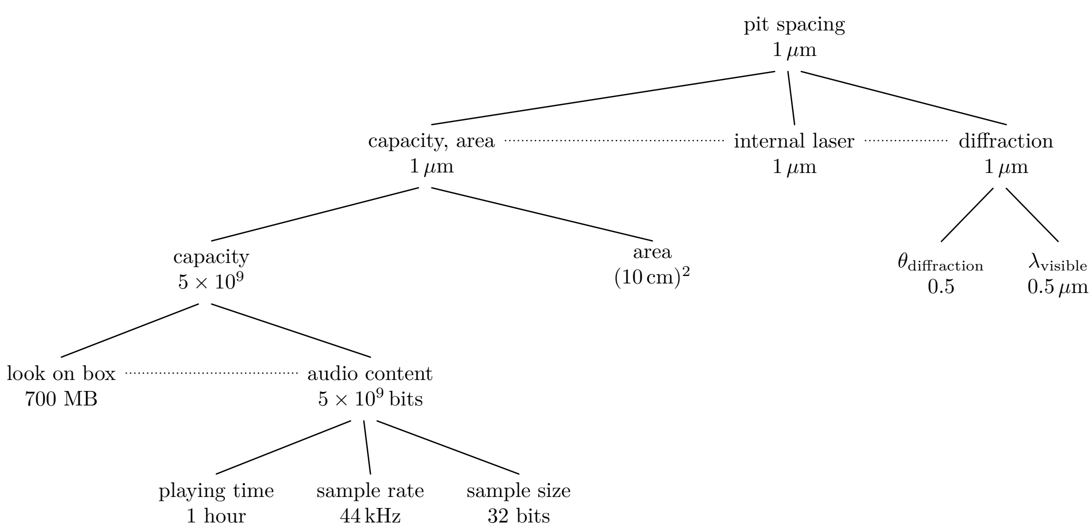
This tree is far more compact than the sentences, equations, and paragraphs of the original analysis in **Section 2.1**. The comparison becomes even stronger by including the alternative estimation methods in **Section 2.2**: (1) the wavelength of the internal laser, and (2) diffraction to explain the shimmering colors of a CD.

Draw a tree that includes these methods.

The wavelength method depends on just quantity, the wavelength of the laser, so its tree has just that one node. The diffraction method depends on two quantities, the diffraction angle and the wavelength of visible light, so its tree has those two nodes as children. All three trees combine into a larger tree that represents the entire analysis:



This tree summarizes the whole analysis of **Section 2.1** and **Section 2.2** – in one figure. The compact representation make it possible to grasp the analysis in one glance. It makes the whole analysis easier to understand, evaluate, and perhaps improve.

2.4 Example 2: Oil imports

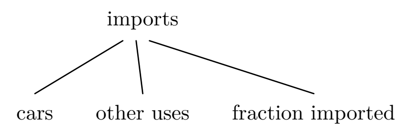
For practice, here is a divide-and-conquer estimate using trees throughout:

How much oil does the United States import (in barrels per year)?

One method is to subdivide the problem into three quantities:

- estimate how much oil is used every year by cars;
- increase the estimate to account for non-automotive uses; and
- decrease the estimate to account for oil produced in the United States.

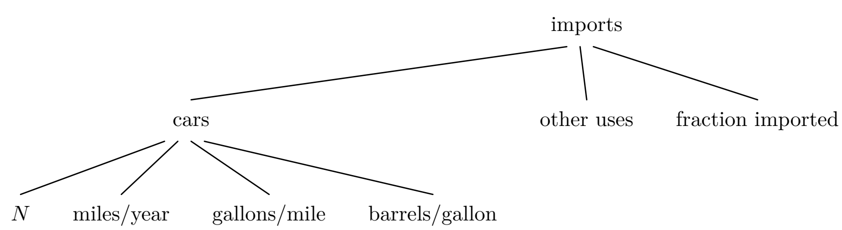
Here is the corresponding tree:



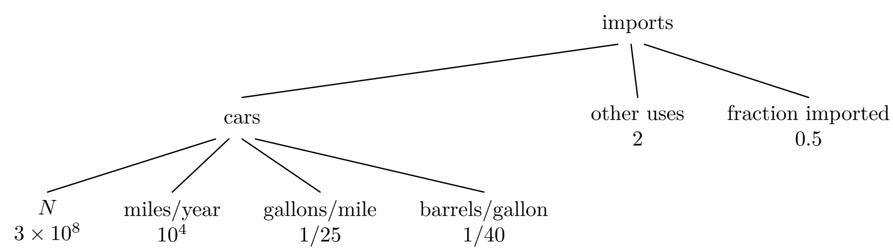
The first quantity requires the longest analysis, so begin with the second and third quantities. Other than for cars, oil is used for other modes of transport (trucks, trains, and planes); for heating and cooling; and for manufacturing hydrocarbon-rich products (fertilizer, plastics, pesticides). To guess the fraction of oil used by cars, there are two opposing tendencies: (1) the idea that the non-automotive uses are so important, pushing the fraction toward zero; (2) the idea that the automotive uses are so important, pushing the fraction toward unity. Both ideas seem equally plausible to me; therefore, I guess that the fraction is roughly one-half; and, to account for non-automotive uses, I will double the estimate of oil consumed by cars.

Imports are a large fraction of total consumption, otherwise we would not read so much in the popular press about oil production in other countries, and about our growing dependence on imported oil. Perhaps one-half of the oil usage is imported oil. So I need to halve the total use to find the imports.

The third leaf, cars, is too complex to guess a number immediately. So divide and conquer. One subdivision is into number of cars, miles driven by each car, miles per gallon, and gallons per barrel:



Now guess values for the unnumbered leaves. There are 3×10^8 people in the United States, and it seems as if even babies own cars. As a guess, then, the number of cars is $N \sim 3 \times 10^8$. The annual miles per car is maybe 15,000. But the N is maybe a bit large, so let's lower the annual miles estimate to 10,000, which has the additional merit of being easier to handle. A typical mileage would be 25 miles per gallon. Then comes the tricky part: How large is a barrel? One method to estimate it is that a barrel costs about $100, and a gallon of gasoline costs about \$2.50, so a barrel is roughly 40 gallons. The tree with numbers is:



All the leaves have values, so I can propagate upward to the root. The main operation is multiplication. For the 'cars' node:

$$3 \times 10^8 \text{ cars} \times \frac{10^4 \text{ miles}}{1 \text{ car–year}} \times \frac{1 \text{ gallon}}{25 \text{ miles}} \times \frac{1 \text{ barrel}}{40 \text{ gallons}} \sim 3 \times 10^9 \text{ barrels/year}.$$

The two adjustment leaves contribute a factor of $2 \times 0.5 = 1$, so the import estimate is

$$3 \times 10^9 \text{ barrels/year}.$$

For 2006, the true value (from the US Dept of Energy) is 3.7×10^9 barrels/year – only 25 higher than the estimate!

2.5 Theory 3: Estimating accuracy

How does divide-and-conquer reasoning produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities.

To explain that telegraphic answer, I will analyze a short estimation problem using divide-and-conquer done in slow motion, then apply the lessons to the oil-imports estimate.

The slow-motion problem is to estimate area of a sheet of A4 paper. On first thought, even looking at a sheet I have no clue about its area! On second thought, I know something. For example, the area is certainly more than