What kinds of teaching and learning produce world-class experts?

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Expert perceive mathematics differently from novices

Try this integral:

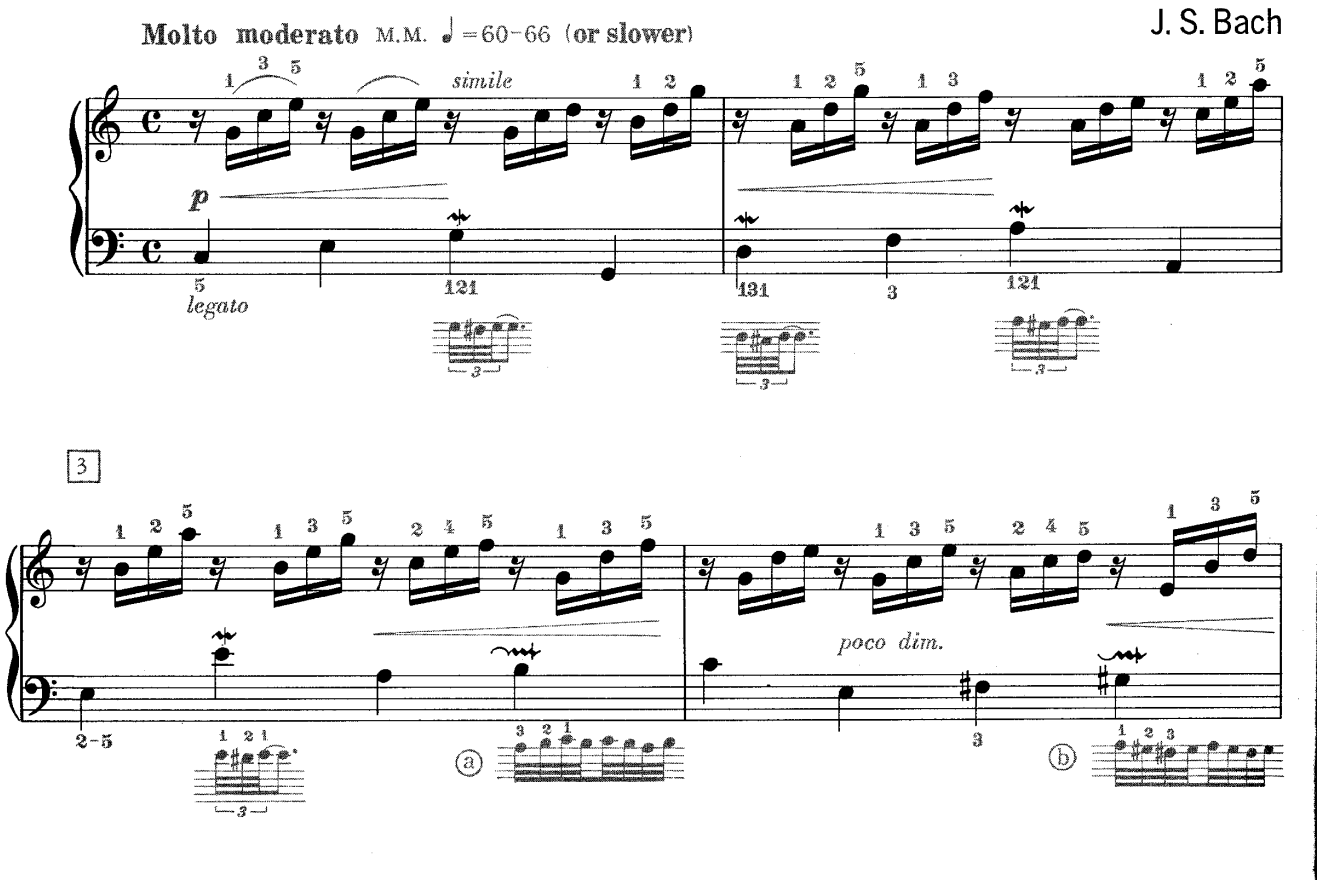
$$\int_0^\infty \frac{1}{(1+x^2)^6} = \quad ?$$

Experts perceive mathematics differently from novices

Experts know to rewrite, then differentiate under the integral sign:

