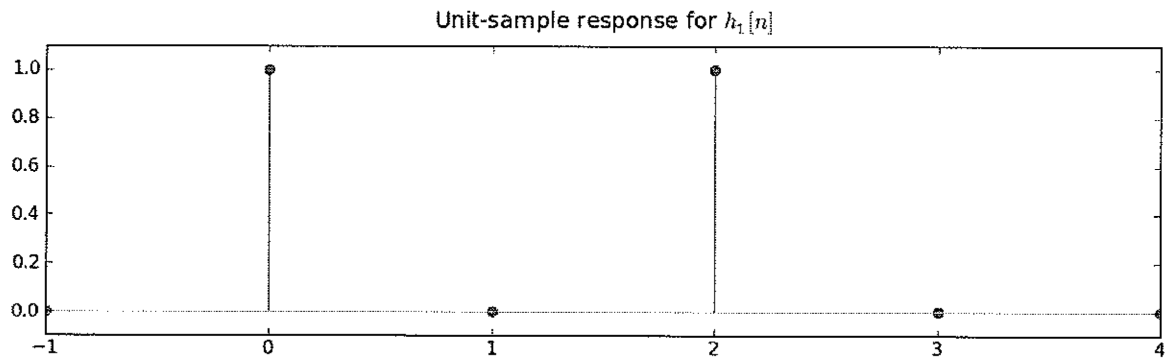
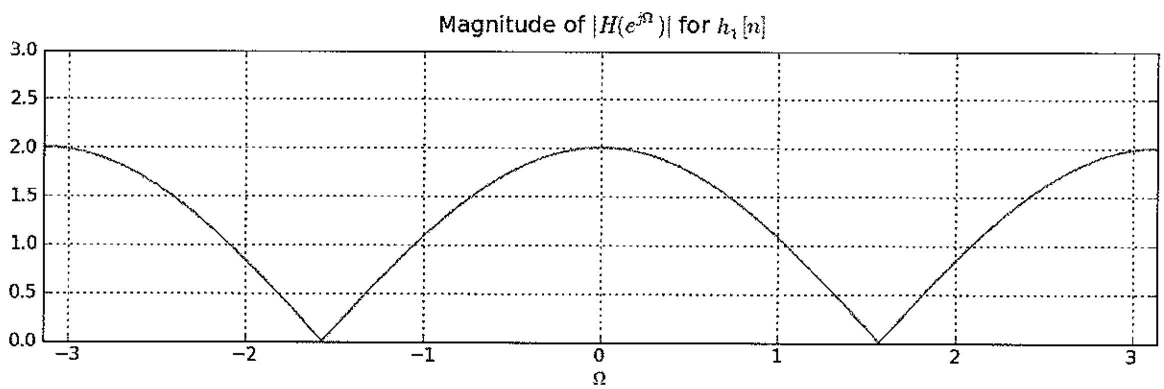
Review Problem Solutions

Review Problem 1

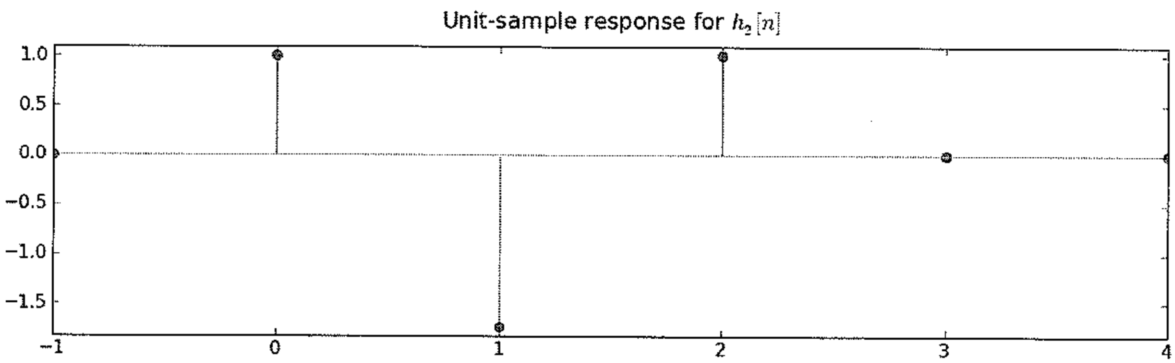
In answering the questions below, please consider the unit sample response and frequency response of two filters, H_1 and H_2, plotted below.

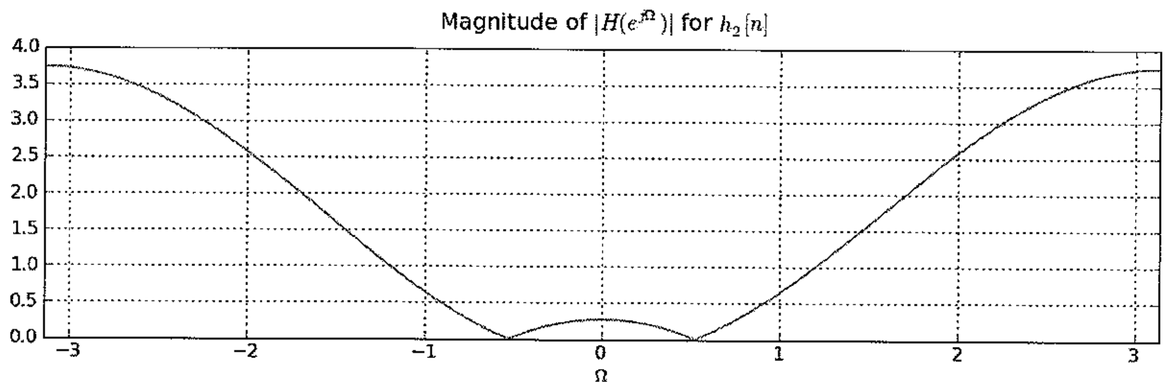




Note, the only nonzero values of unit sample response for H_1 are :

