

6.055J/2.038J (Spring 2009)

Homework 3

Do the following warmups and problems. Due in class on **Friday, 03 Apr 2009**.

Open universe: Collaboration, notes, and other sources of information are **encouraged**. However, avoid looking up answers until you solve the problem (or have tried hard). That policy helps you learn the most from the problems.

Homework will be graded with a light touch: P (made a reasonable effort), D (did not make a reasonable effort), or F (did not turn in).

Warmups

1. Minimum power

In lecture we estimated the flight speed that minimizes energy consumption. Call that speed v_E . We could also have estimated v_P , the speed that minimizes power consumption. What is the ratio v_P/v_E ?

2. Solitaire

You start with the numbers 3, 4, and 5. At each move, you choose any two of the three numbers – call the choices a and b – and replace them with $0.8a - 0.6b$ and $0.6a + 0.8b$. The goal is to reach 4, 4, 4. Can you do it? If yes, give a move sequence; if no, show that you cannot.

3. Highway vs city driving

Here is a measure of the importance of drag for a car moving at speed v for a distance d :

$$\frac{E_{\text{drag}}}{E_{\text{kinetic}}} \sim \frac{\rho v^2 A d}{m_{\text{car}} v^2}.$$

a. Show that the ratio is equivalent to the ratio

$$\frac{\text{mass of the air displaced}}{\text{mass of the car}}$$

and to the ratio

$$\frac{\rho_{\text{air}}}{\rho_{\text{car}}} \times \frac{d}{l_{\text{car}}},$$

where ρ_{car} is the density of the car (i.e. its mass divided by its volume) and l_{car} is the length of the car.

b. Make estimates for a typical car and find the distance d at which the ratio becomes significant (say, roughly 1). How does the distance compare with the distance between exits on the highway and between stop signs or stoplights on city streets?

4. Mountains

Look up the height of the tallest mountain on earth, Mars, and Venus, and explain any pattern in the three heights.

Problems

5. Raindrop speed

- How does a raindrop's terminal velocity v depend on the raindrop's radius r ?
- Estimate the terminal speed for a typical raindrop.
- How could you check your estimate in part (b)?

6. Bird flight

- For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed v depend on mass M and air density ρ ? In other words, what are the exponents α and β in $v \propto \rho^\alpha M^\beta$?
- Use that result to write the ratio $v_{747}/v_{\text{godwit}}$ as a product of dimensionless factors, where v_{747} is the minimum-energy speed of a 747, and v_{godwit} is the minimum-energy speed of a bar-tailed godwit. Then estimate the dimensionless factors and their product. Useful information: $m_{\text{godwit}} \sim 0.4 \text{ kg}$.
- Use v_{747} , from experience or from looking it up, to find v_{godwit} . Compare with the speed of the record-setting bar-tailed godwit, which made its 11,570 km journey in 8 days, 12 hours.

7. Checking plane fuel-efficiency calculation

This problem offers two more methods to estimate the fuel efficiency of a plane.

- Use the cost of a plane ticket to estimate the fuel efficiency of a 747, in passenger-miles per gallon.
- According to Wikipedia, a 747-400 can hold up to $2 \cdot 10^5 \ell$ of fuel for a maximum range of $1.3 \cdot 10^4 \text{ km}$. Use that information to estimate the fuel efficiency of the 747, in passenger-miles per gallon.

How do these values compare with the rough result from lecture, that the fuel efficiency is comparable to the fuel efficiency of a car?

Optional

8. Inertia tensor

[For those who know about inertia tensors.] Here is the inertia tensor (the generalization of moment of inertia) of a particular object, calculated in a lousy coordinate system:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 4 & 5 \end{pmatrix}$$

- a. Change coordinate systems to a set of principal axes. In other words, write the inertia tensor as

$$\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

and give I_{xx} , I_{yy} , and I_{zz} . *Hint:* What properties of a matrix are invariant when changing coordinate systems?

- b. Give an example of an object with a similar inertia tensor. On Friday in class we'll have a demonstration.

9. Resistive grid

In an infinite grid of 1-ohm resistors, what is the resistance measured across one resistor?

To measure resistance, an ohmmeter injects a current I at one terminal (for simplicity, say $I = 1$ A), removes the same current from the other terminal, and measures the resulting voltage difference V between the terminals. The resistance is $R = V/I$.

Hint: Use symmetry. But it's still a hard problem!

