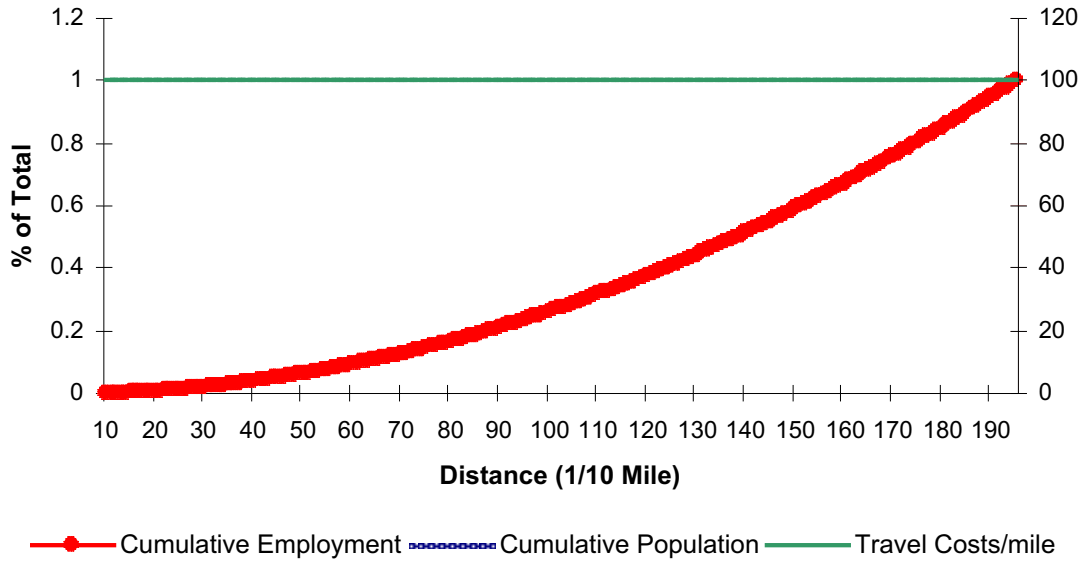
**Figure 11: Land Use and Travel Costs,
2 million inhabitants, mixed use city, low agglomeration**



**Figure 12: Land Rents,
2 million inhabitants, mixed use city, low agglomeration**



**Figure 13: Land Use and Travel Costs,
2 million inhabitants, fully dispersed city**



**Figure 14: Land Rents,
2 million inhabitants, fully dispersed city**

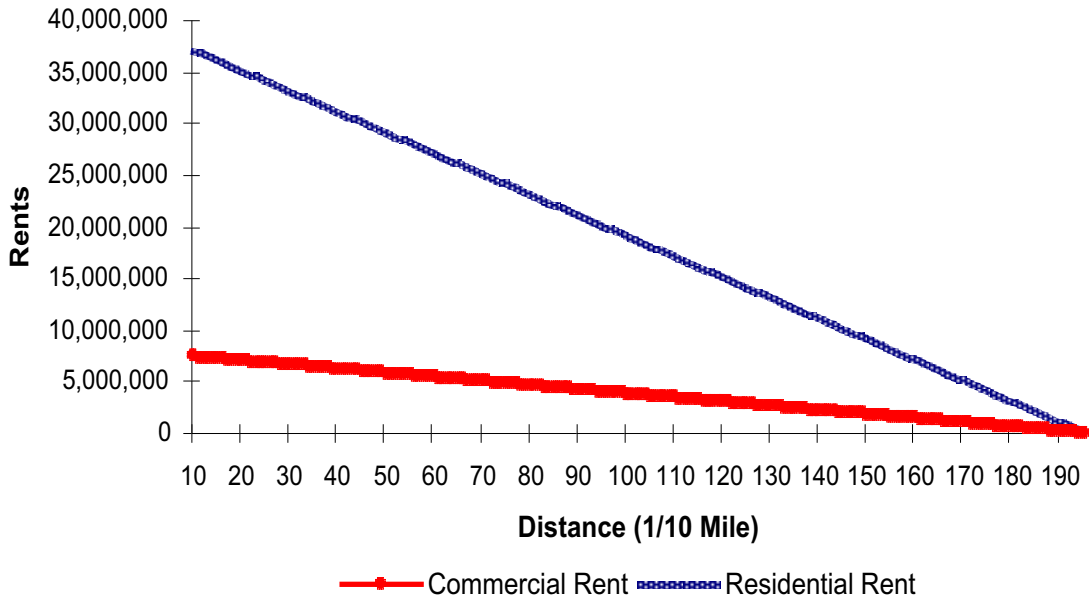


Figure 15: Land Use and Travel Costs, 2 million inhabitants, mixed use city, expanded transport capacity