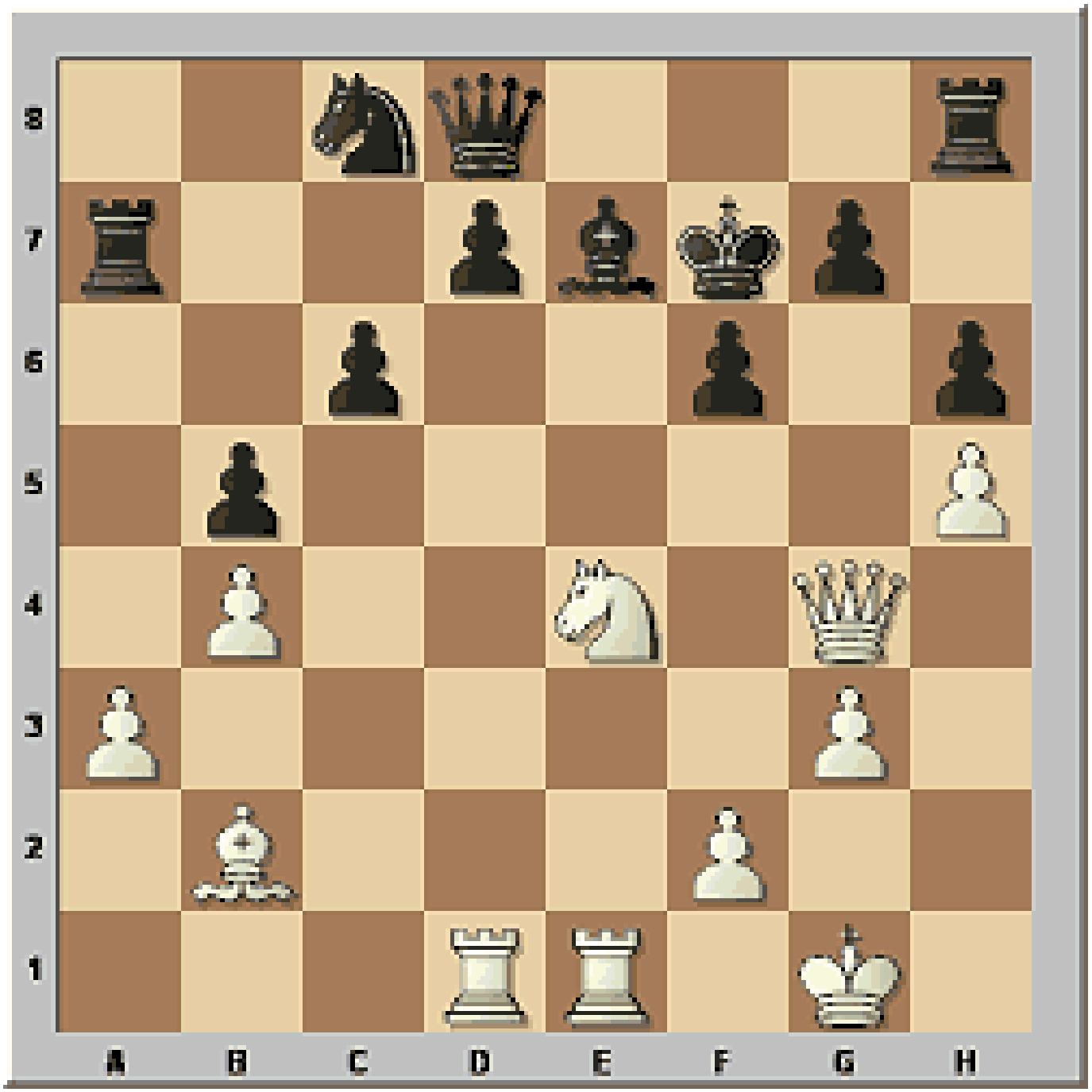
$$\int_0^\infty \frac{1}{(1+x^2)^6} = \frac{63\pi}{512}$$

Experts perceive music differently from novices



Experts perceive chess differently from novices

Try to remember this board:



Positions are hard for non-experts to remember

For example: How many pawns?

