## 2.2 Tree representations

The structure of a divide-and-conquer estimate, with the steps of subdividing, is hierarchical. An ideal representation of hierarchical structure is a tree. Therefore to illustrate this representation, this section redoes our analysis of the pit spacing using trees.

The estimate using the area and capacity of a CDROM is the most elaborate method in **Section 2.1** for finding the pit spacing, so let's represent it as a tree. The root of the tree is 'capacity, area', a tag that reminds us of the method. As the tag suggests, to do the estimate requires finding the capacity and area, so the tree starts with two branches.
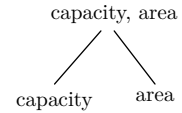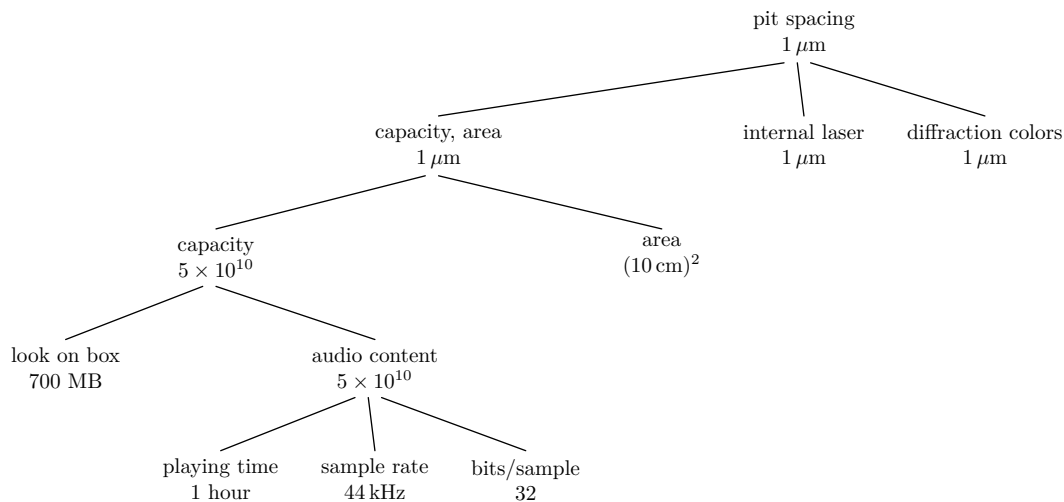
The area is easy to estimate, so the next step is to subdivide the capacity into easier parts. The first method is to look on a CDROM box, which says something like 'capacity 700MB'. A second method is to estimate the bits required to store the audio information that fit on an audio CD, by estimating the playing time, the sampling rate, and the bits per sample, where here the two channels for stereo are included in the bits per sample.
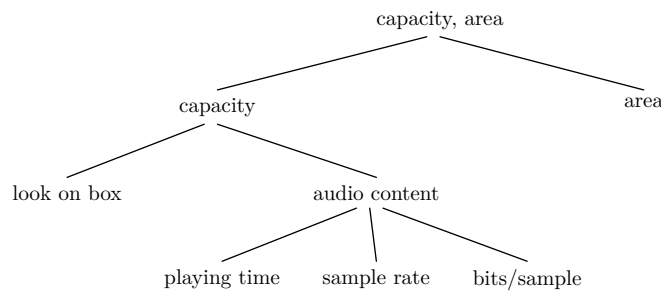
Now fill in the numbers at the leaves and propagate toward the root of the tree. The audio lasts for about an hour, which we estimated as either 20 popular music songs of 3 minutes duration or as Beethoven's Ninth Symphony. The sampling rate is 44 kHz. The samples are 32 bits each including the factor of two for stereo. The tree including these values is:
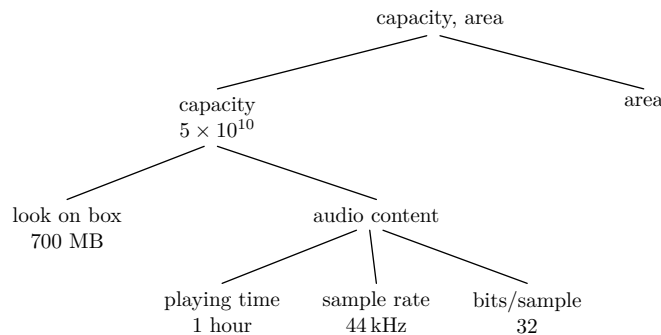
This tree is one subtree of the whole analysis. That analysis included two other methods: knowledge of the laser inside a CD player, and observing the shimmering colors due to diffraction. Including those methods – and finding that the three methods agree – makes the estimate of $1\,\mu$m robust. In pictorial terms, it makes the tree sturdy:

pit spacing
$1\,\mu\mathrm{m}$

capacity, area   internal laser   diffraction colors
$1\,\mu\mathrm{m}$ $1\,\mu\mathrm{m}$ $1\,\mu\mathrm{m}$

capacity   area
5×10^{10} $(10\,\mathrm{cm})^2$

look on box   audio content
700 MB 5×10^{10}

playing time   sample rate   bits/sample
1 hour   44 kHz   32

A tree is well suited for representing divide-and-conquer reasoning. This tree summarizes the whole analysis in one figure. This compact representation make it easier to find and fix mistakes in the numbers or the structure or to see which parts of the estimate are the least reliable (and probably need more subdividing).

2.3 Oil imports

For the next example of divide-and-conquer reasoning, we will make a tree from the beginning. The problem is to estimate how much oil the United States imports, in barrels per year. There are many ways to estimate this number – good news for making robust estimates. Here I estimate it by estimating how much oil cars use, then adjusting that number to account for two items: first, that cars are not the only consumer of oil; and second, that imports are only a fraction of the oil consumed. The starting tree has just three leaves.

imports

cars   other uses   fraction imported
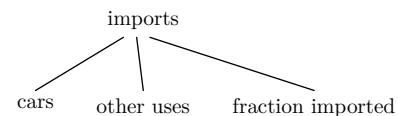
The rightmost two leaves are hard to guess values for, but dividing and conquering does not help. So I'll have to guess them.

Cars are a major consumer of oil, but there are other transport uses (trucks, trains, and planes), and there is heating and cooling. Given how important these other uses are, perhaps cars account for one-half of the oil consumption: a significant fraction leaving room for other significant uses. So I need to double the car result to account for other uses.

Imports are a large fraction of total consumption, otherwise we would not read so much in the popular press about oil production in other countries, and about our growing dependence on imported oil. Perhaps one-half of the oil usage is imported oil. So I need to halve the total use to find the imports.

The third leaf, cars, is too complex to guess a number immediately. So divide and conquer. One subdivision is into number of cars, miles driven by each car, miles per gallon, and gallons per barrel: