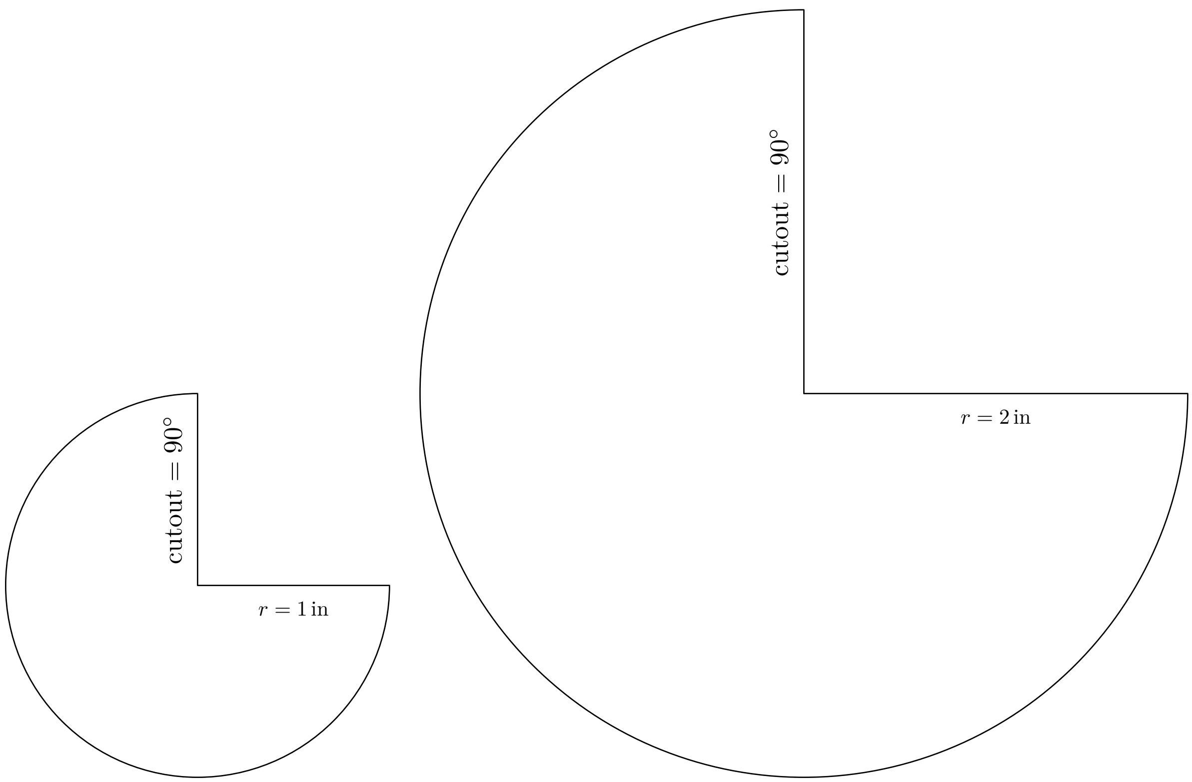
5.4 Drag

This section section contains a proportional-reasoning analysis of drag – using a home experiment – and then applies the results to jumping fleas.

5.4.1 Home experiment using falling cones

Here is a home experiment for understanding drag. Photocopy this page and cut out these templates, then tape the edges together to make a cone:



If you drop the small cone and the big cone, which falls faster? In particular, what is the ratio of their fall times $t_{\text{big}}/t_{\text{small}}$? The large cone, having a large area, feels more drag than the small cone does. On the other hand, the large cone has a higher driving force (its weight) than the small cone has. To decide whether the extra weight or the extra drag wins requires finding how drag depends on the parameters of the situation.

