A tree is well suited for representing divide-and-conquer reasoning. This tree summarizes the whole analysis in one figure. This compact representation make it easier to find and fix mistakes in the numbers or the structure or to see which parts of the estimate are the least reliable (and probably need more subdividing).

2.3 Oil imports

For the next example of divide-and-conquer reasoning, we will make a tree from the beginning. The problem is to estimate how much oil the United States imports, in barrels per year. There are many ways to estimate this number – good news for making ro-

bust estimates. Here I estimate it by estimating how much oil cars use, then adjusting that number to account for two items: first, that cars are not the only consumer of oil; and second, that imports are only a fraction of the oil consumed. The starting tree has just three leaves.
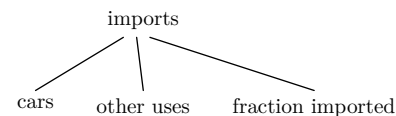
The rightmost two leaves are hard to guess values for, but dividing and conquering does not help. So I'll have to guess them.
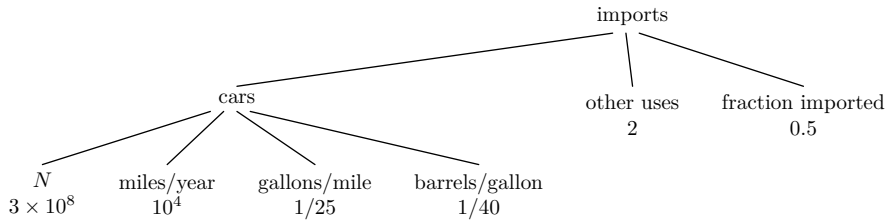
Cars are a major consumer of oil, but there are other transport uses (trucks, trains, and planes), and there is heating and cooling. Given how important these other uses are, perhaps cars account for one-half of the oil consumption: a significant fraction leaving room for other significant uses. So I need to double the car result to account for other uses.

Imports are a large fraction of total consumption, otherwise we would not read so much in the popular press about oil production in other countries, and about our growing dependence on imported oil. Perhaps one-half of the oil usage is imported oil. So I need to halve the total use to find the imports.

The third leaf, cars, is too complex to guess a number immediately. So divide and conquer. One subdivision is into number of cars, miles driven by each car, miles per gallon, and gallons per barrel:

imports
cars   other uses   fraction imported
N   miles/year   gallons/mile   barrels/gallon
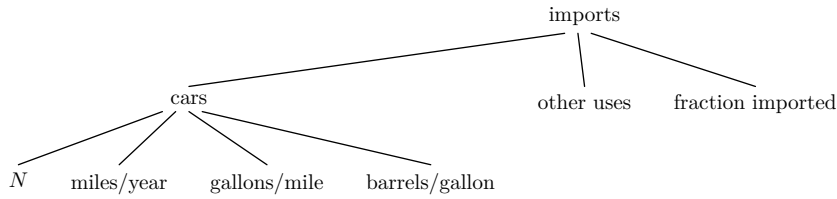
Now guess values for the unnumbered leaves. There are 3×10^8 people in the United States, and it seems as if even babies own cars. As a guess, then, the number of cars is $N \sim 3\times10^8$. The annual miles per car is maybe 15,000. But the N is maybe a bit large, so let's lower the annual miles estimate to 10,000, which has the additional merit of being easier to handle. A typical mileage would be 25 miles per gallon. Then comes the tricky part: How large is a barrel? One method to estimate it is that a barrel costs about \$100, and a gallon of gasoline costs about \$2.50, so a barrel is roughly 40 gallons. The tree with numbers is:

imports
cars   other uses   fraction imported
                2   0.5
N   miles/year   gallons/mile   barrels/gallon
3×10^8 10^4 1/25   1/40

All the leaves have values, so I can propagate upward to the root. The main operation is multiplication. For the 'cars' node:

$$3\times10^8 \text{ cars} \times \frac{10^4 \text{ miles}}{1 \text{ car–year}} \times \frac{1 \text{ gallon}}{25 \text{ miles}} \times \frac{1 \text{ barrel}}{40 \text{ gallons}} \sim 3\times10^9 \text{ barrels/year.}$$

The two adjustment leaves contribute a factor of $2 \times 0.5 = 1$, so the import estimate is

$$3\times10^9 \text{ barrels/year.}$$

For 2006, the true value (from the US Dept of Energy) is 3.7×10^9 barrels/year!

This result, like the pit spacing, is surprisingly accurate. Why? **Section 2.5** explains a random-walk model for it, which suggests that the more you subdivide, the better.

But before discussing that model, try one more example.

2.4 Gold or bills?

Having broken into a bank vault, should you take the \$100 bills or the gold?

The choice depends on how easily and losslessly you can fence the loot and on other issues outside the scope of this book. But we can study one question: Which choice lets you carry away the most money? The weight or the volume may limit how much you can carry and, more importantly for this problem, affect your choice. To make a start, let's assume that you are limited by the weight (actually, the mass) that you can carry, The problem then depends on two subproblems: the value per mass for \$100 bills and for gold. In tree form: