2019 Autonomy Field Exam Core: Principles of Autonomy

This problem exercises your understanding of how to reformulate Linear Programs. You've been given access to a highly experimental **black box** linear program solver that in practice monstrously outperforms everything else on the market (think 100x faster across a standard set of benchmark problems). A downside is that the black box solver only accepts inputs of the standard form:

minimize $c^T x$, subject to $Ax \leq b$ and $x \geq 0$,
where c, x are $n \times 1$ vectors, b is an $m \times 1$ vector, and A is an $m \times n$ matrix

The researchers are willing to add you as a second author on a paper if you generalize their black box solver to accept problems that are not in this input form. You're sure you can do this without modifying the underlying algorithm, by instead introducing new variables and by otherwise transforming the input problem to the black box input form.

Please show how to transform each problem given below into the input form for the black box solver above, such that the solver's solution can be mapped to a correct answer to the original problem. **You may skip one of a or b**, but you **must complete c and d.** In the end you will have completed **three of the four** sub-problems.

a) Convert the following problem to the standard input form accepted by the black box LP solver. [Hint: Think how to represent a variable that is either positive or negative, using only positive variables.]

   minimize $e^T y$, subject to $Cy \leq d$ and y **unrestricted** *(i.e., positive or negative).*

b) Show how to solve the following **feasibility** problem. We are not interested in an optimal solution – we just want to know whether a value of y exists that solves the problem. [Hint: Consider how to formulate a strict inequality using slack variables.]

   Does there exist a y, subject to $Cy < d$, and $y \geq 0$? [Note the **strict inequality**!]

c) Imagine that the researchers reveal to you that their black box algorithm is based on simplex, whereas the current state-of-the-art algorithm is based on a polynomial-time interior point method. Consider the sub-problem a) or b) that you completed, and the transformation you specified. For this sub-problem and transformation, how will the ***worst case*** run time of the researcher's simplex-based black box solver compare to the state-of-the-art interior point method worse, same or better? Explain why?

d) Finally, write pseudocode that demonstrates how you would build a Mixed-Integer Linear Program (MILP) solver, using the above black-box LP solver, given by the researchers. A MILP is the same as an LP except some subset of the variables (i.e. x_1, x_4, x_5) are constrained to be integer values. [Hint: you will have to perform combinatorial search]

2019 Autonomy Field Exam: Estimation Elective

A simplified version of the old LORAN-C navigation is as follows: Three transmitters are located in the (x, y)-plane, say, at

$$(x_1, y_1) = (1, 0)\,\frac{b}{\sqrt{3}}$$
$$(x_2, y_2) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\frac{b}{\sqrt{3}}$$
$$(x_3, y_3) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\frac{b}{\sqrt{3}}$$

where b is the (known) baseline distance between transmitters. The three transmitters each send a pulse simultaneously, but at an unknown time. The pulses propagate at the speed of light to the receiver onboard a ship or aircraft, which measures the three arrival times, z_1, z_2, and z_3. Denote this measurement vector by $\mathbf{z}$. New measurements are available every 0.1 s; denote the kth measurement as $\mathbf{z}[k]$. The received signals have a relatively low signal-to-noise ratio, as low as 0.1, so the uncertainty in the time of arrival of each pulse can be significant, and can be modeled as white Gaussian noise with covariance $\sigma^2 I_3$.

1. For a stationary receiver at an unknown location, describe how one might estimate the receiver position from one or more sets of measurements $\mathbf{z}[k]$.

2. Now suppose that the receiver is on a vehicle with dynamics given by

$$\dot{x}(t) = V\cos\theta(t)$$
$$\dot{y}(t) = V\sin\theta(t)$$
$$\dot{\theta}(t) = w(t)$$

   where w is a white noise process with intensity W. Describe how you would estimate the position of the vehicle. How would your answer depend on the noise variances and intensities in the problem? How would your answer depend on the size of the baseline b? For this question, the baseline might range from a few meters (laboratory scale) to hundreds of kilometers (implementation scale).