$$T \propto \frac{x_0}{x_0} = x_0^0.$$

In other words, the period is independent of amplitude.

5.3 Mountain heights

The next example of proportional reasoning explains why mountains cannot become too high. Assume that all mountains are cubical and made of the same material. Making that assumption discards actual complexity, the topic of ??. However, it is a useful approximation.

To see what happens if a mountain gets too large, estimate the pressure at the base of the mountain. Pressure is force divided by area, so estimate the force and the area.

The area is the easier estimate. With the approximation that all mountains are cubical and made of the same kind of rock, the only parameter distinguishing one mountain from another is its side length l. The area of the base is then l^2.

Next estimate the force. It is proportional to the mass:

$$F \propto m.$$

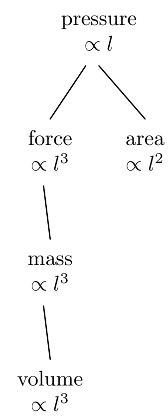
In other words, F/m is independent of mass, and that independence is why the proportionality $F \propto m$ is useful. The mass is proportional to l^3:

$$m \propto \text{volume} \sim l^3.$$

In other words, m/l^3 is independent of l; this independence is why the proportionality $m \propto l^3$ is useful. Therefore

$$F \propto l^3.$$

The force and area results show that the pressure is proportional to l:



$$p \sim \frac{F}{A} \propto \frac{l^3}{l^2} = l.$$

With a large-enough mountain, the pressure is larger than the maximum pressure that the rock can withstand. Then the rock flows like a liquid, and the mountain cannot grow taller.

This estimate shows only that there is a maximum height but it does not compute the maximum height. To do that next step requires estimating the strength of rock. Later in this book when we estimate the strength of materials, I revisit this example.

This estimate might look dubious also because of the assumption that mountains are cubical. Who has seen a cubical mountain? Try a reasonable alternative, that mountains are pyramidal with a square base of side l and a height l, having a 45° slope. Then the volume is $l^3/3$ instead of l^3 *but the factor of one-third does not affect the proportionality between force and length*. Because of the factor of one-third, the maximum height will be higher for a pyramidal mountain than for a cubical mountain. However, there is again a maximum size (and height) of a mountain. In general, the argument for a maximum height requires only that all mountains are similar – are scaled versions of each other – and does not depend on the shape of the mountain.

5.4 Animal jump heights

We next use proportional reasoning to understand how high animals jump, as a function of their size. Do kangaroos jump higher than fleas? We study a jump from standing (or from rest, for animals that do not stand); a running jump depends on different physics. This problem looks underspecified. The height depends on how much muscle an animal has, how efficient the muscles are, what the animal's shape is, and much else. The first subsection introduces a simple model of jumping, and the second refines the model to consider physical effects neglected in the crude approximations.

5.4.1 Simple model

We want to determine only how jump height varies with body mass. Even this problem looks difficult; the height still depends on muscle efficiency, and so on. Let's see how far we get by just plowing along, and using symbols for the unknown quantities. Maybe all the unknowns cancel.