4.4 Cycling

This section discusses cycling as an example of how drag affects the performance of people as well as fleas. Those results will be used in the analysis of swimming, the example of the next section.

What is the world-record cycling speed? Before looking it up, predict it using armchair proportional reasoning. The first task is to define the kind of world record. Let's say that the cycling is on a level ground using a regular bicycle, although faster speeds are possible using special bicycles or going downhill.

To estimate the speed, make a model of where the energy goes. It goes into rolling resistance, into friction in the chain and gears, and into drag. At low speeds, the rolling resistance and chain friction are probably important. But the importance of drag rises rapidly with speed, so at high-enough speeds, drag is the dominant consumer of energy.

For simplicity, assume that drag is the only consumer of energy. The maximum speed happens when the power supplied by the rider equals the power consumed by drag. The problem therefore divides into two estimates: the power consumed by drag and the power that an athlete can supply.

The drag power P_{drag} is related to the drag force:

$$P_{drag} = F_{drag}v \sim \rho v^3 A.$$

It indeed rises rapidly with velocity, supporting the initial assumption that drag is the important effect at world-record speeds.

Setting $P_{drag} = P_{athlete}$ gives

$$v_{max} \sim \left(\frac{P_{athlete}}{\rho A}\right)^{1/3}$$

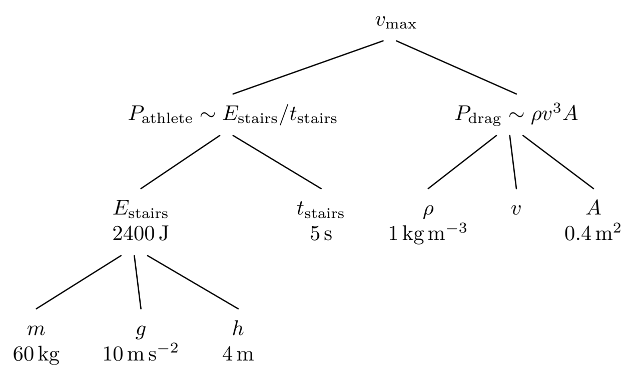
To estimate how much power an athlete can supply, I ran up one flight of stairs leading from the MIT Infinite Corridor. The Infinite Corridor, being an old building, has spacious high ceilings, so the vertical climb is perhaps $h \sim 4\,\mathrm{m}$ (a typical house is 3 m per storey). Leaping up the stairs as fast as I could, I needed $t \sim 5\,\mathrm{s}$ for the climb. My mass is 60 kg, so my power output was

$$P_{author} \sim \frac{\text{potential energy supplied}}{\text{time to deliver it}} = \frac{mgh}{t} \sim \frac{60\,\mathrm{kg} \times 10\,\mathrm{m\,s^{-2}} \times 4\,\mathrm{m}}{5\,\mathrm{s}} \sim 500\,\mathrm{W}.$$

$P_{athlete}$ should be higher than this peak power since most authors are not Olympic athletes. Fortunately I'd like to predict the endurance record. An Olympic athlete's long-term power might well be comparable to my peak power. So I use $P_{athlete} = 500\,\mathrm{W}$.

The remaining item is the cyclist's cross-sectional area A. Divide the area into width and height. The width is a body width, perhaps 0.4 m. A racing cyclist crouches, so the height is maybe 1 m rather than a full 2 m. So $A \sim 0.4\,\mathrm{m}^2$.

Here is the tree that represents this analysis:



Now combine the estimates to find the maximum speed. Putting in numbers gives

$$v_{max} \sim \left(\frac{P_{athlete}}{\rho A}\right)^{1/3} \sim \left(\frac{500\,\mathrm{W}}{1\,\mathrm{kg\,m^{-3}} \times 0.4\,\mathrm{m}^2}\right)^{1/3}.$$

The cube root might suggest using a calculator. However, massaging the numbers simplifies the arithmetic enough to do it mentally. If only the power were 400 W or, instead, if the area were 0.5 m! Therefore, in the words of Captain Jean-Luc Picard, 'make it so'. The cube root becomes easy:

$$v_{max} \sim\sim \left(\frac{400\,\mathrm{W}}{1\,\mathrm{kg\,m^{-3}} \times 0.4\,\mathrm{m}^2}\right)^{1/3} \sim (1000\,\mathrm{m^3\,s^{-3}})^{1/3} = 10\,\mathrm{m\,s^{-1}}.$$

So the world record should be, if this analysis has any correct physics in it, around $10\,\mathrm{m\,s^{-1}}$ or 22 mph.

The world one-hour record – where the contestant cycles as far as possible in one hour – is 49.7 km or 30.9 mi. The estimate based on drag is reasonable!