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GLOBAL COMMENTS

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Another tiny bug you might want to consider taking out of NB is, when you misspell a word, the little red line comes up under the word to tell you. This is nice, except usually when this happens you can right click it and it will show you the correct spelling. It's not a huge glitch, but it'd be nice I think.

I don't see any red lines under misspelled words, so I think it's a browser/platform-specific issue. At least it's telling you when things are wrong.

This definitely took a couple readings to understand completely. I feel like it is a concept fundamental to further exercises, so I won't suggest that it should be softened, but it is definitely one of the more difficult logical processes.

I feel this way too, it was definitely one of the hardest, if not the hardest, sections to read through. Given the depth of the calculations, I feel that it lacked helpful summarization and conclusions of the concept which the previous chapters had done well one.

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COMMENTS ON PAGE 1

Read this section for Thursday's memo. Don't worry about the funny page breaks. (All will change in the summer when I sit down with all your comments...)

so

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Might it be too simple to understand all the essentials of atomic theory though?

Yes, but for a lot of things it is realistically the ONLY one we can study. The rest, even helium, are just too complicated

As far as I remember, it's the only atom we studied in quantum mechanics I. And even that was complicated...

Also, one of our mantras is to start simple, try to get somewhere, and then add complexity.

oooh, I like Feynman lectures, great use of ethos.

If all knowledge were to be destroyed, I really doubt that the future people would believe this statement at face value...

interesting to think about

I wonder what a famous scientist of another background might pick.

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This definitely isn't the one thing I would've thought of, but I also don't know the enormous amount of information that can be found in that one sentence.

It definitely is a core concept to our understanding of the world and universe that we probably take for granted though. It is hard to imagine not having this piece of information in our minds.

It's what all the laws of of physics reign over.

Wouldn't instructions be more helpful?

This seems so informal. But I guess with the going of scientific knowledge, so goes the scientific jargon.

I guess i agree with this statement—everything is based on atoms after all right…it's what everything is made of and the interactions between atoms go a long way in explaining almost everything in the physical world…

Too bad humans have this knack for not believing things until they are proven.

Will we find out why?

Is it bad that my first thought was to relate this to information theory, and to relate the number of bits required to transmit this information to the total amount of information stored?

Nah, it's natural.

Did he just postulate this or did he have some proof?

Yeah, how did he end up hypothesizing this with no way of measuring or seeing them? That takes some serious insight..

Oh, I think they had all sorts of educated guesses back then. Perhaps he just happened to be the lucky one who wrote his thoughts down.

'Who is Democritus?" would be a good side bar note...I'm apparently not letting go of this idea. sorry, if it bothers you

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Has anyone read this? Is it worth reading?

I've read it, and I would say it's worth the read if you have the time. It gives a great overview of the philosophers and theories that came before us.

I've read parts of it. It's interesting but long, if I remember.

Bertrand Russell has a lot of good works.

As far as I know, Bertrand Russell was a great philosopher but not a great historian of philosophy. The book is fun to read but don't trust it to get the views of past philosophers right.

I think this is like the Reader's Digest version of Western Philosophy.

However, with...

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Coming after "classical and quantum mechanics", dimensional analysis seems...quaint. All we know about atoms is described by quantum and classical mechanics, so it seems like dimensional analysis is just tacked on because that's what you want to talk about.

I agree. I think if you know classical and quantum mechanics then dimensional analysis probably isn't particularly necessary.

hm maybe dimensional analysis can serve as an easier way to think of classical and quantum mechanics?

This transition seems really forced. From the intro it sounds like you are going to explain atomic theory using mechanics (because that is the basis behind the really broad topics that are talked about in the quoted passage) but in the next section the focus is on dimensional analysis.

well the whole chapter is on dimensional analysis, so we knew it was coming

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Do you think we could use this for every problem on the "diagnostic pretest"? Why do we need other methods when this one does so well?

I'm not sure what you mean by using Dimensional Analysis for every problem. Is that possible? I am still understanding the Dimensional Analysis. Care to explain?

It's always useful, but more so to check your answer or give you an equation. I don't see the help in estimation, like the sequences or proportionalities.

The sequences on the diagnostic is a good example of where dimensional analysis cannot help you. The goal is a pure number (the size of the nth term) as a function of a pure number (n). Both items are dimensionless, so any function would be dimensionally okay. That is, dimensional analysis doesn't place any restriction on what the function is. You need other techniques to make a full toolbox.

This whole concept seems very useful, if mastered. It could significantly help on tests to use units (I don't but probably should)

I don't know how useful this is a heading though... isn't the whole section on that.

This class integrates so many different aspects of Science and I think that is so interesting.

Do you mean to say "uses the hydrogen atom..."?

Agreed, it sounds awkward as written.

This first sentence is also a bit awkward, mostly because of the "in order...two questions" phrase at the end.

is "it" referring to an atom?

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For some reason I found this phrasing a little confusing and had to do a double-take...

I agree. The phrasing is confusing and breaks up the flow of the sentence.

This and the previous sentence could be combined to keep the flow going - at the moment it brings the flowing style of the prior readings to a staccato pattern.

i think the confusion has to do with the ambiguous "it" and "that"

Perhaps, something along the lines of the following:

"The first question is how big it is, which sets the standard for the size of more complex atoms and molecules"

Why does it set the size for more complex atoms?

Because the more complex atoms are built from the smaller atoms.

I would've liked to see some bullet points. Clears things up. Especially when your asking a 2 part question.

Why are these questions important? or rather why are they so important?

I know it states that it let's you find size of more complex, and things like stiffness, speed and energy content. But without understanding some fairly complex physics that relates bond energy to these things, how do we go about relating things like that on a daily basis?

are we going to take the periodic table into account?

Should this better read "how big it is".

The "it" could be a bit ambiguous. Are we trying to find radius, volume, something else? (I realize they are all related)

I think he's referring to how big is the atom itself (dimensionwise)... but this sentence definitely reads awkwardly.

"how big it is" wouldn't be a question.

I am assuming this will be in a future section? I did not see it in this one.

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Disassemble it nuclearly - or its molecular bonds? Are we just looking at the scope of the Atom

hydrogen is an atom, not a molecule, so there are no molecular bonds. To clarify, perhaps the clause "into its subatomic particles" could be added?

Since there aren't any bonds, how do you disassemble it? and for what reason? I think the termiology is confusing me.

I guess we could disassemble to protons and electrons.

I think just saying "separate the electron from the proton" would make things a lot clearer.

Hydrogen is diatomic as a gas, so it could be the separation of one hydrogen atom from the other in H2. That would be a bond energy as mentioned in the next section. It could also be nucleus/electron separation, which is what happens when it's ionized.

Couldn't it also mean separating the proton from the neutron? I'm not sure why you would want to do that but....

Each lecture always starts off so randomly- but they all link together! Again, you're using the most basic element (literally) to calculate more complex ones

this whole paragraph is awkwardly written.

This wasn't immediately obvious to me, but perhaps I've forgotten chemistry.

I think just means that since hydrogen atoms are prevalent in many complex substances, it's useful to know the energy of a hydrogen bond.

Agreed. I don't think he's saying is the absolute basis of all bonds, just that it's a good basis for comparison and is very important in chemistry and physics

i will argue this point... there are a lot of factors that determine stiffness. this would only be true if we were talking about an absolute pure material in crystalline form

I agree, since all materials are made up of hydrogen-like atoms, they should all have relatively similar bond strength. However, this is not the case even with the same elements. Graphite and diamond are both made from carbon and have very different bond strengths. I'm not sure how plausible this conclusion is without mentioning that there are other factors involved.