$$h_1[0] = 1, h_1[1] = 0, h_1[2] = 1$$

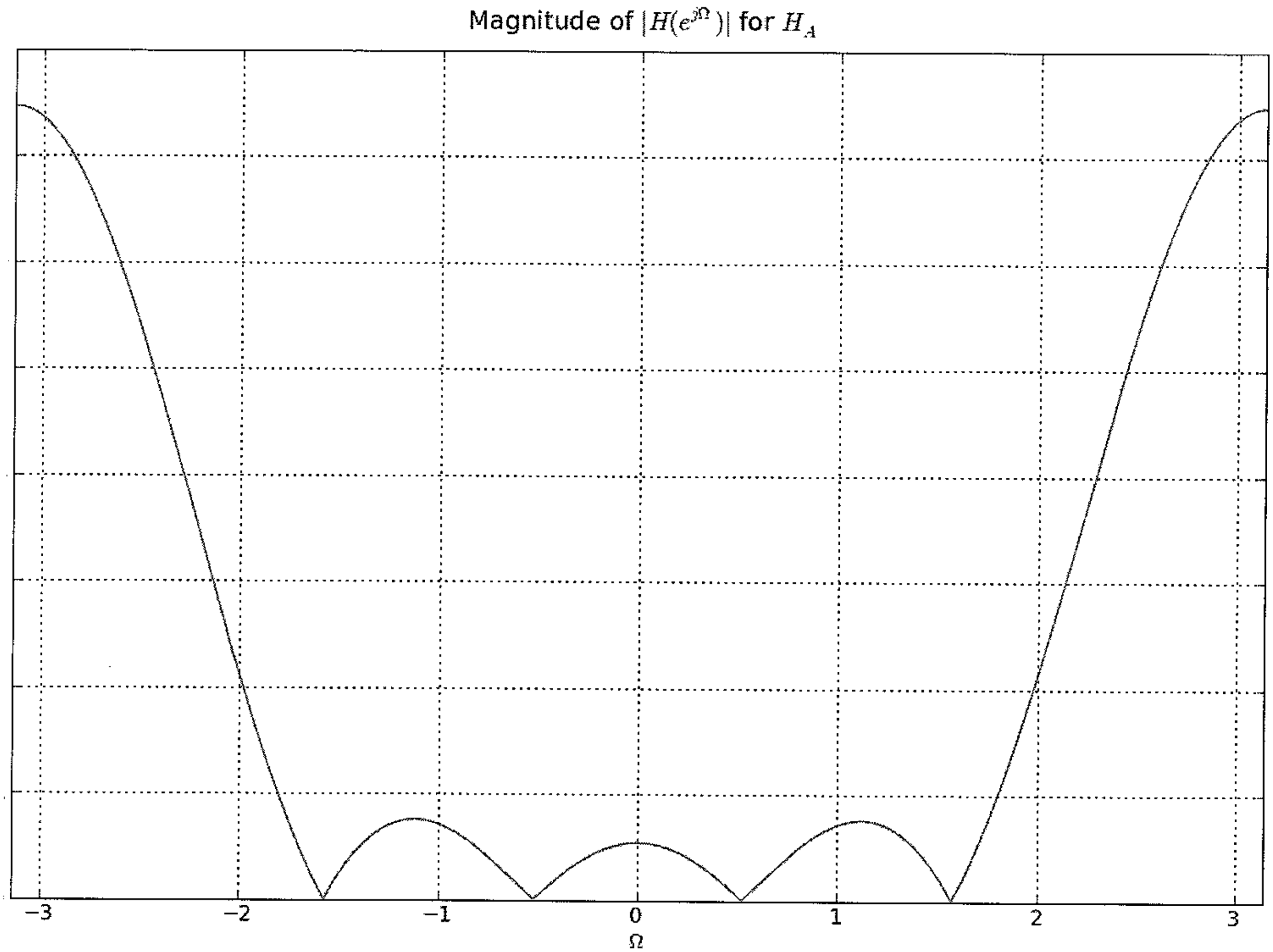


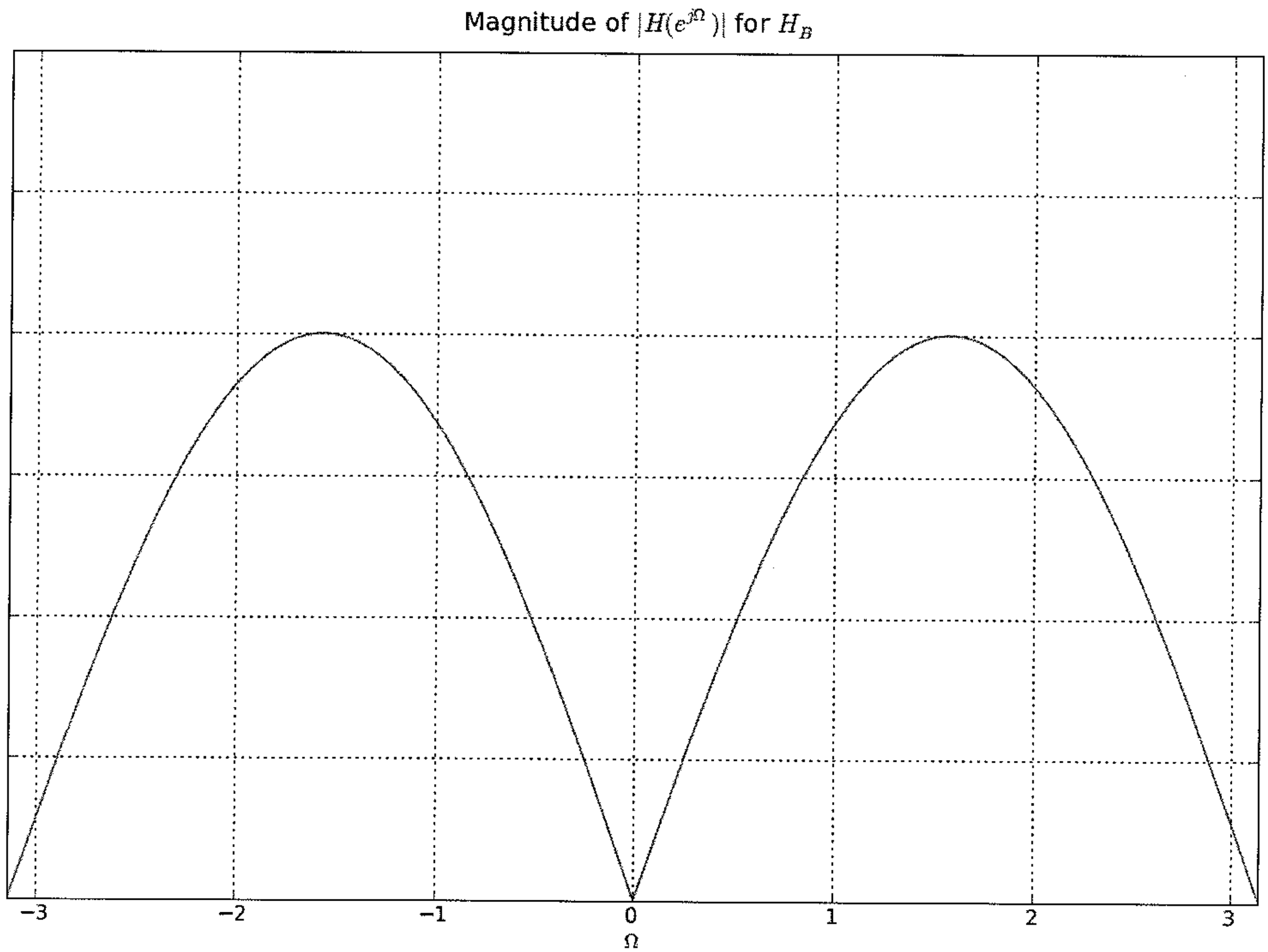


Note, the only nonzero values of unit sample response for H_2 are :

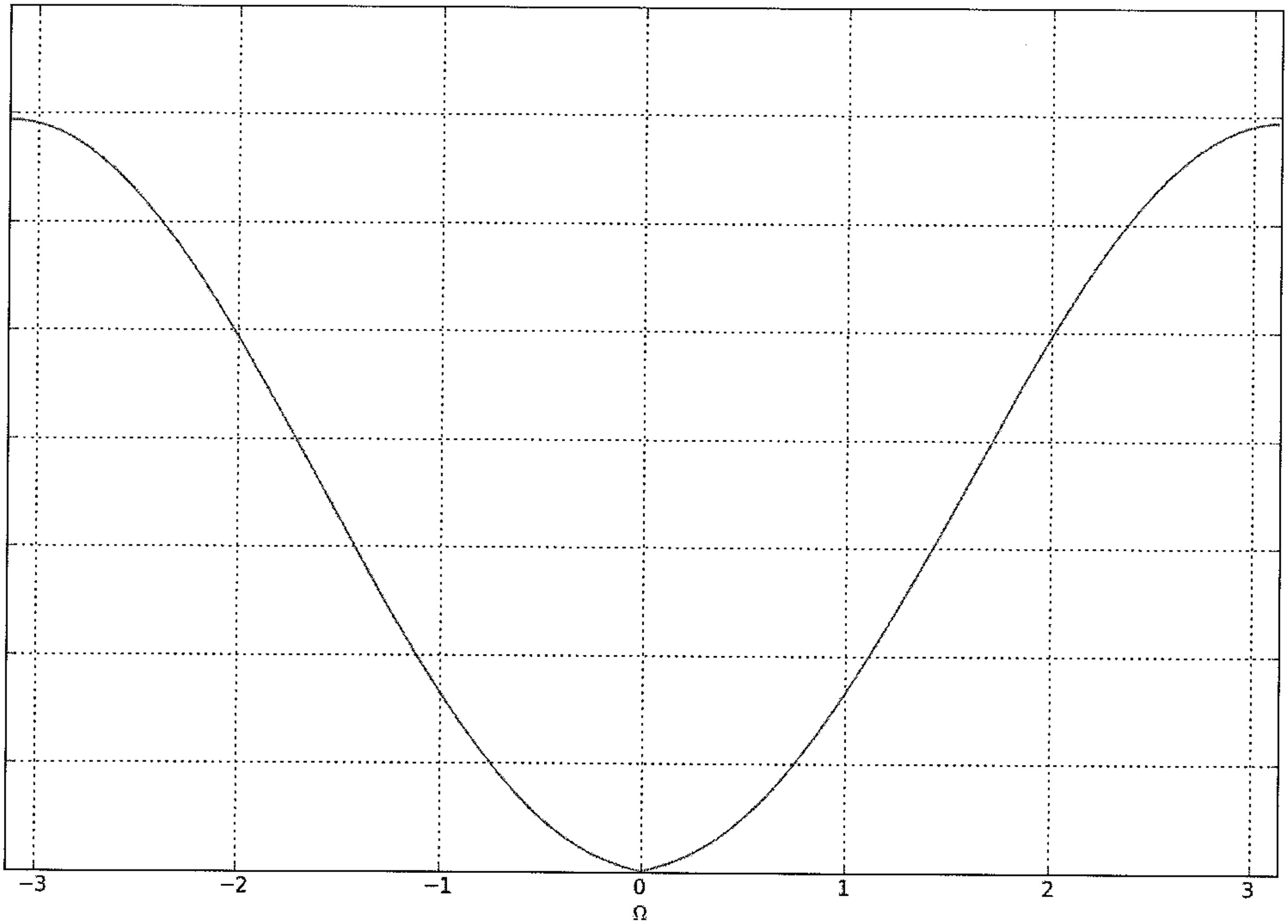
$$h_2[0] = 1, h_2[1] = -\sqrt{3}, h_2[2] = 1$$

In answering the several parts of this review question consider four linear time-invariant systems, denoted A, B, C, and D, each characterized by the magnitude of its frequency response, $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, $|H_C(e^{j\Omega})|$, and $|H_D(e^{j\Omega})|$ respectively, as given in the plots below. This is a review problem, not an actual exam question, so similar concepts are tested multiple times to give you practice

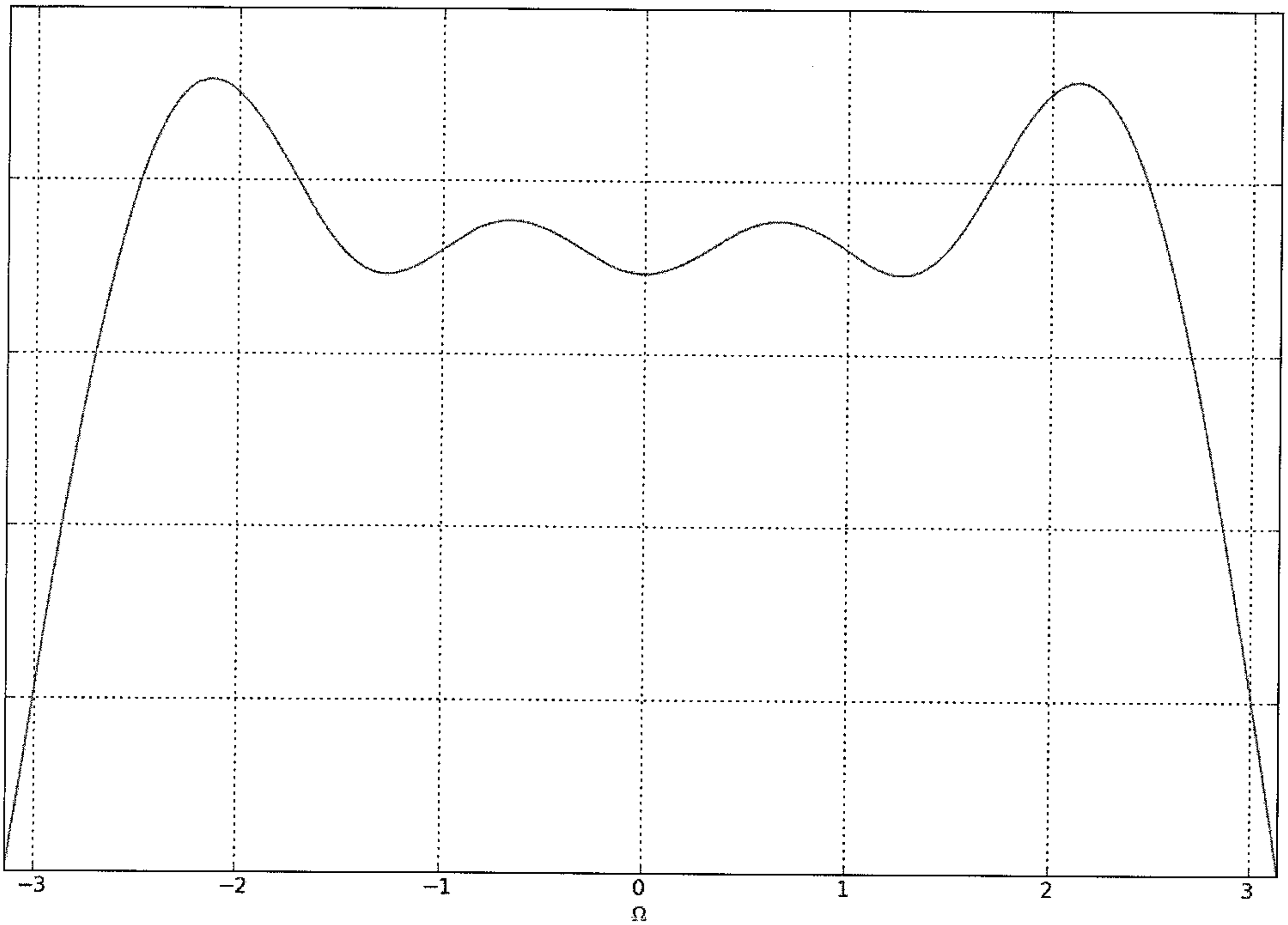



Magnitude of $|H(e^{j\Omega})|$ for H_B
−3
−2
−1
0
1
2
3
Ω

Magnitude of $|H(e^{j\Omega})|$ for H_C



Magnitude of $|H(e^{j\Omega})|$ for H_D



(A) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha\delta[n] - h_1[n]$$

and what is the numerical value of absolute value of α, $|\alpha|$.

Must be H_B as that is the only frequency response that has the same values at zero at 0 and $\pm\pi$ and extremes at $\pm\pi/2$.

$|\alpha| = 2$ as $H_1(e^{j0}) = 2$ but $H_B(e^{j0}) = 0$

(B) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \sum_{m=0}^{m=n} h_1[m]h_2[n-m].$$

and what are the numerical values of $h[3]$ and $H(e^{j0})$?