a. 7

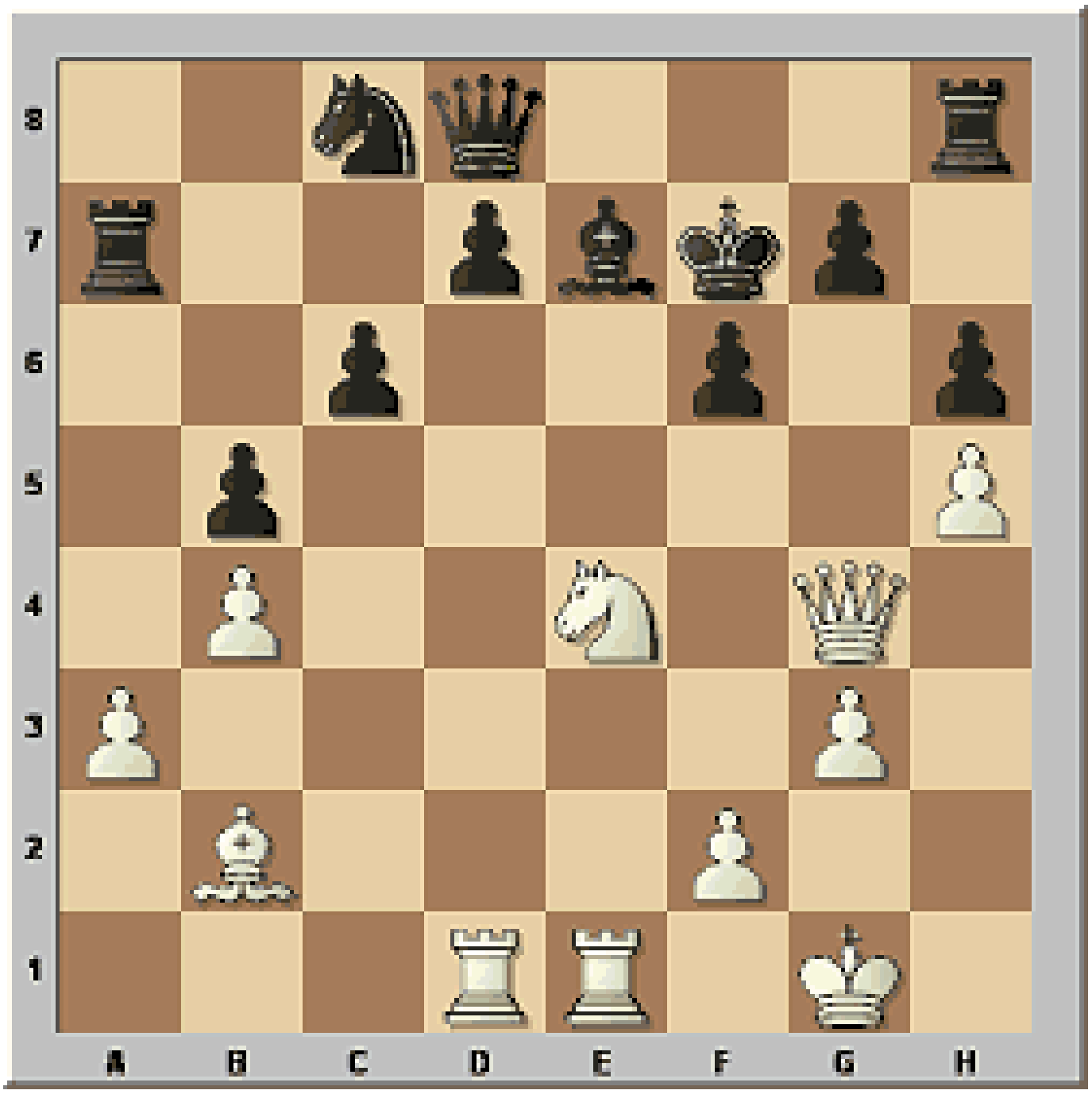
b. 9

c. 11

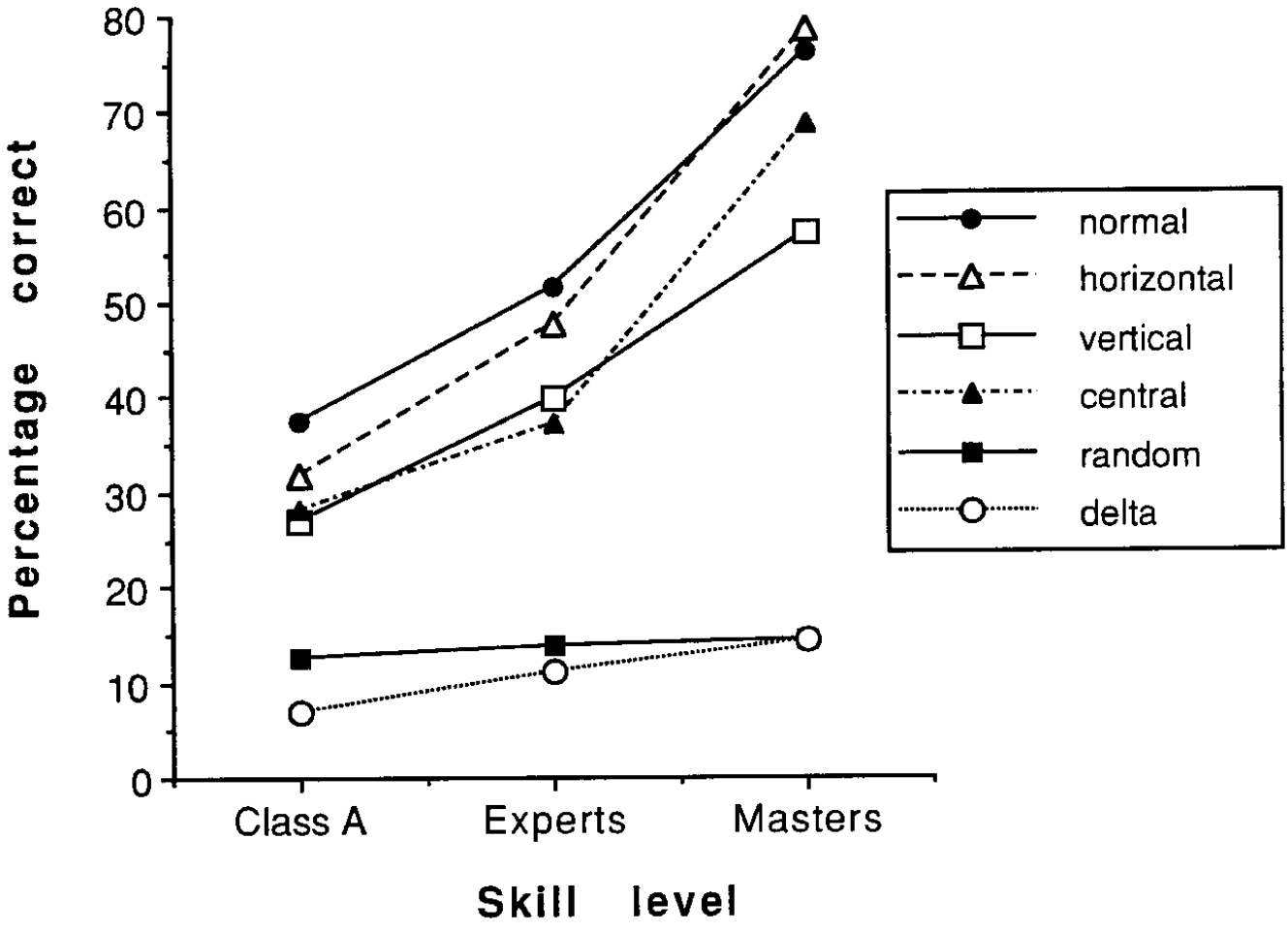
d. 13