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Nice foreshadowing here... gets the reader intrigued about the powerful capabilities of dimension analysis that will soon be explained.

I agree, it makes an impact showing a group of things determined by these energies that are not all completely related.

I agree, it makes an impact showing a group of things determined by these energies that are not all completely related.

I like knowing that there is a reason for learning what I'm learning... that it is useful somehow and applicable to the real world.

I really like this sentence as a way to get the reader excited about why what he or she is about to read is very useful and can be used to derive many important quantities.

I'd use: "All from the basic understanding of hydrogen!"

well it's probably somewhere around half.

what are you referring to?

My earlier statement, about stiffness not solely being dependent on the scaling of bond energies from hydrogen, makes me question the other conclusions like speed of sound and energy from sugar. Its just hard for me to believe that things are that simple.

I would assume that several other factors come in to play to determine these quantities, although we can probably still learn a lot from hydrogen. To me it seems a little too ambitious...

I agree with both those statements, but I think that going straight to the equation takes away from learning how to get variables. Maybe add a bit more and start with a diagram (we have radius, chargy, energy, etc). When it went straight to the equation, I was thinking "I would have never have gotten that right off the bat."

add: "approach"

Is this the first step in answering the question of how big it is?

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Would be useful to have summaries at the end of each chapter w/ a worked through example, at least in the final version.

I thought we were working through examples in the text? Would you like more problems to practice?

What would a more complex model of hydrogen be? I've only ever seen it modeled this way.

You stated that we wanted to find the energy to break an atom, then gave us the equation for the force, then from that got the necessary parameters to get the energy. It seems all very backwards and contrived. Maybe if you figured out another way to get the necessary parameters, and from that got the force or energy, it would make the whole example a lot more legitimate.

Here you use e for the elementary charge, and later you talk about a charge q without relating the two. I feel like this sort of obscures your point and you might want to make a note of this, especially because a casual reader might not recognize that e represents the charge of an electron.

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I thought 8.02 was not a prereq for this class? You might want to mention kqq/r^2 to jog people's memory.

I think this equation is simple enough that a relatively advanced high school student could grasp it (and I think high school and above is the target audience).

I agree with it being simple - while the symbols might initially be scary you can break it down to some constant and r^2

It's not a pre-req, which is why the equation is given here. It's pretty intuitive because all the variables are explained.

8.02 would be a pre-req if instead hte book just said "the magnitude follows the electrostatic force equation" or coulombs law.

I'm pretty sure most of these equations were taught in high school physics class.

You'd be surprised.

I didn't take physics in high school so I hadn't seen it before 8.022. But I agree, it's pretty straight forward and should be left as is.

and we're more focused on learning the method of dimensionless analysis then derivations.

Adding F = this would make it more standard-looking.

Certainly nothing would be lost by adding in the general form for the electrostatic force, and it would probably help make this section more clear, even to people who remember 8.02 well.

it's essentially the same as kqq/r^2 . I think if you've seen it before and didn't remember it, you could just look closely at the equation and realize that, oh, it's just kqq/r^2 .

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This would look a lot neater in cgs...

cgs?

centimeters, grams, seconds

While I am a big proponent of cgs for E&M problems, I have to say that SI gets the point across better than cgs for people who are not physics majors because everyone is familiar with SI.

Very true. The only E&M experience I have ever had has been with cgs and seeing SI things is strange. It's probably best to pick the unit system that most people are familiar with, regardless of how phenomenally stupid that unit system is.

This kinda comes outta nowhere and there's little detail about it...maybe for non-physics students it might be nice to just throw a sentence or two explaining it's meaning or something

I feel like this is a pretty basic formula and that the majority of the class, even non-physics students like myself, have seen this whether it be in high school or 8.02. Besides, the variables are explained in the few sentences after.

Agreed- right now this just looks like a bunch of variables with no meaning to me. It'd be nice to see a derivation or some background of where this is coming from.

Would it help if it read F=(what is there?

I think it would help if it said "F=" and if it had the name somewhere. "Coulomb's law" or something just so people can realize where they've seen this before.

Are we expected to follow the math completely here?

I think it should be mentioned here, in the last reading, and maybe in the beginning of most examples to not just choose relevant variables, but to also make a table. The table helps immensely.

Therefore rather than listing q and ϵ_0 separately, list only $e^2/4\pi\epsilon_0$. And rather than listing r, list a_0 , the common notation for the Bohr radius (the radius of ideal hydrogen). The acceleration of the electron depends on the electrostatic force, which can be constructed from $e^2/4\pi\epsilon_0$ and a_0 , and on its mass m_e . So the list should also include m_e . To find the dimensions of $e^2/4\pi\epsilon_0$, use the formula for force

$\begin{array}{cccc} \omega & T^{-1} & \text{frequency} \\ k & L^{-1} & \text{wavenumber} \\ g & LT^{-2} & \text{gravity} \\ h & L & \text{depth} \\ \rho & ML^{-3} & \text{density} \\ \gamma & MT^{-2} & \text{surface tension} \end{array}$	Var	Dim	What
g LT^{-2} gravity h L depth ρ ML^{-3} density	w	T^{-1}	frequency
h L depth ρ ML density	k	L-1	wavenumber
ρ ML 3 density	g	LT^{-2}	gravity
	h	T	depth
γ MT ⁻² surface tension	ρ	ML ³	density
1 '	γ	MT^{-2}	surface tension

92

$$F = \frac{e^2}{4\pi\epsilon_0} \, \frac{1}{r^2}.$$

Then

$$\left[\frac{e^2}{4\pi\epsilon_0}\right] = \left[r^2\right] \times [F] = ML^3T^{-2}.$$

The next step is to make dimensionless groups. However, no combination of these three items is dimensionless. To see why, look at the time dimension because it appears in only one quantity, $e^2/4\pi\varepsilon_0$. So that quantity cannot occur in a dimensionless group: If it did, there would be no way to get rid of the time dimensions. From the two remaining quantities, a_0 and m_e , no dimensionless group is possible.

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COMMENTS ON PAGE 2

are you going to explain the rest of the variables?

which list? the variables from the thing right above on p1 or this boxed list below? i'm guessing it's the list below but just the word "below" would help

Or the "list of variables we need to think about"

Thought this phrasing was a little awkward but I understand what you mean.

I had to re-read it a few times to understand this phrase.

Using the word 'variable' twice makes it a bit strange. Maybe replace the first one with 'the list below'?

Yeah...I've re read it a few times now and I'm still having trouble understanding it...

You could eliminate e0 with a clever choice of conventions (a la 8.022).

This paragraph doesn't seem entirely clear. I'm not sure how we make the arguments about when q and epsilon naught appear.

are these defined? I don't see them defined.

Can you at least say, explicitly, what these are? I assume q is charge and r radius, but I don't know or remember about epsilon.0. Or what 'e' is, for that matter.

Epsilon_0 is a constant of permittivity, which is the measure of how much resistance is encountered when forming an electric field in a vacuum- its value is about 9*10^-12 F/m

I didn't follow exactly why since q is relevant that it produces a force, that the charge appears only in the combined equation given.

Nor did I. I also don't understand exactly why it is we should include enough variables to generate the force expression. Is it just because we have to start *somewhere*?

I understood this logic in the case of g*sin(Theta) in class, but not here.

Is this quantity produced dimensionless?

Nevermind, its dimensions are determined later on.