Must be H_A as the product $H_1(e^{j\Omega})H_2(e^{j\Omega}) = H(e^{j\Omega})$, and therefore $|H(e^{j\Omega})| = 0$ whenever $H_1(e^{j\Omega}) = 0$ or $H_2(e^{j\Omega}) = 0$

$$H(e^{j\Omega}) = H_1(e^{j0})H_2(e^{j0})$$

$$= 2 \cdot (2 - \sqrt{3}) = 4 - 2\sqrt{3}$$

$h[n] = [1, 0, 1] * [1, -\sqrt{3}, 1]$

$h[3] = 2$

(C) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha\delta[n] - \sum_{m=0}^{m=n} h_1[m]h_2[n-m].$$

and what is the numerical value of absolute value of α, $|\alpha|$.

Since H_A is the freq response for $H_1 * H_2$, H_D must be the solution. It is the only frequency response with enough wiggles. $|\alpha| = 2\cdot(2+\sqrt{3}) = 4+2\sqrt{3}$ from $|H_A(e^{j\pi})| = |H_1(e^{j\pi})||H_2(e^{j\pi})|$

(D) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha\delta[n] - h_2[n]$$

and what is the numerical value of absolute value of α, $|\alpha|$.

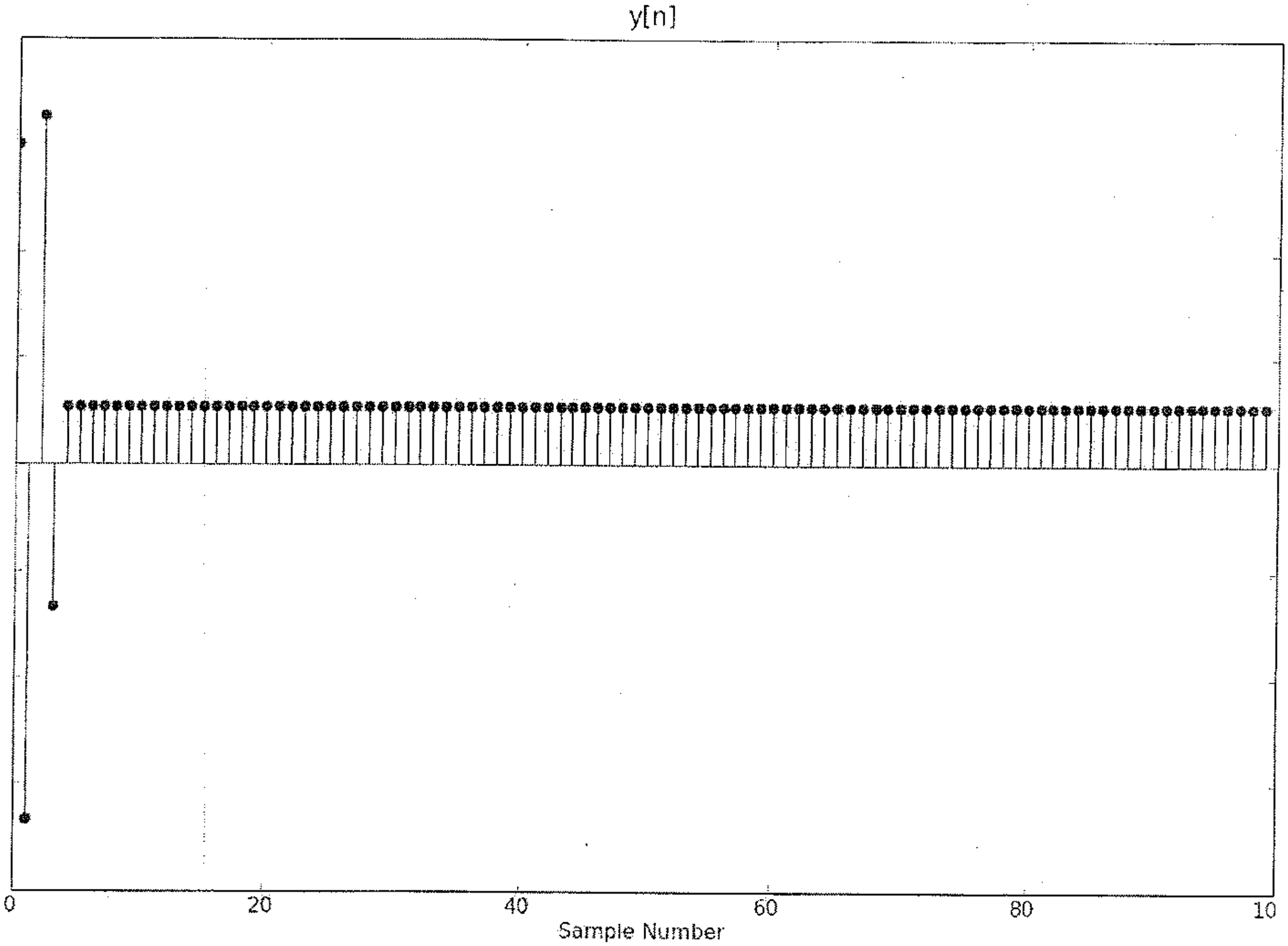
Must be H_C (by elimination) but also because none of the other frequency responses could be generated by a single magnitude shift of $H_2(e^{j\Omega})$.

$H_C(e^{j0}) = 0$ so $|\alpha| = |H_2(e^{j0})| = 2-\sqrt{3}$

(E) Suppose the input to each of the above four systems is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

$$x[n] = \cos\frac{\pi}{6.0}n + \cos\frac{\pi}{2.0}n + 1.0.$$

Which system (A, B, C or D) produced an output, $y[n]$ below, and what is the value of $y[n]$ for $n > 10$?



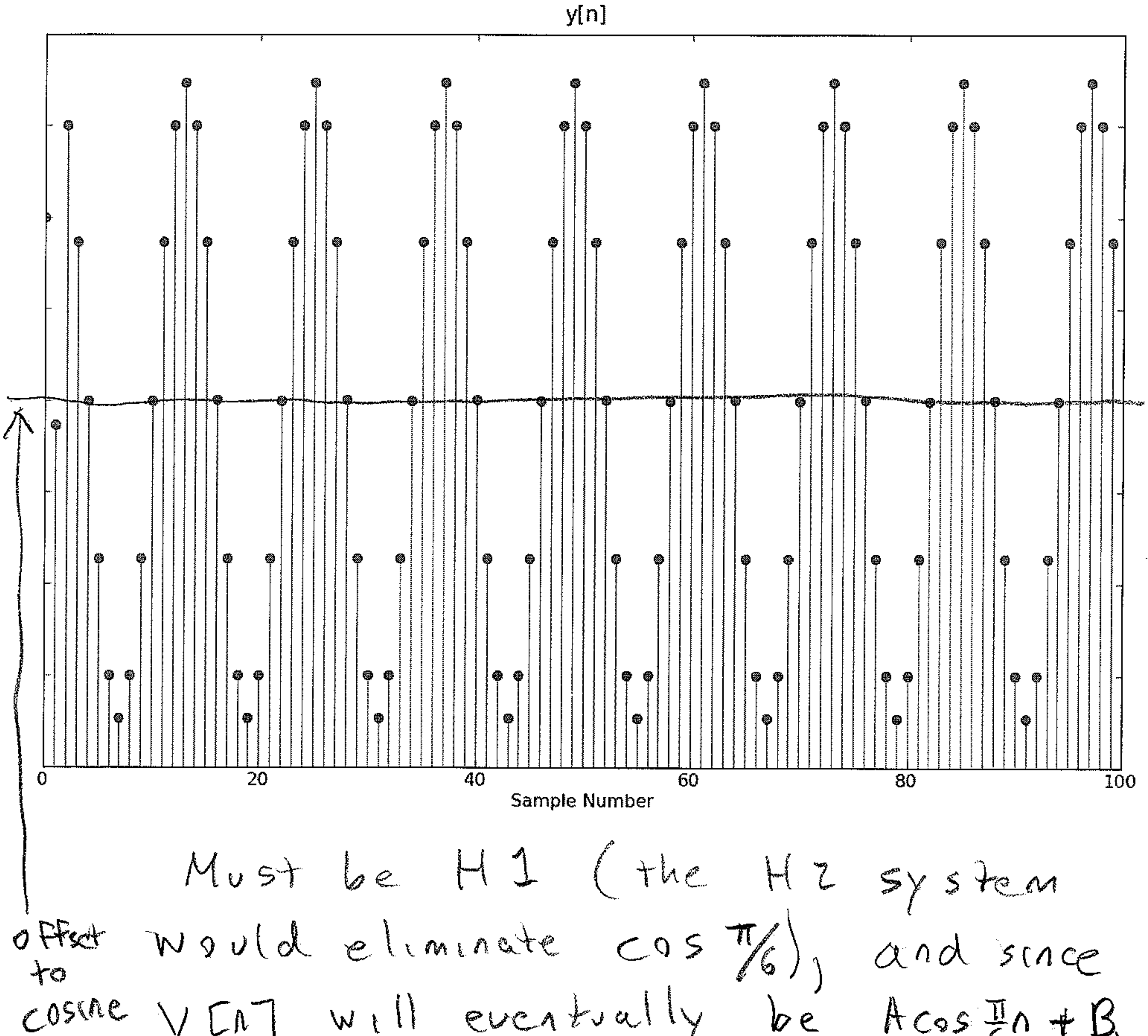
Must be H_A, as $|H_A(e^{j\Omega})| = 0$ for $\Omega = \pm\pi/6$ and $\Omega = \pm\pi/2$.

