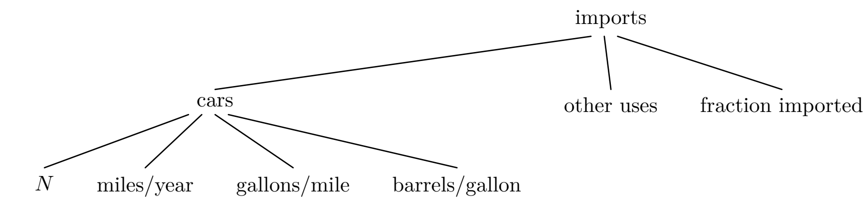
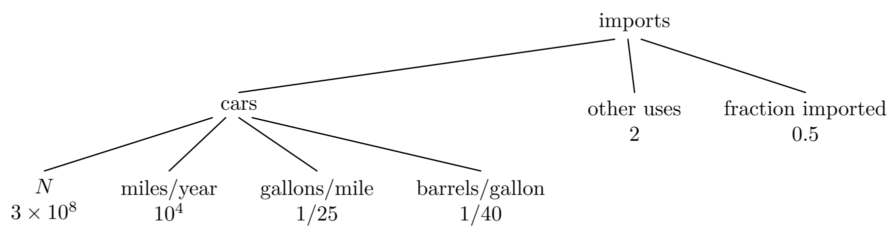


Now guess values for the unnumbered leaves. There are 3×10^8 people in the United States, and it seems as if even babies own cars. As a guess, then, the number of cars is $N \sim 3\times10^8$. The annual miles per car is maybe 15,000. But the N is maybe a bit large, so let's lower the annual miles estimate to 10,000, which has the additional merit of being easier to handle. A typical mileage would be 25 miles per gallon. Then comes the tricky part: How large is a barrel? One method to estimate it is that a barrel costs about \$100, and a gallon of gasoline costs about \$2.50, so a barrel is roughly 40 gallons. The tree with numbers is:



All the leaves have values, so I can propagate upward to the root. The main operation is multiplication. For the 'cars' node:

$$3\times10^8\,\text{cars}\times\frac{10^4\,\text{miles}}{1\,\text{car–year}}\times\frac{1\,\text{gallon}}{25\,\text{miles}}\times\frac{1\,\text{barrel}}{40\,\text{gallons}}\sim3\times10^9\,\text{barrels/year}.$$

The two adjustment leaves contribute a factor of $2\times0.5=1$, so the import estimate is

$$3\times10^9\,\text{barrels/year}.$$

For 2006, the true value (from the US Dept of Energy) is 3.7×10^9 barrels/year!

This result, like the pit spacing, is surprisingly accurate. Why? **Section 2.5** explains a random-walk model for it, which suggests that the more you subdivide, the better.

But before discussing that model, try one more example.

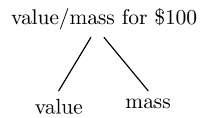
2.4 Gold or bills?

Having broken into a bank vault, should you take the \$100 bills or the gold?

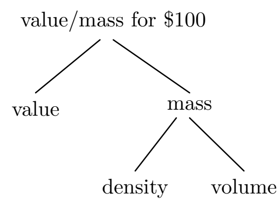
The choice depends on how easily and losslessly you can fence the loot and on other issues outside the scope of this book. But we can study one question: Which choice lets you carry away the most money? The weight or the volume may limit how much you can carry and, more importantly for this problem, affect your choice. To make a start, let's assume that you are limited by the weight (actually, the mass) that you can carry, The problem then depends on two subproblems: the value per mass for \$100 bills and for gold. In tree form:

The value per mass of gold might be a familiar figure from the newspaper or from the financial section of the evening news. It is now (2008) about \$800/oz (oz being the abbreviation for an ounce). As a rough check on the memory – e.g. should the price be \$80/oz or \$8000/oz? – here is another method. When the gold standard was reintroduced as the dollar standard in 1945, gold was set at \$35/oz. Inflation has probably devalued the dollar by at least a factor of 10 since then, so gold should be around \$350/oz now. The half-remembered figure of \$800/oz seems reasonable.