Therefore rather than listing q and e_0 separately, list only $e^2/4\pi\epsilon_0$. And rather than listing r, list a_0 , the common notation for the Bohr radius (the radius of ideal hydrogen). The acceleration of the electron depends on the electrostatic force, which can be constructed from $e^2/4\pi\epsilon_0$ and a_0 , and on its mass m_e . So the list should also include m_e . To find the dimensions of $e^2/4\pi\epsilon_0$, use the formula for force

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It would be nice to see a definition of each before getting into this equation. It would at least be nice to see a description of the equation.

I might be biased since I'm an EE but I think we're getting too hung up on where equations is coming from and not on the point of using dimensional analysis. Someone mentioned if 8.02 should be a prereq for this class, but I mean really 8.02 should've been taken freshmen year.

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I agree, and what is q representing?

I agree, I'm a bit confused with the relevance of each variable.

The table is on the side rather than very integrated in the text. I think if most people could just ignore it and keep reading it. I don't think it's that big of a deal to have it in and someone will find it useful.

Idon't quite understand what this is trying to say

I agree..I'm not quite sure what we're after given the above equation. It seems like everything is there already.

I don't understand what these sentences are trying to explain.

It would be really nice if you mentioned what the units of epsilon_0 are just in case the reader, like myself, has never done E&M in SI units.

It would be really nice if you mentioned what the units of epsilon_0 are just in case the reader, like myself, has never done E&M in SI units.

while e and e_0 may be obvious to a physicist or a chemist,I haven't seen these variables for years. Please define them.

thanks for putting this in, I had forgotten what a Bohr radius was

What makes it ideal?

What

gravity

depth

density

frequency

wavenumber

surface tension

where r is the distance between the proton and electron. The list of variables should include enough variables to generate this expression for the force. It could include q, ϵ_0 , and r separately. But that approach is needlessly complex: The charge q is relevant only because it produces a force. So the charge appears only in the combined quantity $e^2/4\pi\epsilon_0$. A similar argument applies to ϵ_0 .

Var

Dim

 T^{-1}

 LT^{-2}

L

 ML^{-3}

 MT^{-2}

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After reading this whole page, I remembered that in the last reading, you made this list of variables and used it to construct a dimensionless value. However, I didn't remember that at first, so initially I was confused about why we were arbitrarily making this list. Perhaps you could put in a reminder sentence or make a bigger point in the first part from the last reading that making this list is a required step?

I really like the tables... keep them in. It helps me to see quickly what is important and breaks up the text

I would agree with the first comment in this thread.

It is very tempting to read in a linear, up-down fashion. Having the box on the side, decreases its importance in the context of a smooth read. That is, I imagine it is easier to keep reading, rather than to stop the text and carefully examine that table on the side.

Maybe placing that table on its own line, between paragraphs would cue readers to actually look at it the first time through. That way, the table is "connected" within the text, instead of being some reference thing to the side.

yes or maybe adding a caption under the table would be useful.

I still don't understand why this table was included. It actually kept distracting me from the other variables that you were talking about in the reading. I feel that the table on page 93 would have been more suitable here.

Out of every section so far I think dimensional analysis has been my favorite. It seems to be the most quickly applicable and yields the most interesting results, in a way that we would normally never go about solving a problem

I feel like this is the wrong chart for this section...shouldn't the chart here include e & amp; epsilon & amp; r?

After reading this, I understand what all of the variables mean, but I never would have been able to come up with them on my own (at least not all of them). I think this would put a big damper on the accuracy of my analysis - how do I avoid leaving things out?

So why doesn't it?

Therefore rather than listing q and ϵ_0 separately, list only $e^2/4\pi\epsilon_0$. And rather than listing r, list a_0 , the common notation for the Bohr radius (the radius of ideal hydrogen). The acceleration of the electron depends on the electrostatic force, which can be constructed from $e^2/4\pi\epsilon_0$ and a_0 , and on its mass m_e . So the list should also include m_e . To find the dimensions of $e^2/4\pi\epsilon_0$, use the formula for force

Var	Dim	What /
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92

$$F = \frac{e^2}{4\pi\epsilon_0} \frac{1}{r^2}.$$

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Why does this table include things like surface tension (it doesn't seem to appear in our analysis)? Our is this just a reference for common dimensions used in general?

A yank (ctrl-Y in Emacs) bug. I copied it from a later section (on waves), for the template, but didn't update it to use the variables for this problem. Whoops.

Whew. I was looking for this comment; I was very lost trying to figure out where all these figured into atoms.

so simple! I actually get it.

I really like this process, it seems so elegant.

i'm not so sure about it actually. it seems to me we're doing a lot of tricks in order to "find" something we should have known the instant we wrote down the equation.

These are interesting units for this equation. The time unit is usually s^-2, but I guess frequency and 1/s are the same.

T means units of time which are seconds here.

I might be mistaken, but didn't you say in the previous reading/section that you can always make things dimensionless with clever manipulations?

Yeah, I agree, I definitely remember that from the previous reading. And I'm still confused... why can't we make a dimensionless value here?

Keep reading! there are no other time values to cancel

You don't have to use all the variables if something won't cancel out.

where r is the distance between the proton and electron. The list of variables should include enough variables to generate this expression for the force. It could include q, ϵ_0 , and r separately. But that approach is needlessly complex: The charge q is relevant only because it produces a force. So the charge appears only in the combined quantity $e^2/4\pi\epsilon_0$. A similar argument applies to ϵ_0 .

Therefore rather than listing q and ε_0 separately, list only $e^2/4\pi\varepsilon_0$. And rather than listing r, list \mathfrak{a}_0 , the common notation for the Bohr radius (the radius of ideal hydrogen). The acceleration of the electron depends on the electrostatic force, which can be constructed from $e^2/4\pi\varepsilon_0$ and \mathfrak{a}_0 , and on its mass \mathfrak{m}_e . So the list should also include \mathfrak{m}_e . To find the dimensions of $e^2/4\pi\varepsilon_0$, use the formula for force

Var	Dim	What
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I like this explanation - it makes the point very obvious.

Which 3 items are you reffering to? q, r and e_nod? How is not being able to make the force dimensionless important?

I mentioned that the equations didn't need to be explained, but I think the dimensions of the different variables should be given. Especially since the topic is in dimensional analysis. The seconds actually comes from epsilon, which can take the form of several units (farads/meter, J/v^2 , Amps*seconds/V, etc.) In this case I'm guessing the units for epsilon is given by seconds*Coulombs^2/meters^3*kg. then the coulombs cancel out, leaving out the dimensions given.

I also like the explanation here. I wouldn't have seen that they were not going to make a dimensionless group at first though; I think a proper table would have helped with variables. It is MUCH clearer when there is a table of variables.

isn't the easiest way to make dimensionless groups by comparing the hydrogen atom to another element?

the point of dimensionless groups is to understand one quantity at a time, and comparing H to another element would give you a ratio rather than an invariant/constant for hydrogen itself. plus, there is no well defined a0 for other elements...

Wasn't it said in last time's section that any true statement can be written in terms of dimensionless groups? Does this mean this statement isn't true?

It means that you cannot say anything true about hydrogen by using just those variables. Making a true (or meaningful) statement requires adding one or more variables.

I don't understand what a_0 and m_e are referring to.

Sorry, I didn't see the table on page 93.

well, it's in the text too.

where is a_0

Therefore rather than listing q and ε_0 separately, list only $e^2/4\pi\varepsilon_0$. And rather than listing r, list \mathfrak{a}_0 , the common notation for the Bohr radius (the radius of ideal hydrogen). The acceleration of the electron depends on the electrostatic force, which can be constructed from $e^2/4\pi\varepsilon_0$ and \mathfrak{a}_0 , and on its mass \mathfrak{m}_e . So the list should also include \mathfrak{m}_e . To find the dimensions of $e^2/4\pi\varepsilon_0$, use the formula for force

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92

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so is it always that if you can make a dimensionless group you have established all the relevant physics? or are there some times that you would get the wrong answer because you undimensionalized too early

So since the dimensional analysis can't be performed is that proving the model we chose was just wrong?