$y[n]$, $n > 10$ $= |H_A(e^{j0})| \cdot 1 = 4 - 2\sqrt{3}$

(F) Suppose the input to each of the above four systems is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

$$x[n] = \cos\frac{\pi}{6.0}n + \cos\frac{\pi}{2.0}n + 1.0.$$

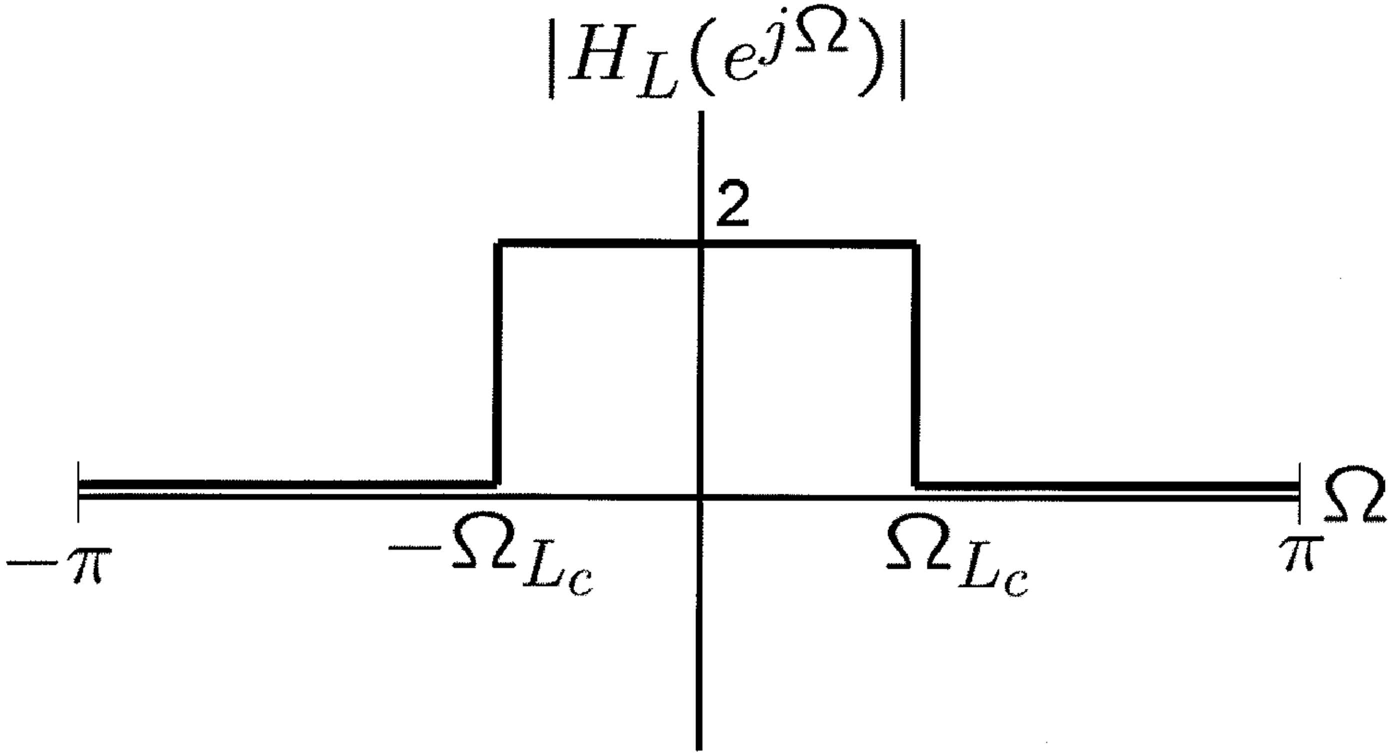
Which system (H1 or H2) produced an output, $y[n]$ below, and what is the value of $y[22]$?

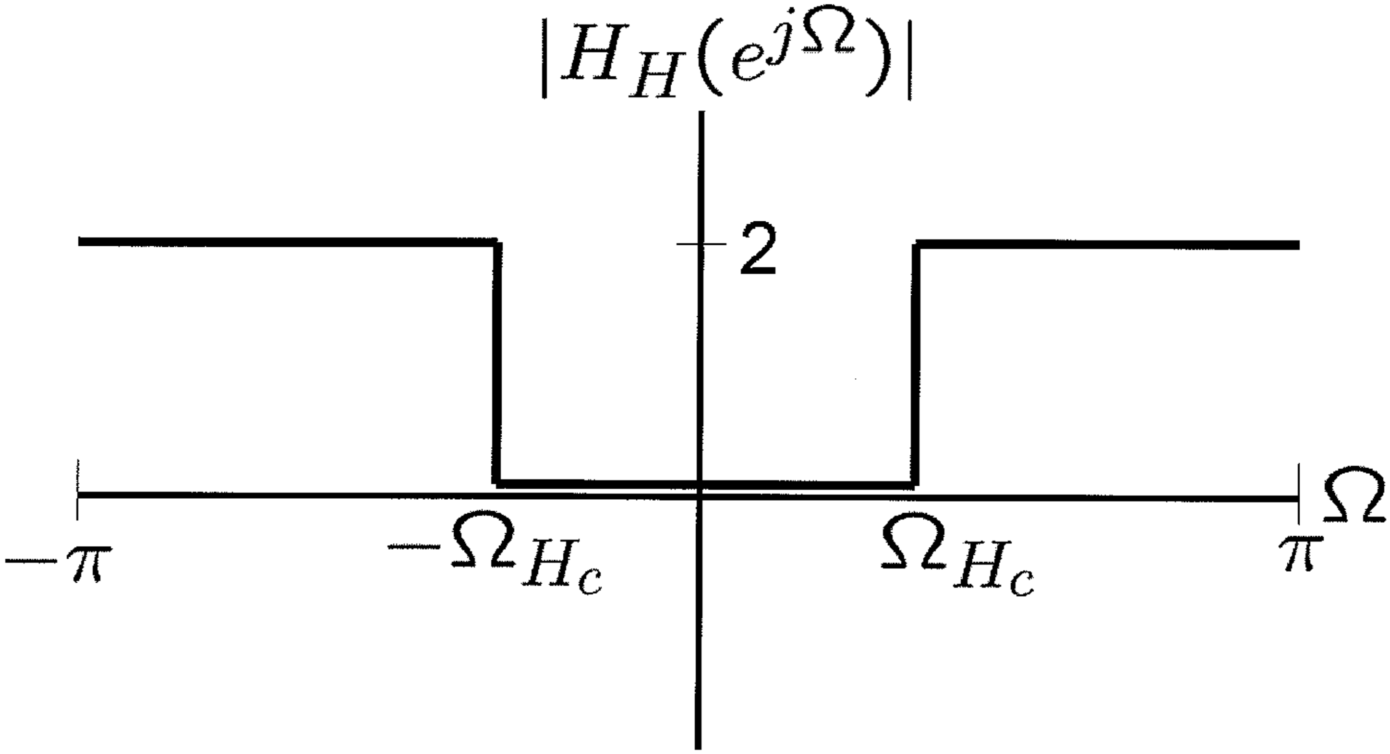


Must be H1 (the H2 system would eliminate $\cos \pi/6$), and since $y[n]$ will eventually be $A\cos\frac{\pi}{6}n + B$, where $B = H_1(e^{j\emptyset}) \cdot 1 = 2 \cdot 1 = y[22] \approx 2$.

Review Problem 2

The questions below refer to two linear time-invariant filters, a low-pass filter, H_L, and a high-pass filter, H_H, whose frequency response magnitudes are plotted below. Please note that in the pass band, each filter has a gain of **TWO**.



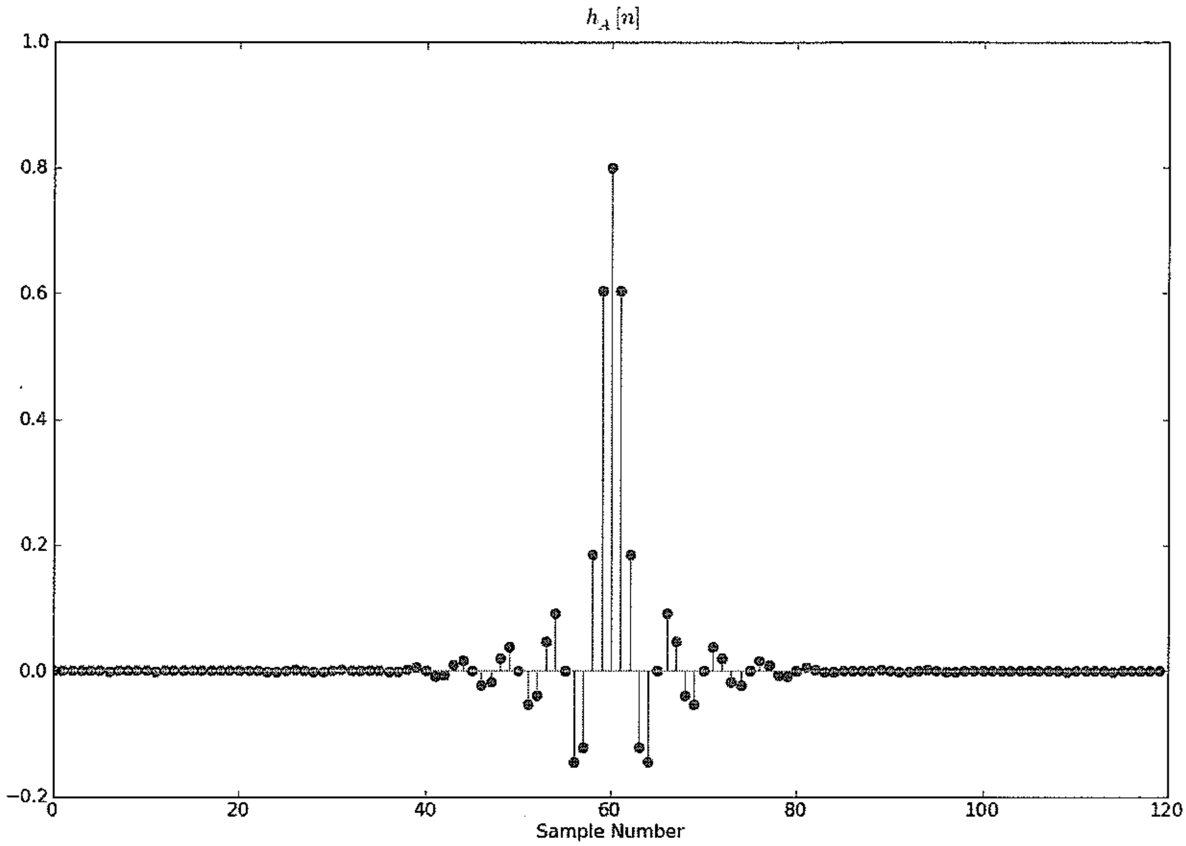

$|H_H(e^{j\Omega})|$
2
$-\pi$
$-\Omega_{H_c}$
Ω_{H_c}
π
Ω

(A) For two unit sample responses with sample values $h_A[n]$ and $h_B[n]$, plotted below, which one could be a high-pass filter and which one could be a low-pass filter? In addition, for what is the value of