However, finding the drag force is a very complicated calculation. The full calculation requires solving the Navier–Stokes equations:

$$(\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v}.$$

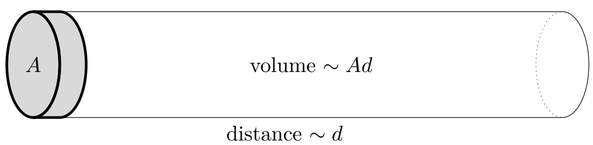
And the difficulty does not end with this set of second-order, coupled, nonlinear partial-differential equations. The full description of the situation includes a fourth equation, the continuity equation:

$$\nabla\cdot\mathbf{v} = 0.$$

One imposes boundary conditions, which include the motion of the object and the requirement that no fluid enters the object – and solves for the pressure p and the velocity gradient at the surface of the object. Integrating the pressure force and the shear force gives the drag force.

In short, solving the equations analytically is difficult. I could spend hundreds of pages describing the mathematics to solve them. Even then, solutions are known only in a few circumstances, for example a sphere or a cylinder moving slowly in a viscous fluid or a sphere moving at any speed in an zero-viscosity fluid. But an inviscid fluid – what Feynman calls 'dry water' – is particularly irrelevant to real life since viscosity is the reason for drag, so an inviscid solution predicts zero drag! Proportional reasoning, supplemented with judicious lying, is a simple and quick alternative.

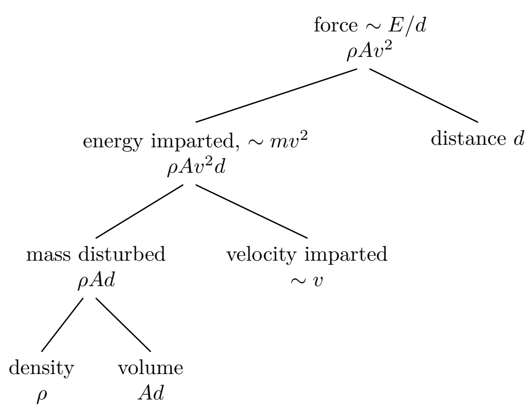
The proportional-reasoning analysis imagines an object of cross-sectional area A moving through a fluid at speed v for a distance d:



The drag force is the energy consumed per distance. The energy is consumed by imparting kinetic energy to the fluid, which viscosity eventually removes from the fluid. The kinetic energy is mass times velocity squared. The mass disturbed is ρAd, where ρ is the fluid density (here, the air density). The velocity imparted to the fluid is roughly the velocity of the disturbance, which is v. So the kinetic energy imparted to the fluid is ρAv^2d, making the drag force

$$F \sim \rho Av^2.$$

The analysis has a divide-and-conquer tree:



The result that $F_{\text{drag}} \sim \rho v^2 A$ is enough to predict the result of the cone experiment. The cones reach terminal velocity quickly – a result discussed later in the book in **Part 3** – so the relevant quantity in finding the fall time is the terminal velocity. From the drag-force formula, the terminal velocity is

$$v \sim \sqrt{\frac{F_{\text{drag}}}{\rho A}}.$$

Since the air density ρ is the same for the large and small cone, the relation simplifies to

$$v \propto \sqrt{\frac{F_{\text{drag}}}{A}}.$$

The cross-sectional areas are easy to measure with a ruler, and the ratio between the small- and large-cone terminal velocities is even easier. The experiment is set up to make the drag force easy to measure: Since the cones fall at their respective terminal velocities, the drag force equals the weight. So

$$v \propto \sqrt{\frac{W}{A}}.$$

Each cone's weight is proportional to its cross-sectional area, because they are geometrically similar and made out of the same piece of paper. With $W \propto A$, the terminal velocity becomes

$$v \propto \sqrt{\frac{A}{A}} = A^0.$$

In other words, the terminal velocity is independent of A, so the small and large cones should fall at the same speed. To test this prediction, I stood on a handy table and dropped the two cones. The fall lasted about two seconds, and they landed within 0.1 s of one another!

5.4.2 Effect of drag on fleas jumping

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

$$E_{\text{drag}} \sim \underbrace{\rho v^2 A}_{F_{\text{drag}}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$