Yes, I think that's what we're saying here.

I'm still unclear as to what "performing dimensional analysis" means in this context. it seems to me like we're trying to find some arbitrary value without knowing why.

Does that mean only the simplest model of a problem can be dimensionless? I assumed its possible to make anything dimensionless but I guess in the context of this problem hydrogen can never be dimensionless.

I really like how in this section, some variables are chosen to use in the dimensional analysis but then it is shown that some important physics concept was forgotten and the model must be reevaluated in order to make an accurate estimation. This is really helpful because in class I was confused on how to know if you have all the right variables, and are not forgetting any.

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$$F = \frac{e^2}{4\pi\epsilon_0} \, \frac{1}{r^2}.$$

Then

$$\label{eq:final_energy} \left[\frac{e^2}{4\pi\varepsilon_0}\right] = \left[r^2\right] \times [\mathsf{F}] = \mathsf{ML}^3\mathsf{T}^{-2}.$$

The next step is to make dimensionless groups. However, no combination of these three items is dimensionless. To see why, look at the time dimension because it appears in only one quantity, $e^2/4\pi\varepsilon_0$. So that quantity cannot occur in a dimensionless group: If it did, there would be no way to get rid of the time dimensions. From the two remaining quantities, a_0 and m_e , no dimensionless group is possible.

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how do you know you have to consider relativity and quantum mechanics, and I didn't think about these at all? so is there a general approach that you can follow when you can't make a dimensionless group?

Yeah, I this didn't strike me as something to consider here. I supposed you could try to increase complexity until you reach a dimensionless group

That seems counterintuitive, although it does work here. Isn't the point of an estimation class to make things simpler? This looks like a very roundabout way to make something simple!

Simpler is relative. Compared to solving Schroedinger's equation, this approach is much simpler. But I take the global point, that maybe this example isn't the best introduction to dimensional analysis.

I've added this small example as the first use of dimensionless groups:

"As a negative example, revisit the comparison between Exxon's net worth and Nigeria's GDP. The dimensions of net worth are simply money. The dimensions of GDP are money per time. These two quantities cannot form a dimensionless group! With just these two quantities, no meanginful statements are possible."

So I understand why those cannot become dimensionless. But if you use a Force equation, then the units are the same on both sides, unlike your example of GDP and Net Worth.

what do you mean by this? you do everything relative to the speed of light?

Maybe I'm the only one, but I don't understand what relativity is? This example seems pretty complex for non-physics students and me trying to figure out the physics is taking away from my learning about dimensional analysis.

You aren't alone.

well it only took Einstein to figure it out...i'm sure one sentence is more than enough to explain..../sarcasm

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So we can simply add any dimensions we see fit in order to obtain these dimensionless quantities?

I don't understand by the speed of light intrinsically encapsulates relativity?

can we see this? i don't quite get it...

Agreed, I don't really get how it produces a size.

Well, c is a rate that has [L] and [T]. When you say size, are you referring to the [L]? This entire paragraph confuses me.

Well, c is a rate that has [L] and [T]. When you say size, are you referring to the [L]? This entire paragraph confuses me.

i agree that i still dont understand how dimensionless group=size

How do we know which size the information gives us? What tells you we can find the size for an electron and not the size of hydrogen.

Is this the radius at which the electron orbits the proton? Because wouldn't that be the size of the Hydrogen atom in this model? Unless you mean the radius of the electron itself....

I am reallyyy confused. Relativity???

Therefore rather than listing q and ε_0 separately, list only $e^2/4\pi\varepsilon_0$. And rather than listing r, list \mathfrak{a}_0 , the common notation for the Bohr radius (the radius of ideal hydrogen). The acceleration of the electron depends on the electrostatic force, which can be constructed from $e^2/4\pi\varepsilon_0$ and \mathfrak{a}_0 , and on its mass \mathfrak{m}_e . So the list should also include \mathfrak{m}_e . To find the dimensions of $e^2/4\pi\varepsilon_0$, use the formula for force

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The next step is to make dimensionless groups. However, no combination of these three items is dimensionless. To see why, look at the time dimension because it appears in only one quantity, $e^2/4\pi\varepsilon_0$. So that quantity cannot occur in a dimensionless group: If it did, there would be no way to get rid of the time dimensions. From the two remaining quantities, a_0 and m_e , no dimensionless group is possible.

The failure to make a dimensionless group means that hydrogen does not exist in the simple model as we have formulated it. I reglected important physics. There are two possibilities for what physics to add.

One possibility is to add relativity, encapsulated in the speed of light c. So we would add c to the list of variables. That choice produces a dimensionless group, and therefore produces a size. However, the size is not the size of hydrogen. It turns out to be the classical electron radius instead. Fortunately, you do not have to know what the classical electron radius is in order to understand why the resulting size is not the size of hydrogen. Adding relativity to the physics – or adding c to the list –

What?? This paragraph is poorly written and races through the material. What exactly are you trying to say here? Why can you arbitrarily add constants? How do you know what the effect of adding those constants is? And what is the point of this dimensionless value if we have to go to these lengths to construct it?

I, too, am curious about the cost-to-benefit ratio here. It seems that the necessary amount of physics knowledge just doubled, all to obtain a dimensionless value?

Yeah, this paragraph sort of lost me.

This confused me as well. I understand the theory behind using c, however, where does it show up in the end product?

Somehow, we need to add a constant that will allow us to remove the time dimension. The speed of light, in some way, is an available variable in all instances and will help us remove time.

I agree....especially the part where it says "produces a dimensionless group, and therefore produce a size"...I don't get this logic.

What is the classical electron radius?

It's the radius of an electron based on classical physics (rather than quantum)

I'm really hoping this section will make more sense after lecture tomorrow, because it sounds very interesting but I can't really understand it as written.

I will try!

allows radiation. So the orbiting, accelerating electron would radiate. As radiation carries energy away from the electron, it spirals into the proton, meaning that in this world hydrogen does not exist, nor do other atoms.

The other possibility is to add quantum mechanics, which was developed to solve fundamental problems like the existence of matter. The physics of quantum mechanics is complicated, but its effect on dimensional analyses is simple: It contributes a new constant of nature \hbar whose dimensions are those of angular momentum. Angular momentum is mvr, so

$$[\hbar] = ML^2T^{-1}.$$

The \hbar might save the day. There are now two quantities containing time dimensions. Since $e^2/4\pi\epsilon_0$ has T^{-2} and \hbar has T^{-1} , the ratio $\hbar^2/(e^2/4\pi\epsilon_0)$ contains no time dimensions. Since

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$$\left[\frac{\hbar^2}{e^2/4\pi\epsilon_0}\right] = ML,$$

a dimensionless group is

$$\frac{\hbar^2}{a_0 m_e (e^2/4\pi\epsilon_0)}$$

It turns out that all dimensionless groups can be formed from this group. So, as in the spring–mass example, the only possible true statement involving this group is

$$\frac{\hbar^2}{a_0m_e(e^2/4\pi\varepsilon_0)} = \text{dimensionless constant.}$$

Therefore, the size of hydrogen is

$$a_0 \sim \frac{\hbar^2}{m_e(e^2/4\pi\varepsilon_0)}.$$

Putting in values for the constants gives

$$a_0 \sim 0.5 \text{Å} = 0.5 \cdot 10^{-10} \text{ m}.$$

It turns out that the missing dimensionless constant is 1: Dimensional analysis has given the exact answer.

COMMENTS ON PAGE 3

I think adding more advanced information like this distracts me from the problem at hand. Maybe towards the end of the section you can mention the different ways you could approach the problem using different methods in physics?

Sorry, I'm a bit lost here. How does adding c to the list allow for radiation?

I agree, lost too....maybe a sentence of explanation?

why it allows for radiation isn't so important to this section. (I think accelerating charges give off radiation, as we learned in 8.02. Here, the electron is in circular motion, thus it has radial acceleration, so it "must" give off light. But if that happened, then it would lose some energy, and start falling inwards towards the nucleus, and it keeps radiating since it's still somewhat circular motion, and so on and so forth, until it demolishes itself in the nucleus. Obviously that's not the case, or else Hydrogen or any other atom would not be stable.)

But back to the point: Sanjoy is searching for a missing variable with a dimension of time in it so that he can use it to set up a dimensionless group.

He speculates that "c" is important in relativity, so maybe it will factor in somehow. The exact way it factors in is no important yet, since we just want to obtain something that works.

After rejecting c, because of the radiation reason, he speculates about using "h_bar", which is an important constant in quantum mechanics. How exactly it factors in isn't important for us to know here. What is important is that we don't have a reason to reject using it, and it has that dimension of time, which we were looking for.

Thus, without knowing much about the field of relativity or about quantum mechanics, you can still use constants from those fields to "arrive" at a relation, via dimensional analysis.

how do you add this by simply adding c to the list of variables?

so confused

me too

This suddenly got way too advanced. The last physics I had was 8.02

allows radiation. So the orbiting, accelerating electron would radiate. As

radiation carries energy away from the electron, it spirals into the proton, meaning that in this world hydrogen does not exist, nor do other atoms.

The other possibility is to add quantum mechanics, which was developed to solve fundamental problems like the existence of matter. The physics of quantum mechanics is complicated, but its effect on dimensional analyses is simple: It contributes a new constant of nature \hbar whose dimensions are those of angular momentum. Angular momentum is $\pi \nu r$, so

$$[\hbar] = ML^2T^{-1}.$$

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a_0	T /	size
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\mathfrak{m}_{e}	M	electron mass
ħ	ML^2T^{-1}	quantum
		1

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It turns out that the missing dimensionless constant is 1: Dimensional analysis has given the exact answer.

That is an interesting point and something that was not intuitive to me at first.

While I suppose I've always considered radiation to be a form of energy loss, it didn't strike me that its effect would cause an electron to spiral into the proton.

I think that a better explanation of what radiation is and how it works would be useful here...how does radiation carrying energy away push the electron in?

I don't really understand what the purpose of this paragraph is. Is it a method that we could use, but are not going to, considering the extensive description of the next possibility?

I agree. I don't think this paragraph has actually taught me anything, about what we're supposed to be learning, it's just made me confused as to why adding in these constants apparently leads to new implications for physics.

I would recommend making a statement earlier in this reading that this section is one of those times when you have to blur your vision and not look too closely at the details of what is going on. Otherwise I think the reader might get really confused and distracted in trying to understand all the quantum physics and laws and stuff and actually end up missing the dimensional analysis lesson that you are trying to teach.

I wish I remembered Quantum better... I'm going to have to review that stuff

some of us have never taken quantum. we've been told we "major in 8.01"

Uh I'm lost, which world? The world in which the model exists?

I don't understand how you reached this conclusion. Your conclusion hinges on the fact that you arbitrarily chose to include the constant c, but couldn't you have chosen any other related constant, and had a different conclusion?

What he is saying is that if c was the proper term to add in to allow the behavior we want, electrons would actually spiral in towards the nucleus and obliterate (since quantum effects are actually what keep electrons bound to nuclei and not spiral in). So c cannot be the correct term. Planck's constant turns out to be the correct value because it is THE related constant and makes it all work. I recommend looking at a basic quantum text (such as Griffiths) for a more indepth explanation of this.

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The other possibility is to add quantum mechanics, which was developed to solve fundamental problems like the existence of matter. The physics of quantum mechanics is complicated, but its effect on dimensional analyses is simple: It contributes a new constant of nature h whose dimensions are those of angular momentum. Angular momentum is mvr, so

$$[\hbar] = \mathbf{M} \mathbf{L}^2 \mathbf{T}^{-1}.$$

The \hbar might save the day. There are now two quantities containing time dimensions. Since $e^2/4\pi\epsilon_0$ has T^{-2} and \hbar has T^{-1} , the ratio $\hbar^2/(e^2/4\pi\epsilon_0)$ contains no time dimensions. Since

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both of these things sound very big and difficult to add

I agree - this requires more outside knowledge than I have i think.

I feel that you just guessed (intelligently) and got lucky.

drum roll.. plank's constant... Where's the wave equation?

But how does this "constant of nature" influence the physics? Before, adding the speed of light had a simple effect on our dimensional analysis, but killed the physical realities of the problem. How do we know that we are not repeating that same mistake here?

Look at any introductory quantum textbook such as Griffiths. What you find is that the angular momentum of an orbit in quantum mechanics is in integer steps of h bar.

This raises a good point, that someone asked in lecture, how do we know when we've found all of the relevant quantities? Sure h bar might be important, but why not another variable we have yet to identify, too?

Well. Physics kind of falls into categories. Classical physics is for general, everday physics. The rest of physics occurs at various 'extremes'. Relativity adds in 'c' and works with physics at extremely high speeds (approaching the speed of light, c). Statistical physics deals with large numbers of particles or interactions, another extreme. And quantum physics deals with extremely small particles and masses, and this is when 'h' becomes important. All of these physics principles are always 'true', so we could always use them, but they only become important (affect orders of magnitude) when they are in these extremes. So basically, you just need to know which extremes you are considering to know which constants you may need.

This makes quite a bit of sense. Thank you.

You might want to consider calling it the Planck constant, just so people who don't have experience with QM can know the name. Also, you should consider saying how h bar is just h/2pi, just for completeness.

How did you get the value for H?

Its a constant... not all constants are dimensionless. h is known as Planck constant

"What" might be a bit informal in this context?

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I love your writing voice – it adds danger and heroism and intrigue (ish) to what were otherwise mundane equations in our textbooks.

Thank you!

My goal, which I'll never fully reach, is that the physical relations and quantities and properties become the actors in a story, and that we, the readers, get drawn into their story.

That's a blurry image of where I'd like to get to, and don't quite see the route. But knowing which examples and phrases go in that direction is part of figuring out the route.

Where does a nod come from?

in the previous page, when he said instead of using a generic "r" for the coulombic force equation, we could use units of length with magnitude a_o, since it is the average distance of the electron from the nucleus in Hydrogen, aka, the "Bohr Radius"

We could put the magnitude here. It's used in the calculation below, but never printed anywhere.

I feel like this table would have been helpful on the previous page, and that things like m.e, etc. should be defined explicitly in the text.

Agreed, I was wondering about the a_0 and m_e on the last page since it references them.

This seems to have been an error: I think it was meant to be where the other table was.

Did I miss why we use electron mass and not nuclear or proton mass?

This is cool but it would have also been nice to see the final dimensionless group if you still used c as a variable.

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I'd like to see this also in the form of M, L, and T. I think it would help a lot with seeing how this is dimensionless to see that everything cancels out.

I agree, I'm trying to see how this is dimensionless and having a lot of difficulty.

I'd also like to see a less constants...I remember in class you talked about a system of measurement where there are no 4pi*epsilons...what was that again?

Does this mean what it did before-that we can raise this group to any arbitrary power?