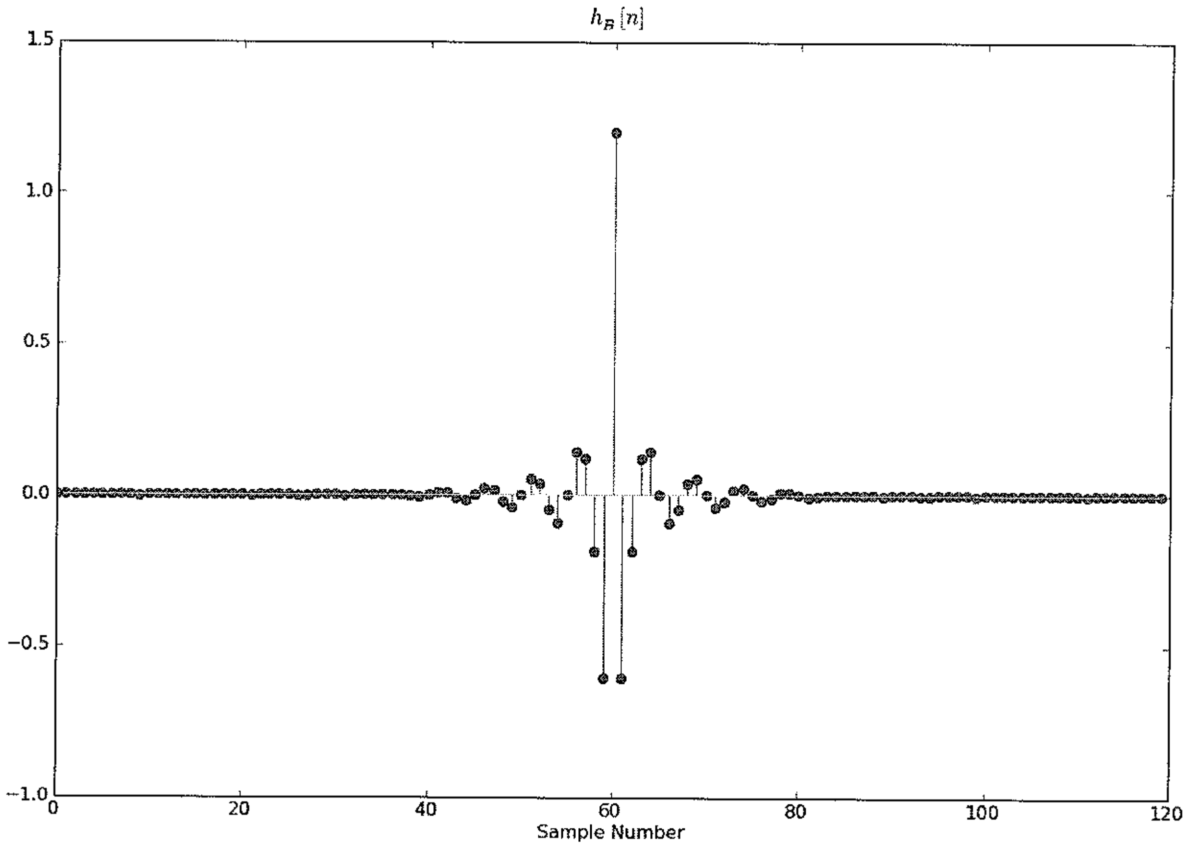
$$\sum_{m=0}^{m=\infty} h_A[m](-1)^m$$

and

$$\sum_{m=0}^{m=\infty} h_B[m](-1)^m?$$



This must be the low pass filter, $\sum h[n] = H(e^{j0})$ looks to be near 2, not zero. Since $\sum h_A[m](-1)^m = H(e^{j\pi})$ and $H(e^{j\pi}) = 0$, $\sum h_A[m](-1)^m = 0$.



This must be the high pass filter, $\sum h_B[n] \simeq 0$ ($h[60] + h[59] + h[61]$ is approximately zero). If so, then $\left|\sum h_B[m](-1)^m\right| = \left|H(e^{j\pi})\right| = 2$.

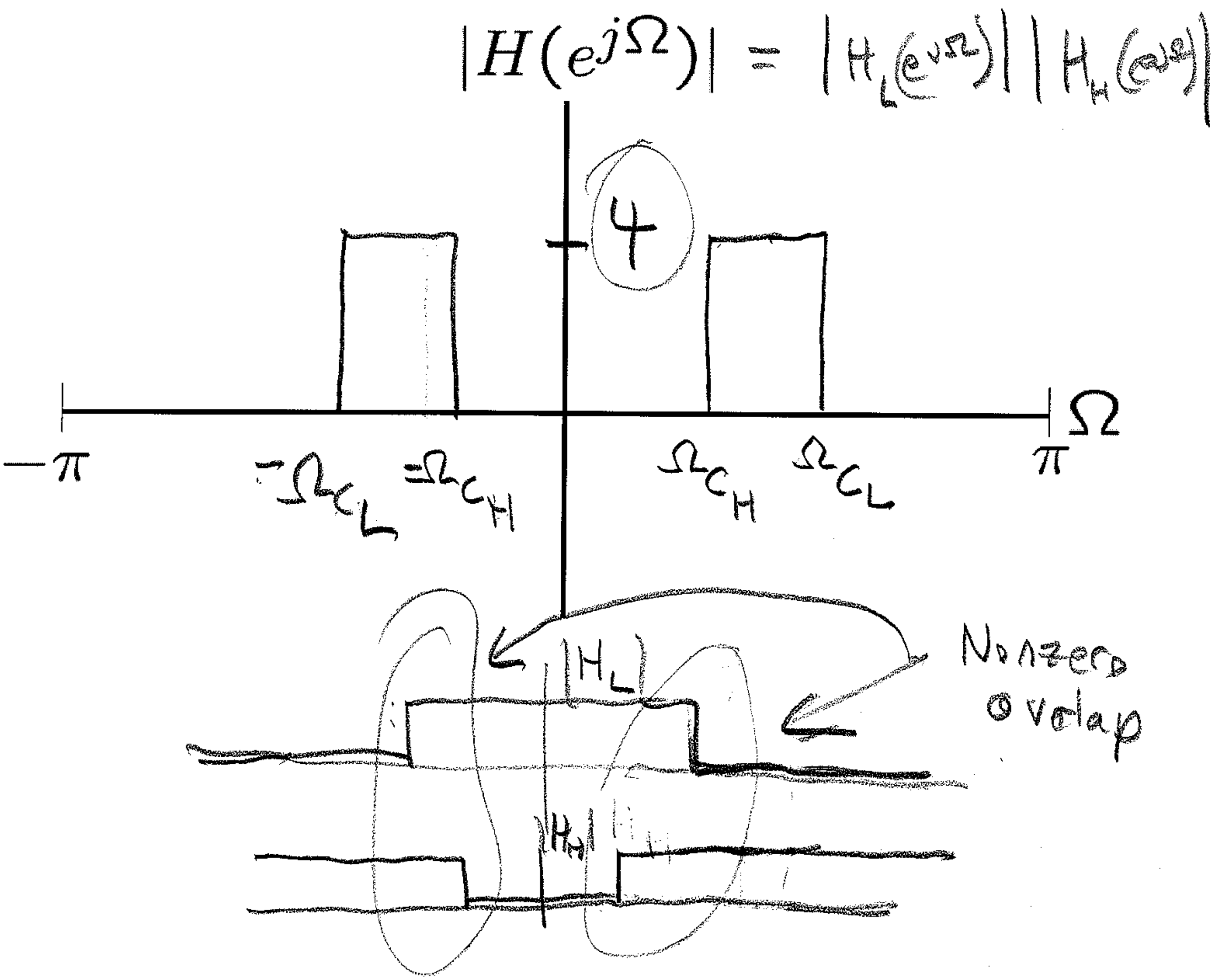
Since the result must be real $\sum h_B[m](-1)^m$ must be either 2

or -2. Since $h[60](-1)^{60} = h[60] \simeq 1.2$ the result must be $\boxed{2}$

(B) On the axes below, please plot the magnitude of the frequency response of the system H, whose unit sample response is given by the convolution of the low-pass and high-pass filter's unit sample responses, as in

$$h[n] = \sum_{m=0}^{m=n} h_L[m]h_H[n-m].$$

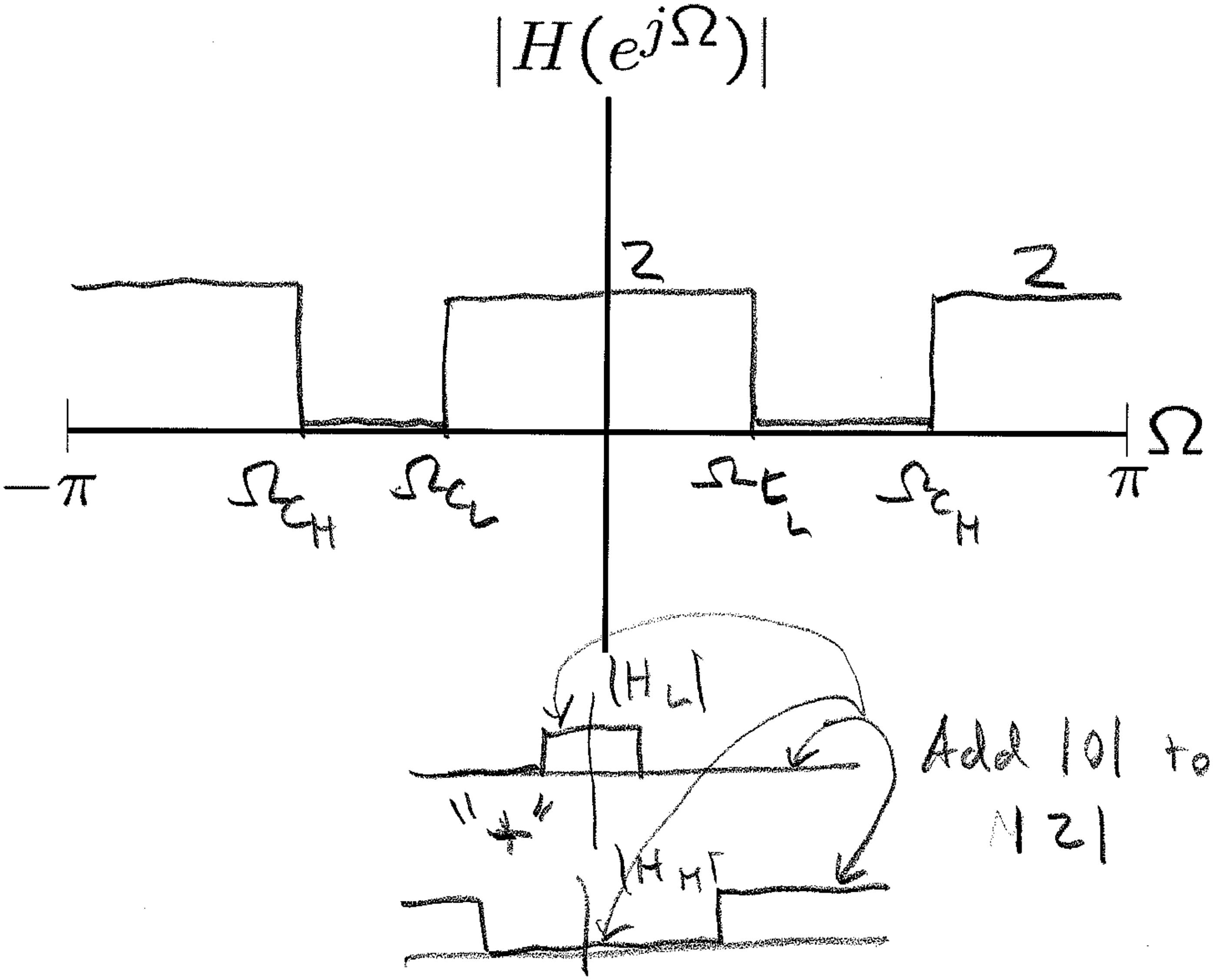
For your plot, please assume $\Omega_{c_L} > \Omega_{c_H}$, and clearly label important magnitude values and key frequency points.



(C) On the axes below, please plot the magnitude of the frequency response of the system H, whose unit sample response is given by the sum of the low-pass and high-pass filter's unit sample responses, as in

$$h[n] = h_L[n] + h_H[n].$$

For your plot, please assume $\Omega_{c_L} < \Omega_{c_H}$ (the opposite case from part B) and clearly label important magnitude values and key frequency points.