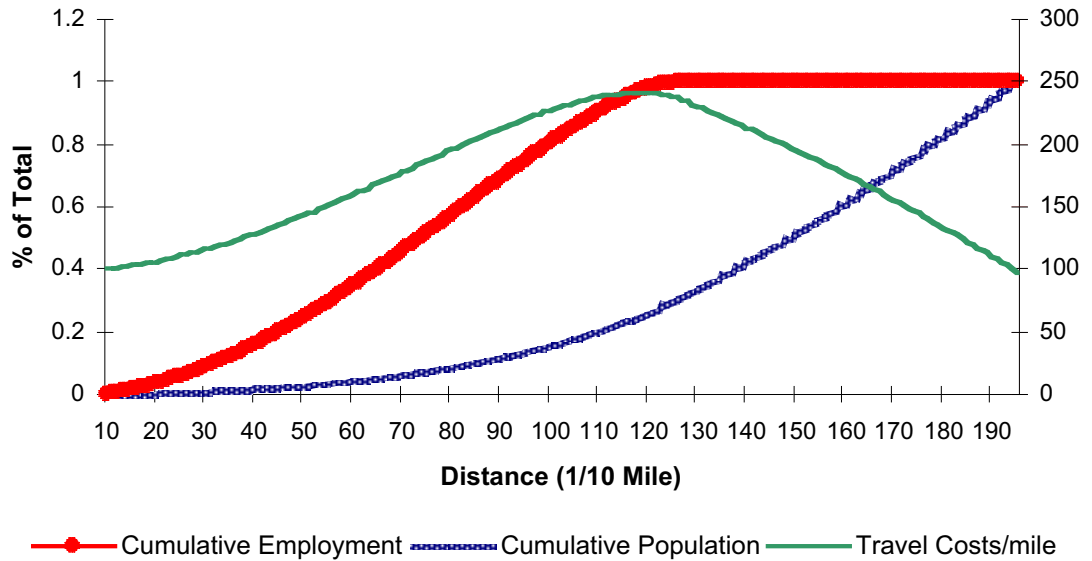
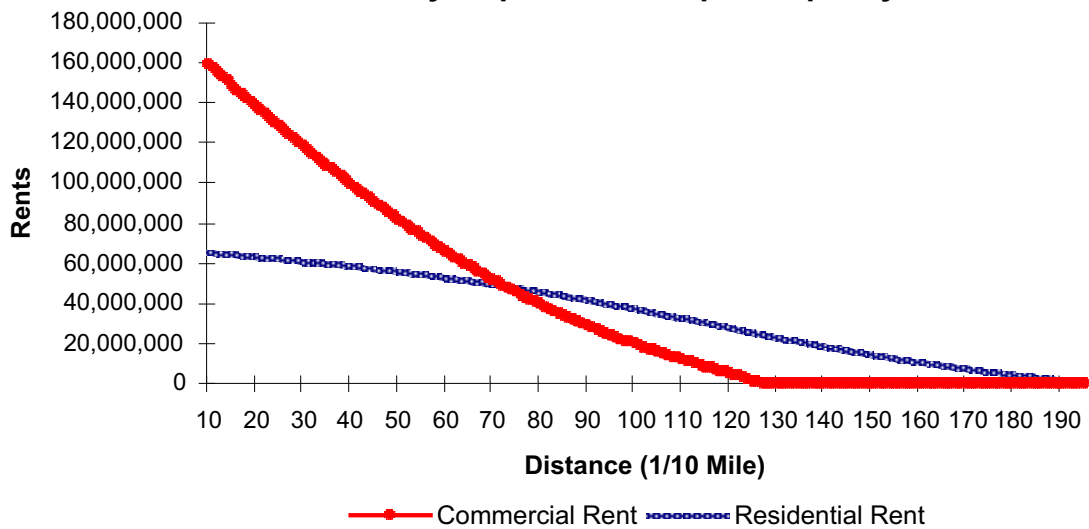
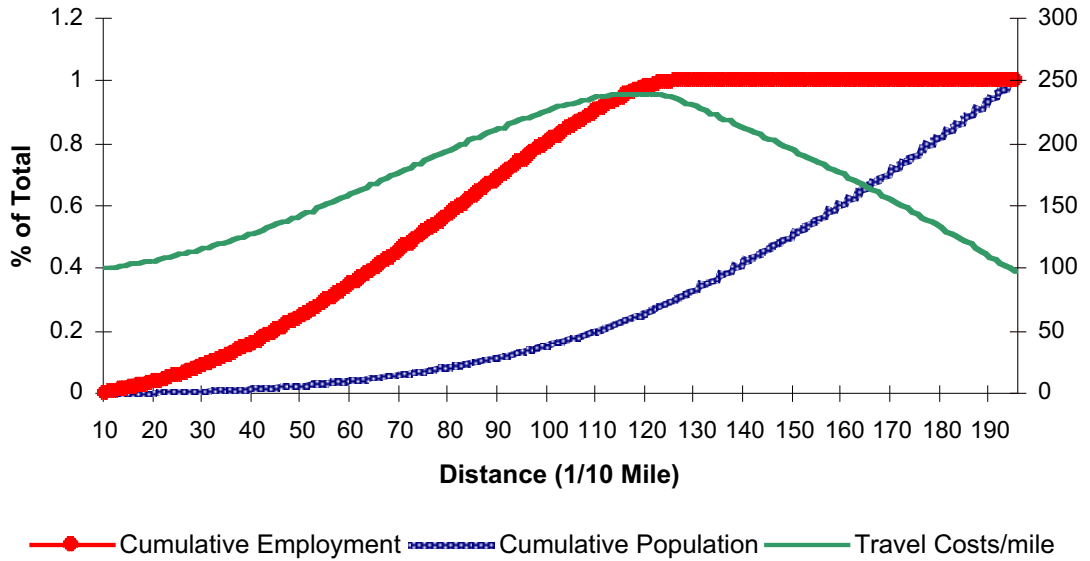


Figure 16: Land Rents, 2 million inhabitants, mixed use city, expanded transport capacity



**Figure 17: Land Use and Travel Costs,
1 million inhabitants, mixed use city, low agglomeration**



**Figure 18: Land Rents,
1 million inhabitants, mixed use city, low agglomeration**