I don't remember this example.

We did it in lecture and it was in the reading too. T^2*k/m was the constant there.

It's in the reading directly before this one.

This is interesting and clearly very difficult to come up with ourselves - what are your thoughts for a general approach on finding dimensionless groups?

My previous class that did dimensionless groups had a method that wasn't too bad, my guess is that he'll teach it in lecture once we get into this topic.

Read the previous few paragraphs on how this was approached.

Though I will agree, the way it's written it's not immediately clear to anyone skimming what the though process behind the equation is.

i think a simpler example to start with would have been helpful (although this was fairly fascinating). it's just a bit hard to understand a concept when the roadblocks are brought up before we ever see the first solution

I agree. This example seems pretty complex and thus loses it application purposes. Maybe a simpler example?

The thing is, when talking about atomic physics, this _is_ the simplest example.

Yea I agree, I think the only way I might have gotten this on my own is by backtracking the units to make sure the entire thing is dimensionless at the end.

So once we introduce h, then we can write the above expression as dimensionless...so it is still true

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isn't there a constant multiplying this value?

Could you also have a table that included these constants somewhere on this page? It'd be nice to see them and know what they represent.

That's pretty neat.

this is using constants which were derived in the study of the atom though? isn't it like going around in a large circle?

But it's taking physical values and making them dimensionless. You still don't know if there was some non dimensional constant to begin with (i.e. K=200 or something)

We actually did this in 8.04 to familiarize ourselves with dimensional analysis. We had a whole unit on it.

What you you mean by "it turns out to be 1" How did you calculate this?

It means that that the answer for ao is very close to the true answer, so we're not missing some dimensionless constant of 2pi or G or 1 billion

ha that always impresses me

Overall, this section was probably one of the most technical reads in the course thus far, but after reading it a second time I understand the point you were trying to make. Still, however, I think the large amount of technical-heavy material in this section makes it a little intimidating.

I definitely agree. This section definitely required me to read over it a couple of times before I understood. I like how you vary between more technical examples and more everyday examples though.

I think highlighting the process a little more helps. All the text is hard to sift through.

I agree with these points as well, it took me far longer to go through these short paragraphs because of jumps in concepts, although only later did I realize the main point of dimensional analysis given what you know.

Why is this important then? This doesn't seem to give us any new information

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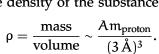
Pretty cool!

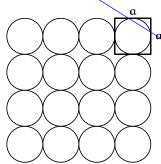
sweeeet. I like it when the approximations are exact agreed, but i also like when i can reproduce them easily! it is easy!

This was another really good example. I think this section has been the best one so far, and possibly the most useful (for me).

Hydrogen has a diameter of TA. A useful consequence is the rule of thumb is that a typical interatomic spacing is 3A. This approximation gives a reasonable approximation for the densities of substances, as this section explains.

Let A be the atomic mass of the atom; it is (roughly) the number of protons and neutrons in the nucleus. Although A is called a mass, it is dimensionless. Each atom occupies a cube of side length a \sim 3 Å, and has mass Am_{proton}. The density of the substance is





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You do not need to remember or look up m_{proton} if you multiply this fraction by unity in the form of N_A/N_A , where N_A is Avogadro's number:

$$\rho \sim \frac{A m_{proton} N_A}{(3 \, \mathring{A})^3 \times N_A}.$$

The numerator is $A\,g$, because that is how N_A is defined. The denominator is

$$3 \cdot 10^{-23} \, \text{cm}^3 \times 6 \cdot 10^{23} = 18.$$

So instead of remembering m_{proton} , you need to remember N_A . However, N_A is more familiar than m_{proton} because N_A arises in chemistry and physics. Using N_A also emphasizes the connection between microscopic and macroscopic values. Carrying out the calculations:

$$\rho \sim \frac{A}{18} \, g \, cm^{-3}.$$

Comments on page 4

Is this an exact statement? I don't mean to nit-pick, but the fact that you don't say about or equivalent could confuse someone as to whether it is exact, a definition, or simply an approximation.

1A = 0.1 nanometers (which equals 1*10^-10 meters)

Yes, it does. But he explained this above where he said the radius was .5 Angstroms. Which he multiplied by 2 to get the diameter.

do you mean "of the rule of thumb"? otherwise, I'm confused.

Why is this a consequence of hydrogen having a diameter of 1A? I'd appreciate it if someone could please elaborate for me.

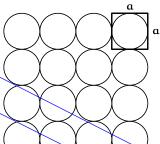
I agree. I have no idea why Hydrogen having a diameter of 1 Angstrom means that the typical interatomic spacing is on the order of 3 Angstroms. Also, your sentence has too many copies of the word "is" in it.

Consequence is probably a poor word choice for it but it means that from that we can guess what the typical diameter might be, and since hydrogen is the smallest is gets, 1 A would be the lower bound (there's a later note that explains this). However, this could be explained a bit better (I was confused until I read the later note... I thought it meant that you have hydrogen with a 1 A diameter and in additional to that there is the space of 3 A between it and the next hydrogen... which doesn't make sense.)

Can we justify the 3 angstrom interatomic spacing? Assuming that the van Der Waals radius is 0.5 angstroms does not yield 3 angstrom separation in an obvious way. In the last section, you picked 1 as the dimensionless quantity. Taking a hint from quantization, do you just pick 2 for larger atoms and average out to 1.5?

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Let A be the atomic mass of the atom; it is (roughly) the number of protons and neutrons in the nucleus. Although A is called a mass, it is dimensionless. Each atom occupies a cube of side length $a \sim 3 \, \text{Å}$, and has mass Am_{proton}. The density of the substance is



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$$\rho = \frac{mass}{volume} \sim \frac{Am_{proton}}{(3 \text{ Å})^3}.$$

You do not need to remember or look up m_{proton} if you multiply this fraction by unity in the form of N_A/N_A , where N_A is Avogadro's numbers

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$$\rho \sim \frac{A}{18} \, g \, cm^{-3}.$$

How did we get 3 from 1?

The 1 was the diameter of hydrogen. The 3 is typical interatomic spacing.

The 3 and 1 represent 2 different values- 3A is the typical space between all atoms while 1A is the diameter for Hydrogen specifically.

So hydrogen atoms are spaced by only 1A when most other atoms are typically spaced between 3A? Am I following you correctly?

Why doesn't this vary significantly based on the size of the atom? Or is three just the overall average.

As you say, it is based on the size of the atom. Hydrogen is at the very small end, and uranium is at the large end. 3 Angstroms is a good average size to use for the common atoms in ordinary substances. As a *very* rough approximation, think of the diameter as 1 Angstrom per shell. (The number of shells is the row number in the periodic table.)

Too many 'is's in this sentence.

This paragraph's wording is a little bit confusing.

I think the confusion also comes from introducing the diameter size earlier with the unit Angstroms.

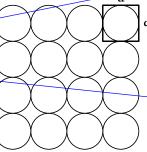
I realize A is commonly used for atomic mass, but maybe in this particular example, where you just introduced angstroms and are talking about atoms with diameters equal to a.... using M for mass might be a little less confusing.

or maybe even 'N' for number of protons/neutrons. Just not 'A'

I saw this too...why can't we use M instead? Armstrong and A for mass is a bit confusing. you can't use M because the number is not actually a mass. I wasn't confused by Angstroms & A...I think they are different enough to work. if you want something other than 'A', 'N' would probably be the best option.

Hydrogen has a diameter of 1Å. A useful consequence is the rule of thumb is that a typical interatomic spacing is 3Å. This approximation gives a reasonable approximation for the densities of substances, as this section explains.