e. none of the above

The position has 11 pawns

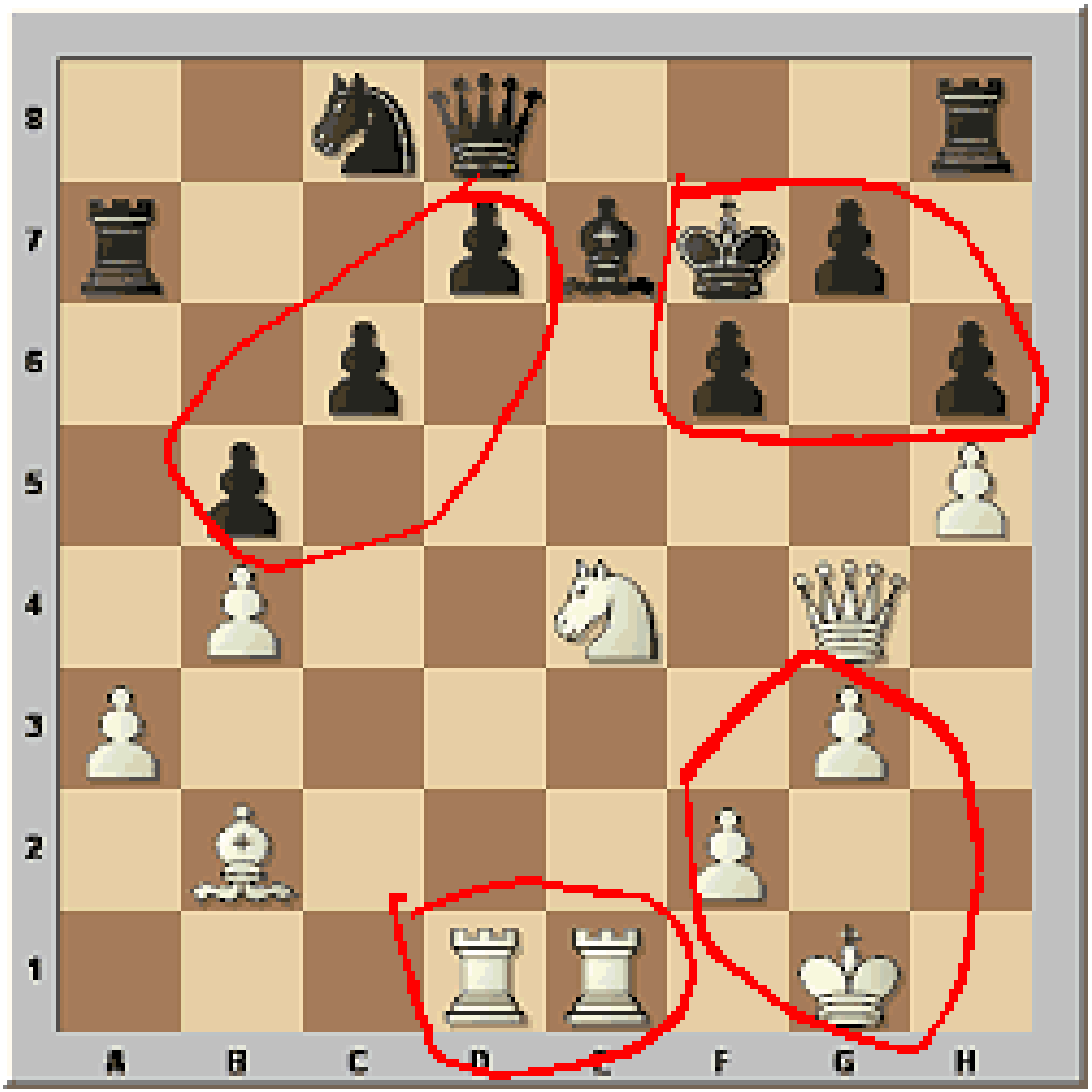


But chess experts do no