(D) Will your plot in part C change if H's unit sample response is the difference of the low-pass and high-pass filter's unit sample responses, as in

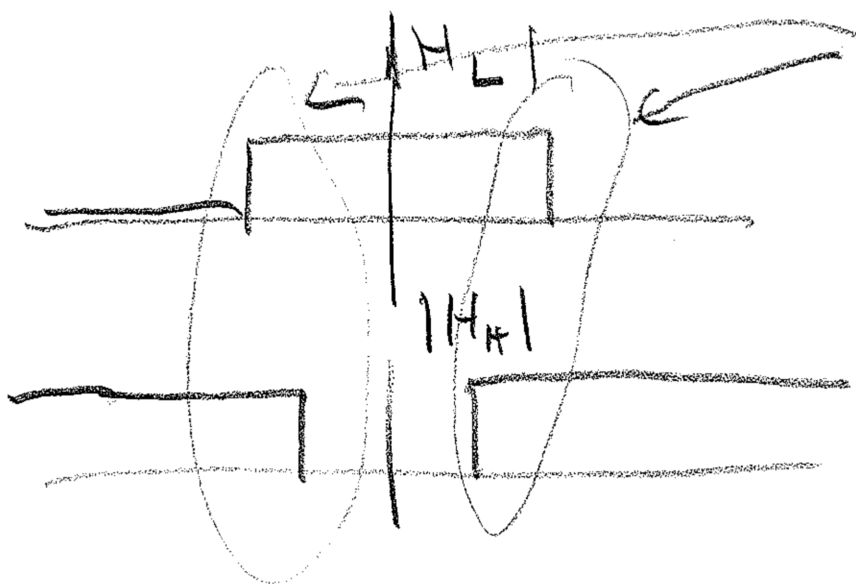
$$h[n] = h_L[n] - h_H[n]?$$

Please still assume $\Omega_{c_L} < \Omega_{c_H}$.

No we were adding zero magnitudes to magnitudes of 2.

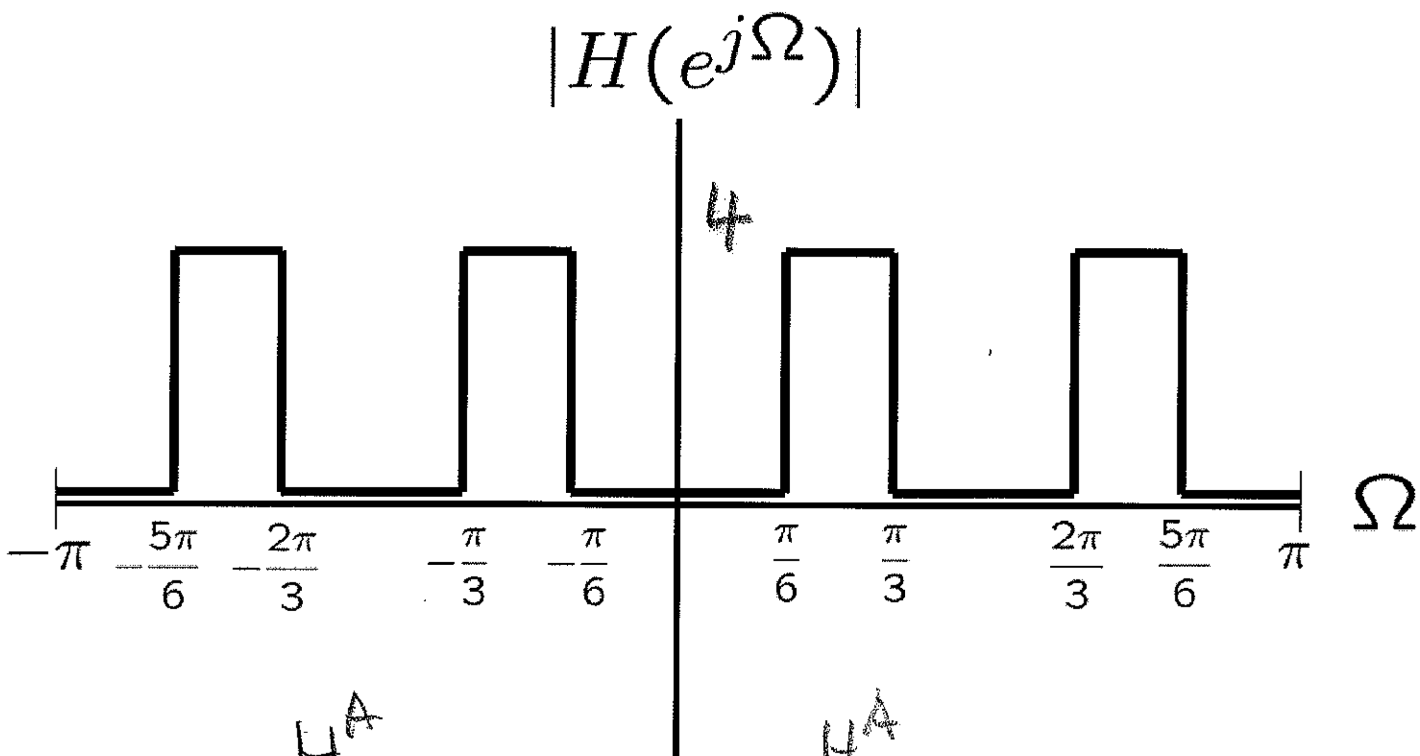
(E) If all you know are the magnitudes of the frequency responses for H_L and H_H, do you have enough information to answer part C if $\Omega_{c_L} > \Omega_{c_H}$? Why or why not?

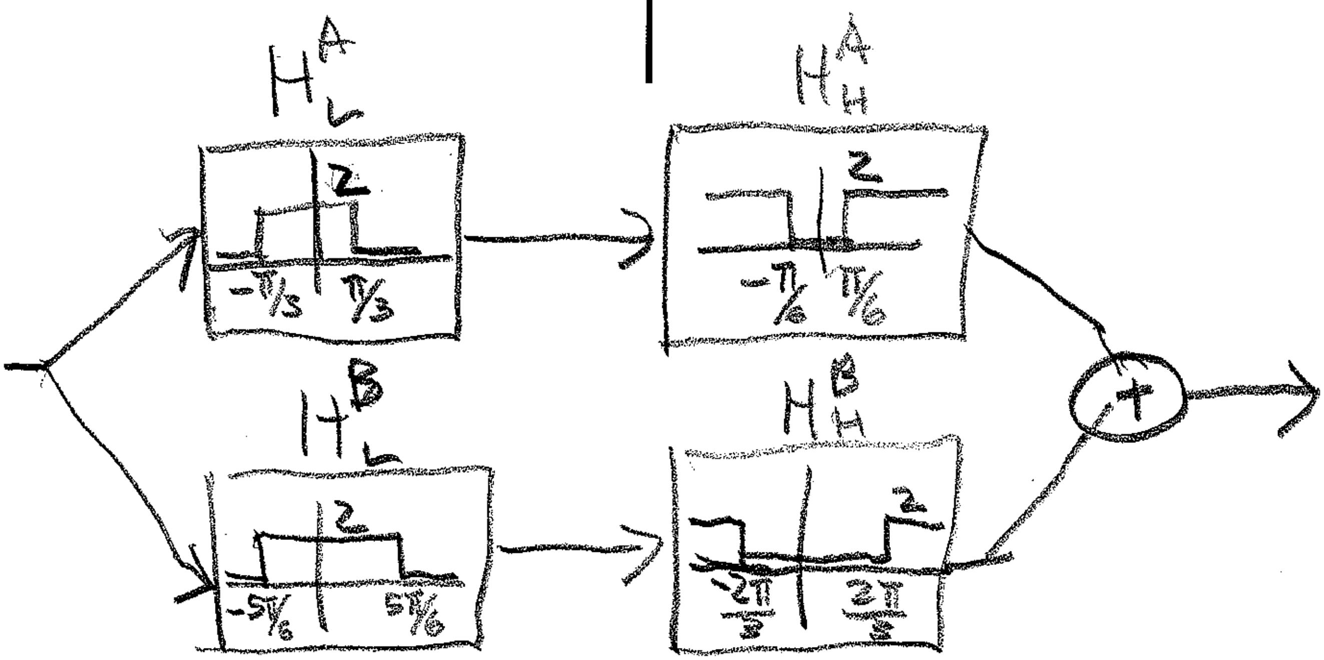
No



$|H_L|$ & $|H_H|$ nonzero here, but $|H_L| + |H_H| \neq |H_L + H_H|$

(F) Suppose you have two low-pass filters, one with cut-off frequency $\Omega_{c_L}^{A}$ and a second with cut-off frequency $\Omega_{c_L}^{B}$, and two high-pass filters, one with cut-off frequency $\Omega_{c_H}^{A}$ and a second with cut-off frequency $\Omega_{c_H}^{B}$. Draw a diagram that shows how you would combine these four filters, and give values for $\Omega_{c_L}^{A}$, $\Omega_{c_L}^{B}$, $\Omega_{c_H}^{A}$, and $\Omega_{c_H}^{B}$, to generate a filter with the frequency response given below.

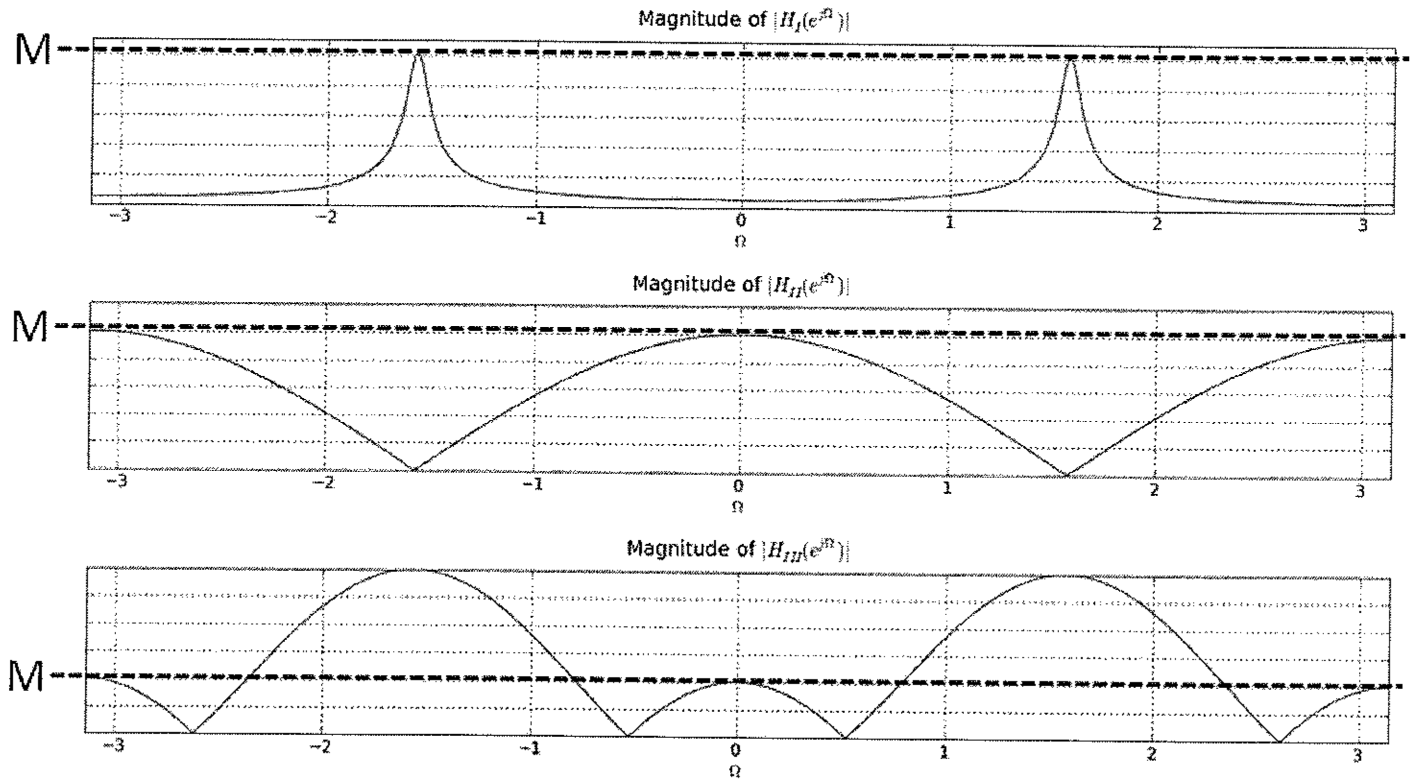




$$h[n] = \sum h_L^A[m]\, h_H^A[n-m] + \sum h_L^B[m]\, h_H^B[n-m]$$

Review Problems 3

In answering the following questions, please refer to the following three plots of the magnitude of three frequency responses, $|H_I(e^{j\Omega})|$, $|H_{II}(e^{j\Omega})|$, and $|H_{III}(e^{j\Omega})|$, given below.