Let A be the atomic mass of the atom; it is (roughly) the number of protons and neutrons in the nucleus. Although A is called a mass, it is dimensionless. Each atom occupies a cube of side length a ~ 3 Å, and has mass Amproton. The density of the substance is



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$$\rho = \frac{mass}{volume} \sim \frac{Am_{proto}}{(3 \, \mathring{A})^3}$$

You do not need to remember or look up m_{proton} if you multiply this fraction by unity in the form of N_A/N_A , where N_A is Avogadro's number:

$$\rho \sim \frac{Am_{proton}N_A}{(3\,\mathring{A})^3\times N_A}.$$

The numerator is $A\,g$, because that is how N_A is defined. The denominator is

$$3 \cdot 10^{-23} \, \text{cm}^3 \times 6 \cdot 10^{23} = 18.$$

So instead of remembering m_{proton} , you need to remember N_A . However, N_A is more familiar than m_{proton} because N_A arises in chemistry and physics. Using N_A also emphasizes the connection between microscopic and macroscopic values. Carrying out the calculations:

$$\rho \sim \frac{A}{18} \, g \, cm^{-3}.$$

If it's not a mass, don't call it a mass. Just introduce it as the number of protons and neutrons (more or less).

I disagree, I think that most people are familiar with the concept of atomic mass.

If A is a mass then it must have units of mass. Otherwise it is not a mass.

I agree. This sentence makes no sense.

A is an atomic mass

Saying A is the atomic mass, then saying its not, then multiplying it by m_proton is a little confusing to follow at first. Would it make more sense to simply say A for an atom is roughly the number of protons and neutrons? This way it would make more sense to simply multiply by the mass. I understand why it is explained this way...it makes perfect sense but I feel it could be explained a little bit simpler and easier for someone to follow who isn't familiar with atomic properties (although I'm sure most all of this would then be overwhelming).

I think this makes sense... but maybe that's just because I'm not unfamiliar with the term "atomic mass".

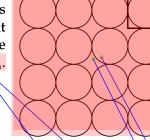
I found the use of 'A' for atomic mass and 'a' for side length a bit distracting at first...the first time i read it my brain didn't want to parse it correctly.

Let a A be the "atomic mass" of a particular atom, roughly equal to the number of protons and neutrons in the nucleus of the atom. Although A is called a mass, it is actually dimensionless; in reality, an atom has a mass of Am(proton). Each atom occupies a cube of side length a 3Å, making the density of a substance:

are we saying this is dimensionless (i.e.) as part of a dimensionless group or it is it just dimensionless by itself

Hydrogen has a diameter of 1Å. A useful consequence is the rule of thumb is that a typical interatomic spacing is 3Å. This approximation gives a reasonable approximation for the densities of substances, as this section explains.

Let A be the atomic mass of the atom; it is (roughly) the number of protons and neutrons in the nucleus. Although A is called a mass, it is dimensionless. Each atom occupies a cube of side length $a \sim 3 \, \text{Å}$, and has mass $\frac{\text{Amproton}}{\text{The density of the substance is}}$



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$$\rho = \frac{mass}{volume} \sim \frac{Am_{proton}}{(3 \, \text{Å})^3}.$$

You do not need to remember or look up m_{proton} if you multiply this fraction by unity in the form of N_A/N_A , where N_A is Avogadro's number:

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Why is it dimensionless?

It appears that it is more of a count (the number of protons and neutrons), and when the unit of mass is needed, A is multiplied by the mass of a proton.

It;s more of just a number. Since we approximate the weight of neutrons and protons to be the same, it's more of just giving a number of how many. This isn't entirely true in my opinion, since that's the definition of 'amu' or atomic mass unit.

If this is true, then its confusing in the paragraph that it says to let A be teh atomic mass of the "atom". If a is just a count, then shouldn't A be considered to be something like Avogadro's number?

I believe it's just convention to call it "mass".

the mass is approximately equal to the number of protons and neutrons, so it's just a count. but my question is, if this really is a mass, it must be convertible into other mass units like grams. if this is true, it can't be dimensionless anymore right?

Although 'atomic mass' is often given in AMU, I think, which are actually units of mass.

Is it still valuable while dimensionless or is it only useful after converting it back to the dimensioned version?

It comes up on this page but the idea is that you are using the mass of the proton also so you can just use A as a completely dimensionless variable that depends on the element.

Is m proton the mass of a proton or is it the mass of the protons in the atom? I am assuming its the mass of a proton but I just wanted to clarify.

I think this could be much smaller and still get the point across.

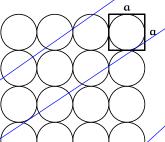
I feel like I have seen this drawing before...where it asks you what is the area of wholes in between? I don't know if this is relevant but it is still interesting.

It looks like you mean A is a scaling factor on the mass of a proton, and not that A is actually a mass itself.

If this is true, I would recommend using a different letter since the A might be confused with the angstrom symbol.

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You do not need to remember or look up m_{proton} if you multiply this fraction by unity in the form of N_A/N_A , where N_A is Avogadro's number:

$$\rho \sim \frac{A \frac{m_{proton} N_A}{(3 \, \mathring{A})^3 \times N_A}}{(3 \, \mathring{A})^3 \times N_A}.$$

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Don't you still have to remember Avogrado's number now?

Oh sorry, you say that below.

I like this. Despite being a little unclear at first, this is a really neat trick to move on in the problem.

I am entirely confused on how this does anything.

Although you mention what NA is a few lines below, it might be useful to just include its value here in parentheses.

He defines this value when he uses it about 2 lines further down. I think that this is sufficient.

I agree with 10:14. We only really care about the numerical value of N_A when we plug in numbers.

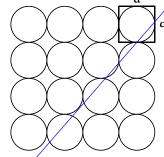
Personally, I would prefer that he would state a value immediately after he defines/discusses a new variable.

So what does a mole of protons weight?

It's almost exactly 1 gram. One mole of carbon-12 atoms weight exactly 12 grams, and carbon-12 has the almost the same mass as 12 protons (the 6 neutrons and 6 protons add up to roughly 12 protons, and the 6 electrons are very light).

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I had to read this sentence a couple times to really understand it, I don't know if it should be phrased better or if I just personally couldn't wrap my head around it.

i got confused because i thought g = gravity but i think it's grams here. it'd probably make sense to say Na * mproton = 1g so we understand that this is why it turns to A g.

This is because a mole of hydrogen atoms just weighs 1 gram as given by the periodic table, correct?

Yeah I feel like it would make it a little more clear if gram is actually written out instead of writing g.

Absolutely thought that was a 'g' for acceleration. Perhaps put it in parenthesis like (in grams)

me too, at first, I thought he meant gravity.

actually, I've done this on the TI-89 too, if you type just g for gram, it will give you the gravitational acceleration instead of gram. "gm" is the symbol used for grams on the calculator.

I'd like to be reminded in the text what Avogadro's number is, with units.

No estimating here?

Why do you need to estimate to multiple 3 by 6?

no units here? should be cm³

We only estimated when multiplying for large numbers and powers of 10. Basic arithmetic would be unnecessary estimation

if it is easier to not estimate, you should give the accurate answer

why is there a "however" starting this sentence? I think you can just get rid of the however.

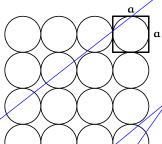
The however makes sense because at first it seems like he's contradicting himself a little but this sentence explains how he's not.

So in the end this doesn't save us a constant, but allows us to use a more familiar/memorable constant?

add: "frequently"

Hydrogen has a diameter of 1Å. A useful consequence is the rule of thumb is that a typical interatomic spacing is 3Å. This approximation gives a reasonable approximation for the densities of substances, as this section explains.

