Problem set 2

Turn in Wednesday 2008-01-23 in class. Turn in only the 'Problems' section. The other sections are for your own practice.

Warmups

Warmup problems are quick problems for you to check your understanding; ***don't turn them in.***

1. The half-life $\tau_{1/2}$ of a radioactive substance is the time until only one-half of the substance remains. How is $\tau_{1/2}$ related to the time until $1/e$ of the substance remains?

2. You are a ship navigator back in the old days when clocks were the only way to measure longitude. If your expensively constructed clock has lost 10 minutes after, say, a sea voyage of 1 month, by roughly how many degrees will you be in error about your longitude? More importantly, by roughly how many miles will you be in error about your position? Should you be worried? (Assume that you are at 45° latitude.)

Problems

Turn in solutions to these problems.

3. Estimate the size (in dollars/year) of the US diaper market. These market-sizing questions are often asked in management-consulting interviews.

4. Estimate the integrals by replacing the smooth curve with a rectangle, using the FWHM (full-width, half-maximum) heuristic (Chapter 15 of the notes) to choose the rectangle. How accurate is each estimate?

a. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$. [Exact answer: π.]

b. $\displaystyle\int_{-\infty}^{\infty} e^{-x^4}\,dx$. [Exact answer: $\Gamma(1/4)/2 \approx 1.813$.]

5. The period of a pendulum is approximately

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{\theta_0^2}{16}\right),$$

where θ_0 (measured in radians!) is the angle at which the pendulum is started.

Roughly how many seconds per day does a pendulum clock with $\theta_0 = 10°$ lose compared to a pendulum clock with $\theta_0 = 5°$, if both have the same string length l? [See Problem 2 if you want to know whether the loss of those seconds is significant for navigation.]

Bonus problems

Bonus problems are more difficult but ***optional*** *problems for those who are curious.*

6. Guess the functional form of $T(\theta_0)$ for $\theta_0 \approx \pi$.