(A) Suppose the input to a linear time invariant system is the sequence

$$x[n] = 2 + \cos\frac{5\pi}{6}n + \cos\frac{\pi}{6}n + 3(-1)^n$$

What is the maximum value of the sequence x, and what is the smallest positive value of n for which x achieves its maximum?

$\max_n \left(\cos \frac{\pi}{6} n\right) = 1 \quad n=0,\ n=12, \ldots$

$\max_n \left(\cos \frac{5\pi}{6} n\right) = 1 \quad n=0,\ n=12$

$\max_n 3(-1)^n = 3 \quad n$ is even

$\max_m x[m] =$ $2+1+1+3=7$

Smallest $n > 0$ for which $x[n] = \max_m x[m]$ $n = 12$

(B) Suppose the sequence X from part A is the input to a linear time invariant system described by one of the three frequency response plots above (I, II or III). If y is the resulting output and is given by

$$y[n] = 8 + 12(-1)^n,$$

which frequency response plot describes the system, and what is the value of M in the plot you selected? Be sure to justify your selection.

Frequency response at $\Omega = \pi/6$ and $\Omega = \frac{5\pi}{6}$ must be zero. Could only be III.

$y[n] = \underbrace{H(e^{j0})}_{=4} \cdot 2 \cdot (1)^n + \underbrace{H(e^{j\pi})}_{=4} 3 \cdot (-1)^n$

Frequency response plot (I, II, or III) = III

Numerical value of M = 4

(C) Suppose the unit sample response of a linear time-invariant system has only three nonzero *real* values, $h[0]$, $h[1]$, and $h[2]$. In addition, suppose these three *real* values satisfy the three equations:

$$\begin{aligned} h[0] + h[1] + h[2] &= 5 \\ h[0] + e^{-j\frac{\pi}{2}\cdot 1}h[1] + e^{-j\frac{\pi}{2}\cdot 2}h[2] &= 0 \\ h[0] + e^{j\frac{\pi}{2}\cdot 1}h[1] + e^{j\frac{\pi}{2}\cdot 2}h[2] &= 0. \end{aligned}$$

Which of the above plots, I, II or III, is a plot of the magnitude of the frequency response of this system, and what is the value of M in the plot you selected? Be sure to justify your selection.

$$H(e^{j\Omega}) = \sum_{m=0}^{2} h[m]e^{-j\Omega m}$$

1st eqn $\quad \Omega = 0 \quad H(e^{j0}) = 5 = M$

2nd eqn $\quad \Omega = \pi/2 \quad H(e^{j\pi/2}) = 0$

3rd eqn $\quad \Omega = -\pi/2 \quad H(e^{-j\pi/2}) = 0$

frequency response plot (I, II, or III) = Must be II

Numerical value of M = 5

(D) For the system given in part C, if $y[n] = \sum_{m=0}^{2} h[m]x[n-m]$ and $x[n] = e^{j\frac{\pi}{6}n}$ for all n, please determine the complex numerical value for

$$\frac{y[n]}{e^{j\frac{\pi}{6}n}}.$$

It might be helpful to know that the numerical values of $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = 0.5$, $\cos\frac{\pi}{3} = 0.5$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

$$y[n] = H(e^{j\pi/6})\,x[n] = H(e^{j\pi/6})\,e^{j\frac{\pi}{6}n}$$

$$\Rightarrow \frac{y[n]}{e^{j\frac{\pi}{6}n}} = H(e^{j\pi/6})$$

$H(e^{j\pi/6}) = h[0] + h[2]e^{-j\pi/3}$

Find $h[0]$, $h[1]$, $h[2]$

$h[0] + h[1] + h[2] = 5$

$h[0] - j\,h[1] - h[2] = 0$

$h[0] + j\,h[1] - h[2] = 0$

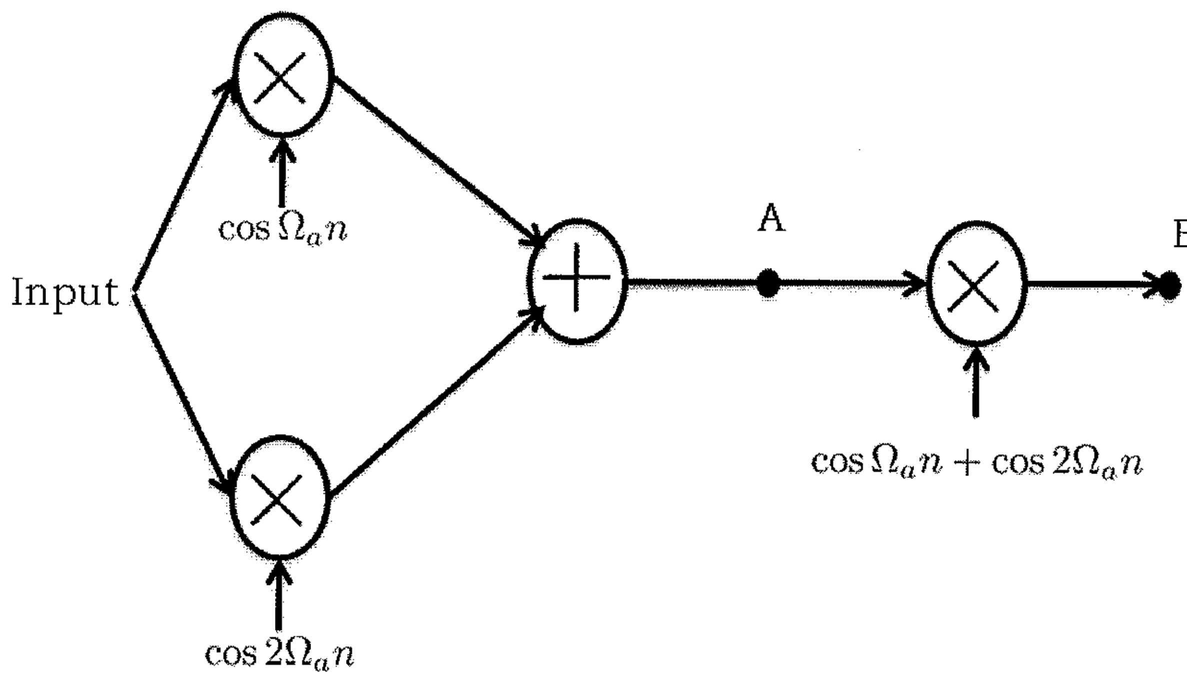
$h[0] = h[2] = 2.5 \quad h[1] = 0$

$\frac{y[n]}{e^{j\frac{\pi}{6}n}} = 2.5 + 2.5(\cos\pi/3 + j\sin(-\pi/3))$

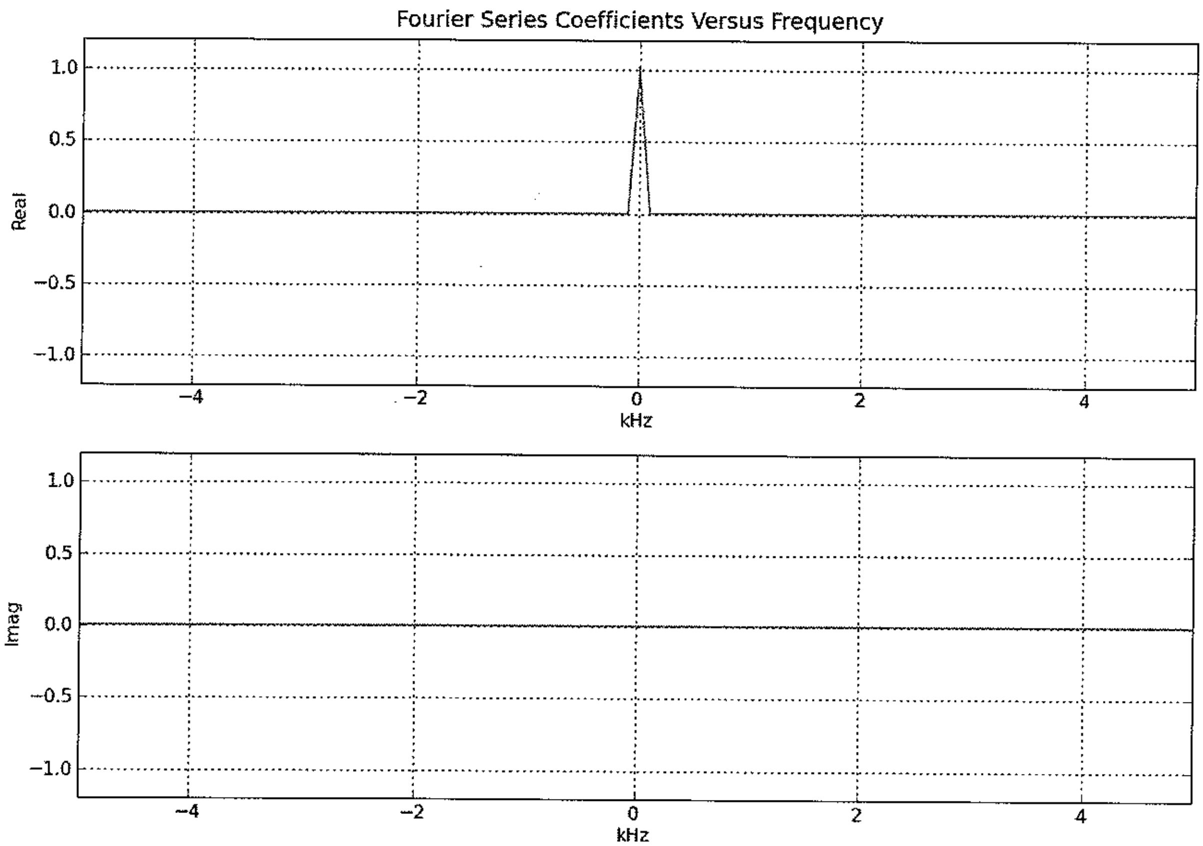
$= 3.75 - j\left(\frac{5\sqrt{3}}{4}\right)$

Review Problem 4

Consider the simple modulation-demodulation system below, where all signals are assumed periodic with period $N = 10000$, and the sampling frequence, f_s, is 10000 samples per second. In addition, $\Omega_a = 2\pi \frac{f_a}{f_s} = \frac{1000*2*\pi}{10000}$.