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$$\rho \sim \frac{A}{18} \text{ g cm}^{-3}.$$

This section is great! Back in the days when I competed in science competitions, I would often use dimensional analysis to guess the answer when I did not understand the physics. It helped me when in a few cases!

I like this jump, it really shows how number's like Avogadro's number that we learned back in high school can pop up and still be useful

This conclusion is really cool and makes a lot of sense in the approximation world. It reminds me of the period tables with size of atoms plotted

I also really like when these methods that I would never think of eventually reach an equation that makes sense to me. It's also nice to see in the table below that it's a good approximation.

The table compares the estimate against reality. Most everyday elements have atomic masses between 15 and 150, so the density estimate explains why most densities lie between 1 and 10 g cm⁻³. It also shows why, for materials physics, cgs units are more convenient than SI units are. A typical cgs density of a solid is 3 g cm⁻³, and 3 is a modest number and easy to remember and work with. However, a typical SI density of a solid 3000 kg m⁻³. Numbers such as 3000 are unwieldy. Each time you use it,

i			
	Element	Pestimated	ρ _{actual}
	Li	0.39	0.54
	H ₂ O	1.0	1.0
	Si	1.56	2.4
	Fe	3.11	7.9
	Hg	11.2	13.5
	Au	10.9	19.3
	U	13.3	18.7
-			

you have to think, 'How many powers of ten were there again?' So the table tabulates densities using the cgs units of $g \, \text{cm}^{-3}$. I even threw a joker into the pack – water is not an element! – but the density estimate is amazingly accurate.

COMMENTS ON PAGE 5

Your lengthier readings usually have sections or subparts and a lost of forecasting of what is to come. This passage seemed to read more like a physics textbook and thus was harder to follow, at least for me.

A bit too high with respect to the other line its on, formatting issue

i feel like it would be useful to include an example or two of what these densities mean in real-life terms, so that we can get a feel what these values mean.

All the estimates seem to be 10^0.5 off that seems kind of significant based on the estimatation we have done in this class.

Well, I don't think we would worry about a power of 2 here and there, but it does seem that we have a consistent underestimate of rho, which does suggest that we've missed something. Fortunately, whatever it is isn't too important.

this is very true i think the units in cgs are more intuitive or we are more familiar with

$$cgs = ??$$

cgs = centimeters, grams, seconds.

Why don't you just use cgs throughout this section? (as suggested on the first page, it would also simplify the force equation)

I think it depends on your background for which units you are more familiar with. As a MechE, I know m/kg/s much better (although not in a Physics context).

Just be thankful we aren't using miili-inch - Slug -pound system.

When I first wrote (don't ask how long ago), it was in cgs. But I found that almost all teaching, even for electromagnetism, is in SI units (meters, kilograms, seconds). That was especially true in England, but it's true here too. So I thought that the correct choice for the long term is to use SI units even though they are, for electromagnetism, messier than cgs units. (In cgs units, the 4*pi*epsilon_0 is replaced by 1.)

The table compares the estimate against reality. Most everyday elements have atomic masses between 15 and 150, so the density estimate explains why most densities lie between 1 and 10 g cm⁻³. It also shows why, for materials physics, cgs units are more convenient than SI units are. A typical cgs density of a solid is 3 g cm⁻³, and 3 is a modest number and easy to remember and work with. However, a typical SI density of a solid 3000 kg m⁻³. Numbers such as 3000 are unwieldy. Each time you use it,

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		$\overline{}$

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I don't know if the typical reader will be acquainted with cgs units. Would it be useful to include a brief note on what they are, or should we just assume the reader can look it up if he is confused?

I didn't know and I had to look it up

Yea, I feel that even mentioning in parenthesis that cgs is centimeter, gram, seconds would be useful. Solid Works is the only reason why I recognized this.

Thanks for the clarification - I had no idea what cgs stood for. Consequently, I agree this could use a little explanation in the text.

I wasn't familiar with cgs either. We always used kilograms/meters/seconds in physics.

Woah...who would have thought. Why are some of these extremely close and others very off?

If I remember correctly there is some interaction with the way orbitals fill up that causes atom sizes to shrink across a period even though each the size increases going down a group (http://en.wikipedia.org/wiki/Atomic_radius#Calculated_atomic_radii)

And the density will always be too small, as we are approximating an atom as a box, and the volume is in the denominator. And so, if you have something spaced smaller than average, then the volume will be really way small.

Yeah, all the estimates are lower than the actual. Can we do something about this?

It's the opposite effect to what we found when estimating the maximum cycling speed. There, v was proportional to power(1/3), so even large errors in estimating power turned into small errors in estimating v.

Here, however, we are estimating density, which depends on diameter 3. So even small errors in estimating the diameter (i.e. small deviations from the 0.3 nm baseline value) produce a large change in the density.

Is there a reason that this one is the worst? Does it have anything to do with iron's magnetism?

probably a propagation in error of interatomic spacing

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you have to think, 'How many powers of ten were there again?' So the table tabulates densities using the cgs units of $g \, cm^{-3}$. I even threw a joker into the pack – water is not an element! – but the density estimate is amazingly accurate.

...of a solid "is" 3000 kg m^-3.

I was wondering if we were going to have to make special considerations for molecules instead of elements when we were deriving the equation before but I guess it makes sense that we don't need to. We never needed to assume it was just one element.

It would make more sense if it held for only one element! I wonder if CO2 is as close as H2O...

I don't see how this could possibly apply to gases. Typical gas densities are 10^-3 g/cm^3. So A would have to be a small fraction of 1.

I assumed that these densities were talking about solids until water came up (I realize water is a weird case though with regards to liquid/solid densities). Is it fair to assume that the liquid and the solid densities of most of these elements are close enough?

even though water is polar? does this make it less than ideal?

I was wondering about this as well

I wasn't thinking about that, but now I am. putting myself on the thread so I will get an email when someone answers the question for us.

Being polar doesn't affect the intermolecular spacing much in water. The molecules are still as closely packed as possible (basically, until their electron clouds touch and repel each other).

But in ice, being polar is responsible for the open structures that water molecules form – which is why ice is less dense than water (unlike most liquids, which contract when they freeze). So for ice, the polarity does slightly affect the average spacing.

The estimate for water is actually absurdly accurate, much closer than any of the others, even though it was not what we were modeling. Interesting.

The table compares the estimate against reality. Most everyday elements have atomic masses between 15 and 150, so the density estimate explains why most densities lie between 1 and $10 \, \mathrm{g \, cm^{-3}}$. It also shows why, for materials physics, cgs units are more convenient than SI units are. A typical cgs density of a solid is $3 \, \mathrm{g \, cm^{-3}}$, and 3 is a modest number and easy to remember and work with. However, a typical SI density of a solid 3000 kg m⁻³. Numbers such as 3000 are unwieldy. Each time you use it,

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I have some problems with this section. Firstly, it was difficult for me to understand, but when go back to reread it, I don't find Topic sentences, or even a decent conclusion that summarizes the Section. I understand this is hard material to teach, but you can't expect the reader to reread and reread your sections until they make sense. You should reiterate the points of interest yourself.

Agreed. Some basic introductions and conclusions, like you have in the previous sections, would really help make this section clearer.

Agreed!

In the beginning of 2.006 Prof. Brisson taught dimensional analysis using the MLT quantities that you described here. He had a succinct method that was very easy to understand and was applicable to the questions of scaling we saw before. I think you should talk to him and get his notes on it, because this section was entirely too confusing for me, and I know how to do it!

Thanks for the suggestion. I've just asked him.