better than novices in random positions

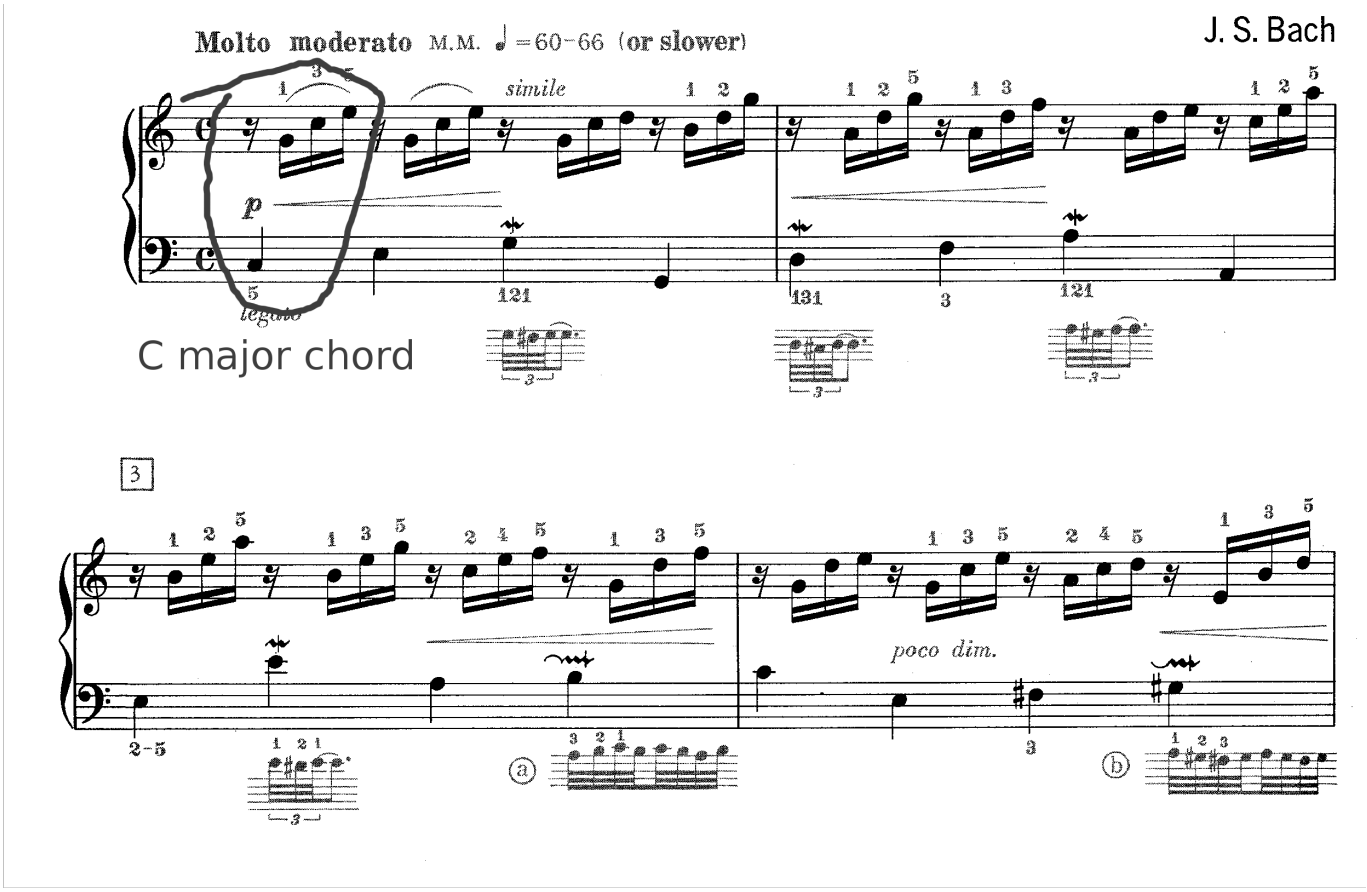


Source: (Gobet and Simon, 1996)

Experts use chunks to understand chess positions

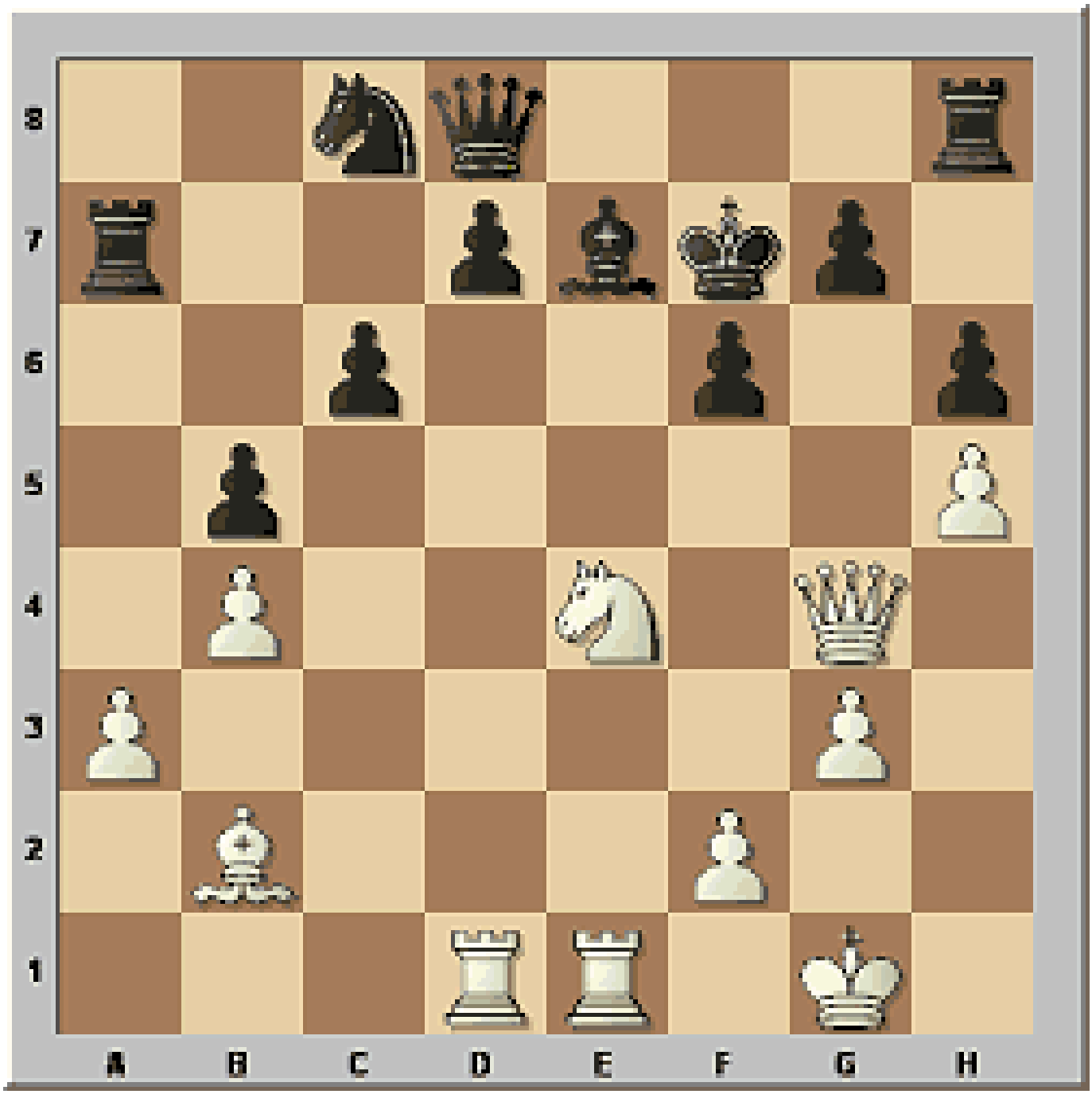


Experts use chunks to understand music



Developing chess expertise requires a special kind of practice

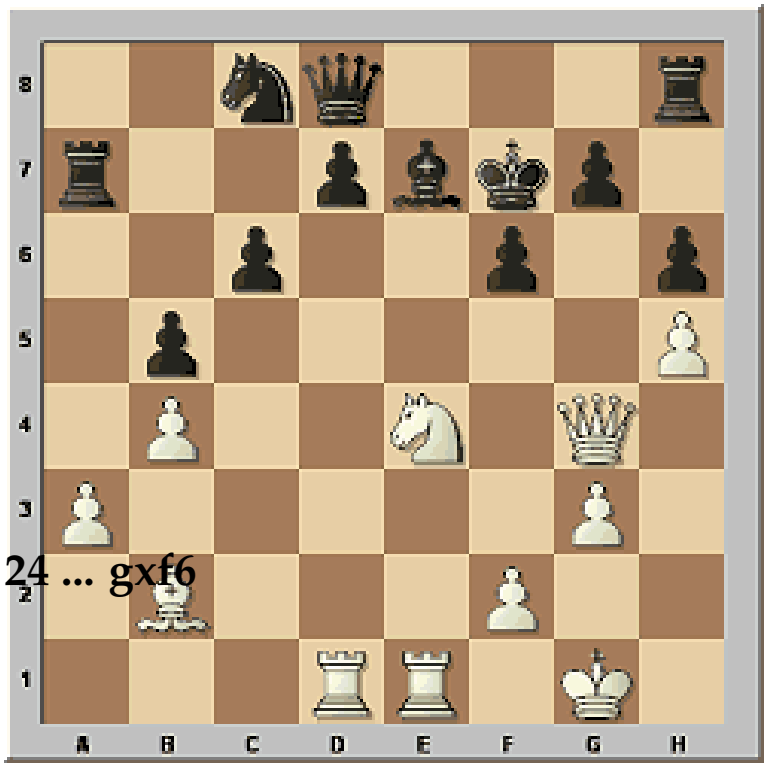
You ask yourself: What move would you (white) make?



