Question 1

We are interested in the design of control systems for an airplane that can take off vertically and transition to forward flight, as shown in the figure on the left. For this purpose, we designed a laboratory experiment, which is depicted in figure on the right. In this experiment, we built a flat plate airplane connected to a propeller with an elevator-like control surface. The elevator angle can be controlled with a servo motor. We pinned the experimental equipment on one end as shown in the figure. The propellor turns at a constant speed, maintaining constant air flow towards the elevator. Changing the elevator angle produces a force to push the plate in the opposite direction, as shown in the figure.

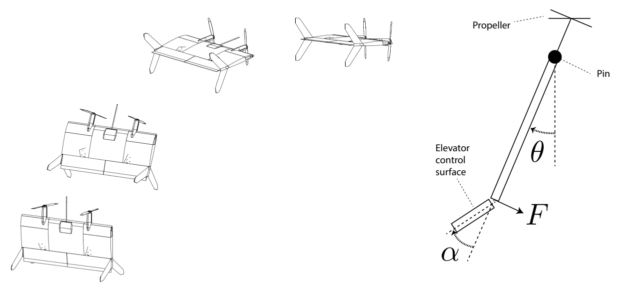

Figure 1: *On the left, a "tailsitter" aircraft is shown. The aircraft sits on its tail, takes of vertically, and transitions to horizontal flight. On the right, a laboratory experiment to gain insight to this system is shown.*

The angles θ and α are shown in the figure. We model the behavior of the system around $\theta = 0$ and $\alpha = 0$ with the following linear differential equation:

$$\ddot{\theta}(t) = -c_1\theta(t) - c_2\dot{\theta}(t) + c_3\alpha(t),$$

where c_1, c_2, c_3 are positive constants. In this case, the term $-c_1\theta(t)$ models the effect of gravity, the term $-c_2\dot{\theta}(t)$ models the effect of air drag acting on the flat plate as turns around its pin, and the term $-c_3\alpha(t)$ models the effect of the force acting on the plate via the elevator.

Please answer the following questions:

1. Develop a state space model for this system. Identify the state variables. Write the dynamics in state-space form.

2. What can you say about the relationship between this system and a pendulum? Do the mathematical descriptions of the two systems differ?

3. What can you say about the open-loop stability of this system?

4. Suppose we can only measure $\theta(t)$. For sake of simplicity, suppose $c_1 = c_2 = c_3 = 1$. Please design a feedback control system, involving an estimator and a controller, so that the settling time of the system is roughly no more than one second.

5. Now suppose we can measure $\theta(t)$ and $\dot{\theta}(t)$. How would you design a control system that minimizes the following cost function:

   $$\int_0^\infty x(t)^2 + \rho\, u(t)^2\, dt$$

   Please write down the equations for this design as a function ρ. (But you do not need to solve them.) What can you say about the settling time of the system as a function of ρ? Is this control system a P, PD, or a PID control?

6. How do you think about this experiment with a pinned flat plate in Figure (b) compares with the transition flight of the aircraft shown in Figure (a), from a feedback control system design point of view? How are they similar? How do they differ?

Question 2

A simplified version of the old LORAN-C navigation is as follows: Three transmitters are located in the (x, y)-plane, say, at

$$
\begin{aligned}
(x_1, y_1) &= (1, 0)\,\frac{b}{\sqrt{3}} \\
(x_2, y_2) &= \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\frac{b}{\sqrt{3}} \\
(x_3, y_3) &= \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\frac{b}{\sqrt{3}}
\end{aligned}
$$

where b is the (known) baseline distance between transmitters. The three transmitters each send a pulse simultaneously, but at an unknown time. The pulses propagate at the speed of light to the receiver onboard a ship or aircraft, which measures the three arrival times, z_1, z_2, and z_3. Denote this measurement vector by $\mathbf{z}$. New measurements are available every 0.1 s; denote the kth measurement as $\mathbf{z}[k]$. The received signals have a relatively low signal-to-noise ratio, as low as 0.1, so the uncertainty in the time of arrival of each pulse can be significant, and can be modeled as white Gaussian noise with covariance $\sigma^2 I_3$.

1. For a stationary receiver at an unknown location, describe how one might estimate the receiver position from one or more sets of measurements $\mathbf{z}[k]$.

2. Now suppose that the receiver is on a vehicle with dynamics given by

$$
\begin{aligned}
\dot{x}(t) &= V \cos\theta(t) \\
\dot{y}(t) &= V \sin\theta(t) \\
\dot{\theta}(t) &= w(t)
\end{aligned}
$$

   where w is a white noise process with intensity W. Describe how you would estimate the position of the vehicle. How would your answer depend on the noise variances and intensities in the problem? How would your answer depend on the size of the baseline b? For this question, the baseline might range from a few meters (laboratory scale) to hundreds of kilometers (implementation scale).