6.055J/2.038J (Spring 2009)

Homework 1

Do the following warmups and problems. Due in class on ***Tuesday, 17 Feb 2009****.*

Open universe: *Collaboration, notes, and other sources of information are* ***encouraged.*** *However, avoid looking up answers until you solve the problem (or have tried hard). That policy helps you learn the most from the problems.*

Homework will be graded with a light touch: P (made a reasonable effort), D (did not make a reasonable effort), or F (did not turn in).

Warmups

1. One or few

Use the 1 or 'few' method of multiplication (and division) to do the following calculations mentally, and compare the approximate result with the actual answer:

a. A random multiplication problem generated by a short Python program:

$$161 \times 294 \times 280 \times 438.$$

The actual product is $\approx 5.8 \cdot 10^9$.

b. Earth's surface area $A = 4\pi R^2$, where the radius is $R \sim 6 \cdot 10^6$ m. The actual surface area is $5.1 \cdot 10^{14}$ m^2.

c. $10! \equiv 10 \times 9 \times \cdots \times 1$. The actual product is $3,628,800$.

2. Two-to-one-odds ranges

Give your two-to-one-odds range for the following quantities *without* doing a divide-and-conquer estimate:

a. Mass of a full 747 jumbo jet. In **Problem 4** you use divide and conquer to make a more precise estimate.

b. Mass of air in the 6.055J/2.038J classroom. In **Problem 5** you use divide and conquer to make a more precise estimate, and then you evaluate the precision of the new estimate.

3. Combining plausible ranges

If the plausible range for x and for y is $1 \ldots 10$ (and x and y are independent), what is the midpoint of the plausible range for the product xy? (The midpoint of a plausible range is the geometric mean of its endpoints.) What is the plausible range for xy?

Problems

4. 747

Estimate the mass of a full 747 jumbo jet, explaining your estimate using a tree. Then compare data online against your estimate from this problem and from **Problem 2.a**. We'll use the mass later in the course when we estimate how much energy it costs to fly.

5. Air mass

a. Use divide-and-conquer to estimate the mass of air in the 6.055J/2.038J classroom and explain your estimate with a tree. If you have not yet seen the classroom, try harder to attend lecture!

b. Give your plausible ranges (your 2-to-1-odds ranges) for the leaf nodes in your tree, and use those ranges to estimate your plausible range for the mass of air in the room.

c. Compare this new plausible range with the initial range you gave in **Problem 2.b**.

6. Your turn to create

Invent – but do not solve! – an estimation question that divide and conquer would help solve. To give you an idea for the kind of problems that work well, the classic of this genre, due to Fermi, is 'How many piano tuners are there in Chicago?' Other examples are the problems that we solved in lecture.

Particularly interesting or instructive questions might appear on the website or as examples in lecture or the notes (let me know should you *not* want your name attributed in case your question gets selected).