Chess expertise develops from reflective analysis

Then compare with grandmaster moves and analysis:

Kasparov–Ulf Andersson (Tilburg 1981)



24 ... gxf6

24 Nxf6!!

White's army is fully mobilized and well coordinated, while Black's rooks are still unconnected, he has glaring light-square weaknesses and his king is vulnerable. In such a situation a tactical solution is the logical outcome.

The pretty point of White's idea is 24...Bxf6 25 Qg6+ Kf8 26 Bxf6 gxf6 27 Re6!, winning.

25 Qg6+ Kf8

26 Bc1!

Kasparov plays for mate, but even the endgame after 26 Rxe7 Qxe7 27 Bxf6 Qh7 28 Bxh8 Qxg6 (28...Qxh8 loses to 29 Re1 Ne7 30 Qd6) 29 hxg6 is hopeless for Black; e.g., 29 ... Ne7 30 Bd4 and Bc5.

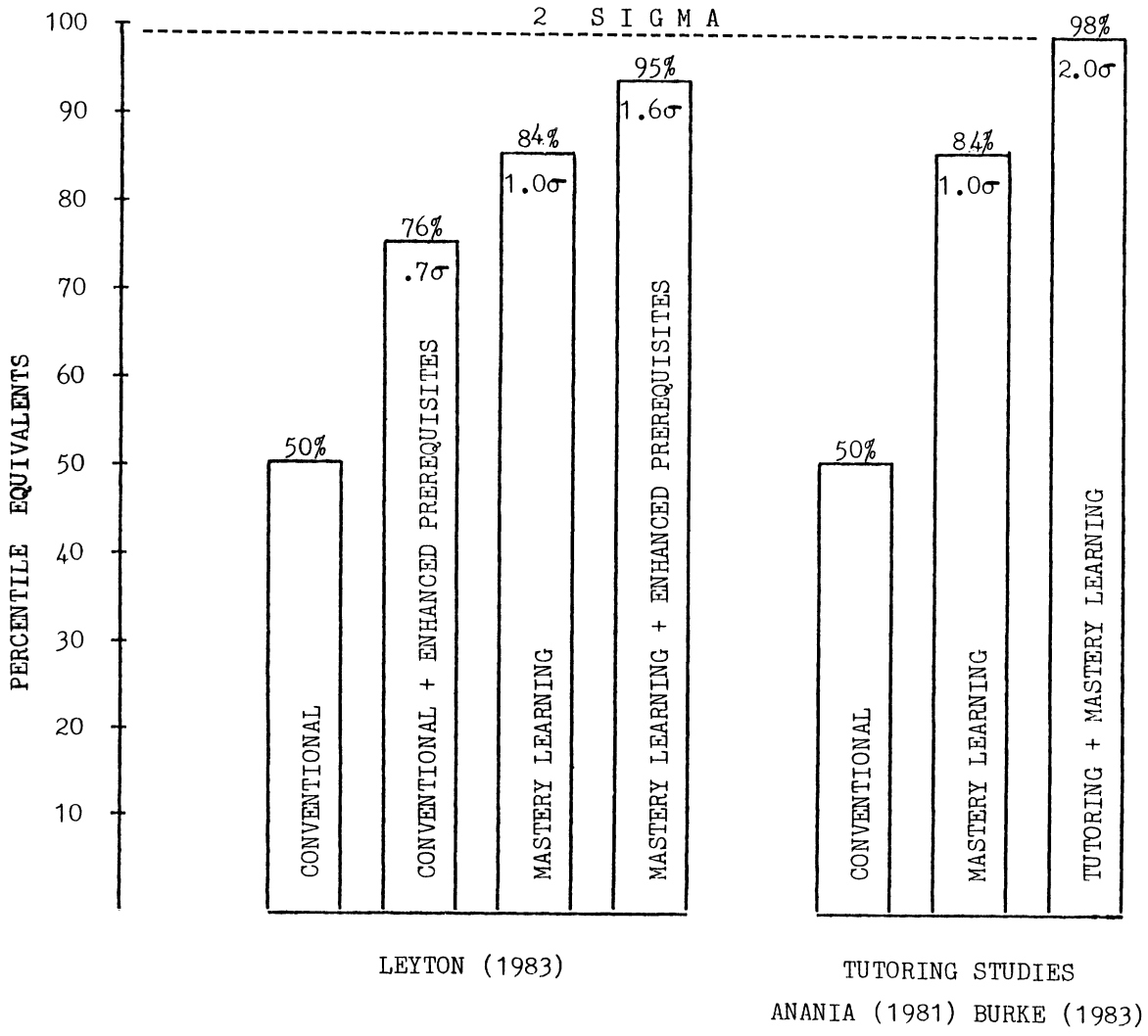
Deliberate practice is essential to developing expertise