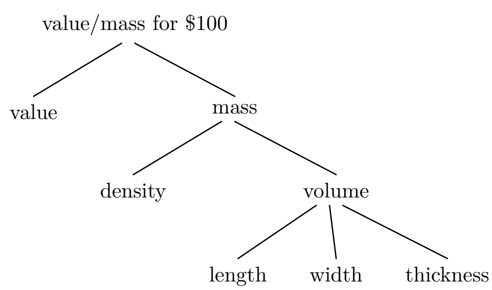
Finding the value per mass of a dollar bill starts with this tree:



The value is specified in the problem as \$100, but the mass needs work. It breaks into the volume times the density, so the value per mass tree becomes:

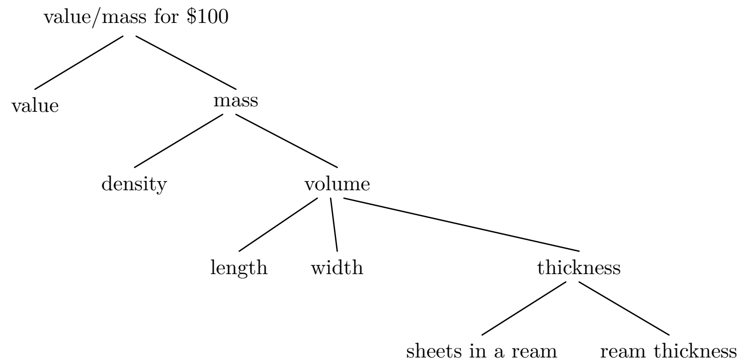


The volume breaks into length times width times thickness, so the tree grows:



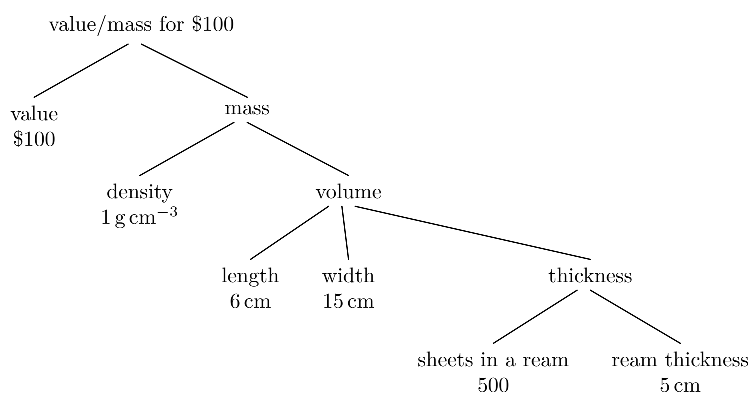
To find the length and width of a bill, lay a ruler next to a dollar bill or guess that a bill measures 2 or 3 inches by 6 inches or 6 cm × 15 cm. To develop a feel for sizes, make a guess and then, if you feel uneasy, check your answer with a ruler. As your feel for sizes develops, you will need the ruler less frequently.

Guessing the thickness of a bill is harder than guessing the length or width. However, as George Washington Plunkitt, onetime boss of Tammany Hall, said: 'I seen my opportunities and I took 'em.' Pretend that a dollar bill is made from ordinary paper. To find its thickness, look around. Next to the computer used to compose this book sits an inkjet printer; next to the printer is a ream of printer paper. If we know how thick the ream is and how many sheets it has, then we know the thickness of one sheet. You might call this technique multiply and conquer. The general lesson is that tiny values, those much below typical human experience, need to be magnified to make them easy to estimate. Large values, those much above typical human experience, need to be broken into smaller parts to make them easy to estimate. With this last step of magnifying the sheet's thickness, the full tree for the value per mass of the bill becomes:



The ream (500 sheets) is roughly 5 cm thick. The only missing leaf value is the density of a bill. To find the density, use what you know: Money is paper. Paper is wood or fabric, except for many complex processing stages whose analysis is far beyond the scope of this book. When a process, here papermaking, looks formidable, forget about it and hope that you'll be okay anyway. More important is to get an estimate; correct the egregiously inaccurate assumptions later (if ever). How dense is wood? Wood barely floats, so its density is roughly that of water, which is $\rho \sim 1\,\mathrm{g\,cm^{-3}}$. So the density of a $100 bill is roughly $1\,\mathrm{g\,cm^{-3}}$.

Here is a tree including all the leaf values:



Now propagate the leaf values upward. The thickness of a bill is roughly 10^{-2} cm, so the volume of a bill is roughly

$$V \sim 6\,\mathrm{cm} \times 15\,\mathrm{cm} \times 10^{-2}\,\mathrm{cm} \sim 1\,\mathrm{cm}^3.$$

So the mass is

$$m \sim 1\,\mathrm{cm}^3 \times 1\,\mathrm{g\,cm^{-3}} \sim 1\,\mathrm{g}.$$

How simple! Therefore the value per mass of a $100 bill is $100/g. To choose between the bills and gold, compare that value to the value per mass of gold. Unfortunately our figure for gold is in dollars per ounce rather than per gram. Fortunately one ounce is roughly 27 g so $800/oz is roughly $30/g. Moral: Take the $100 bills but leave the $20 bills.

2.5 Random walks