Study of $n = 375$ tournament players' chess rating:

$$
\begin{aligned}
&\mathbf{195} \times \log(\text{total hours serious study}) \\
+\ &\mathbf{33} \times \log(\text{total hours tournament play}) \\
+\ &\mathbf{9} \times \text{years private instruction} \\
+\ &\mathbf{4} \times \text{years group instruction} \\
\hline
= \quad & \text{chess rating}
\end{aligned}
$$

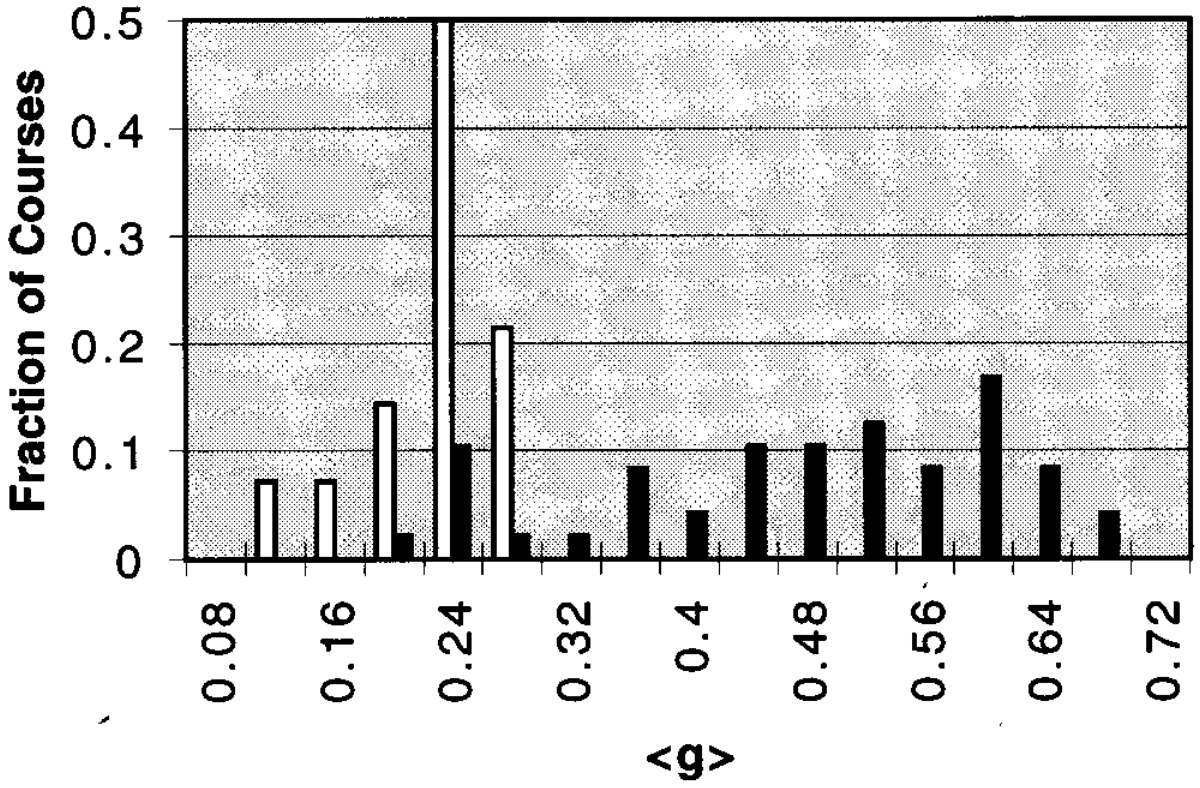
Source: (Charness et al., 2005)

Tutorial teaching is hard to beat, partly because it incorporates lots of deliberate practice



Source: (Bloom, 1984)

Interactive classroom teaching incorporates deliberate practice and increases learning



Source: (Hake, 1988)

Guessing and approximating are forms of deliberate practice

Go forth and practice. . . deliberately!

References

Gobet, F. and Simon, H. A. (1996). Recall of random and distorted chess positions: Implications for the theory of expertise. *Memory & Cognition, 24*(4), 493-503.

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