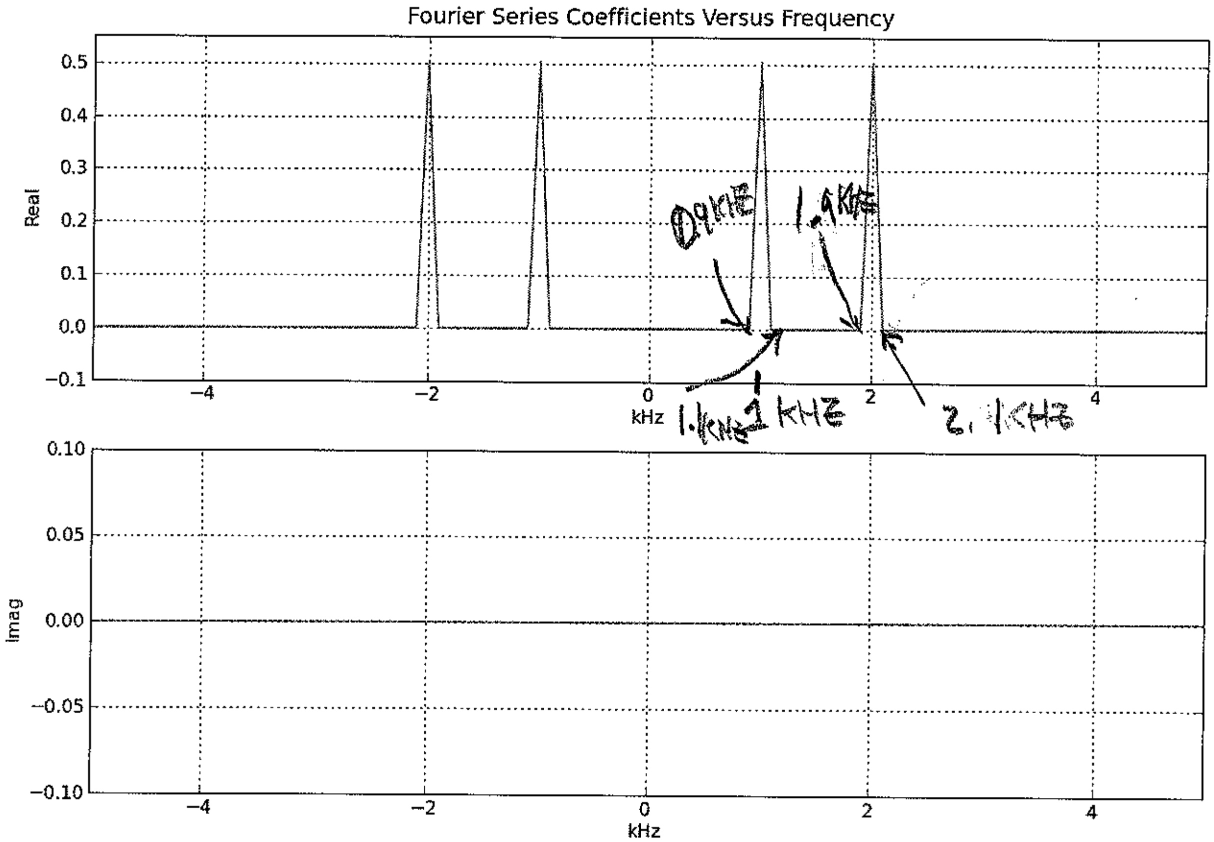


The Fourier Series coefficients versus frequency for the input to the modulation-demodulation system are plotted below for the case $N = 10000$ and $f_s = 10000$. Note that the Fourier coefficients are nonzero only for $-100 \leq f_k \leq 100$.

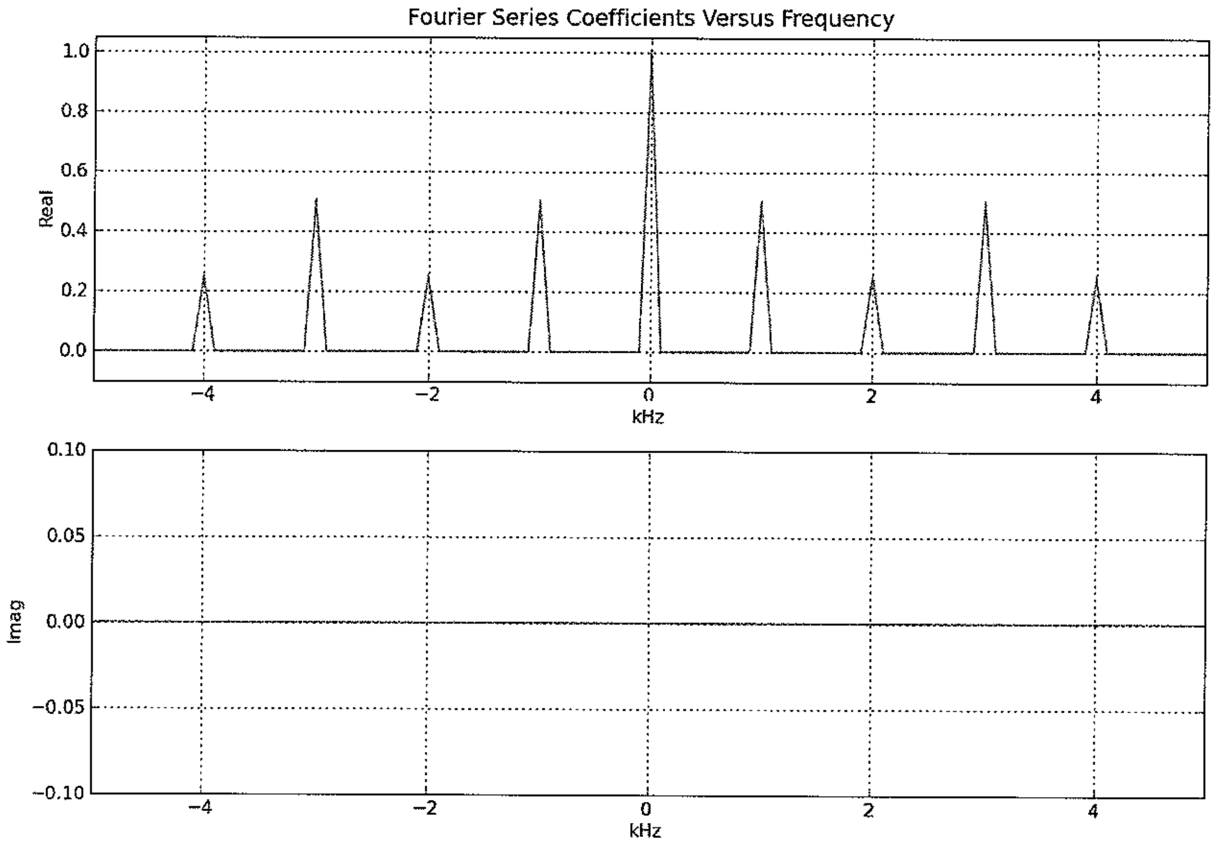


On the two sets of axes below, please plot the Fourier series coefficients versus Ω for the signals at location **A** and **B** in the above diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of Fourier Coefficients of signal at Point A



Plot of Fourier Coefficients of signal at Point B



Review problem 6

In this modulation problem you will be examining periodic signals and their associated discrete-time Fourier series (DTFS) coefficients. Recall that a periodic signal $x[n]$ with period N has DFTS coefficients given by

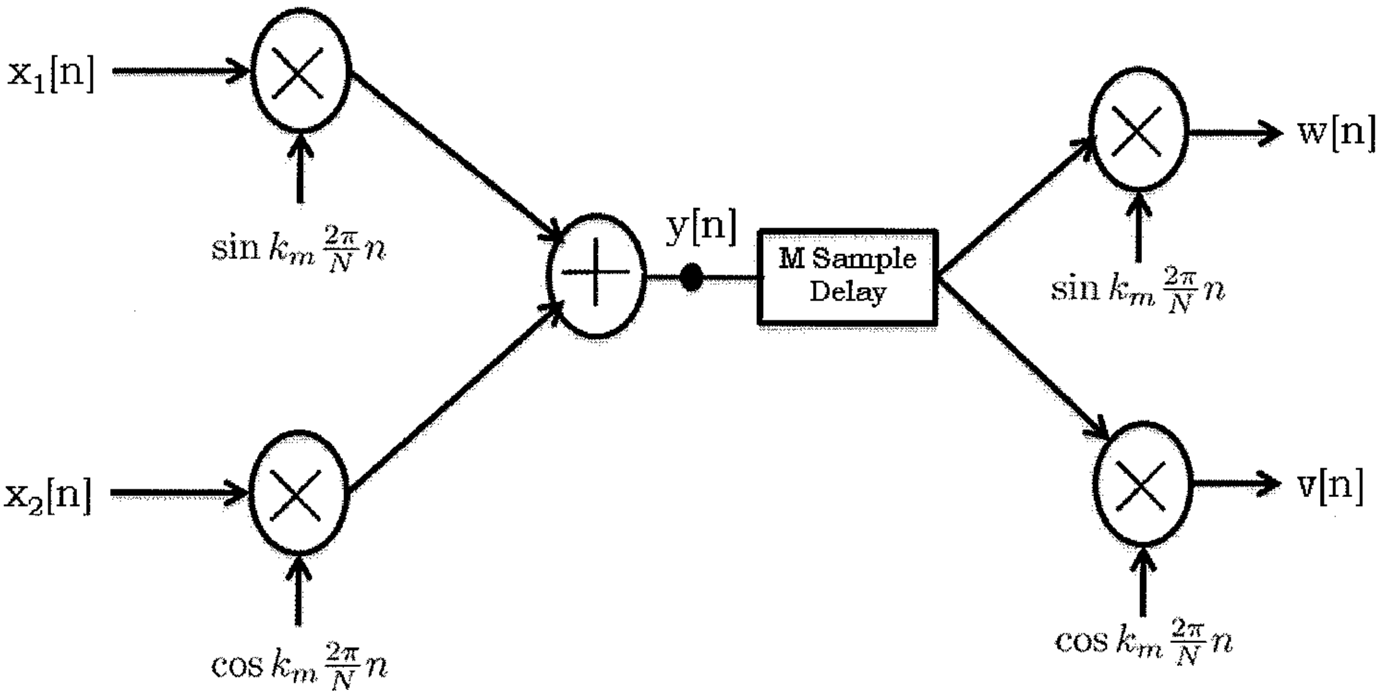
$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

and that the signal $x[n]$ can be reconstructed from the DFTS coefficients using

$$x[n] = \sum_{k=-K}^{K-1} X[k]e^{j\frac{2\pi}{N}kn}$$

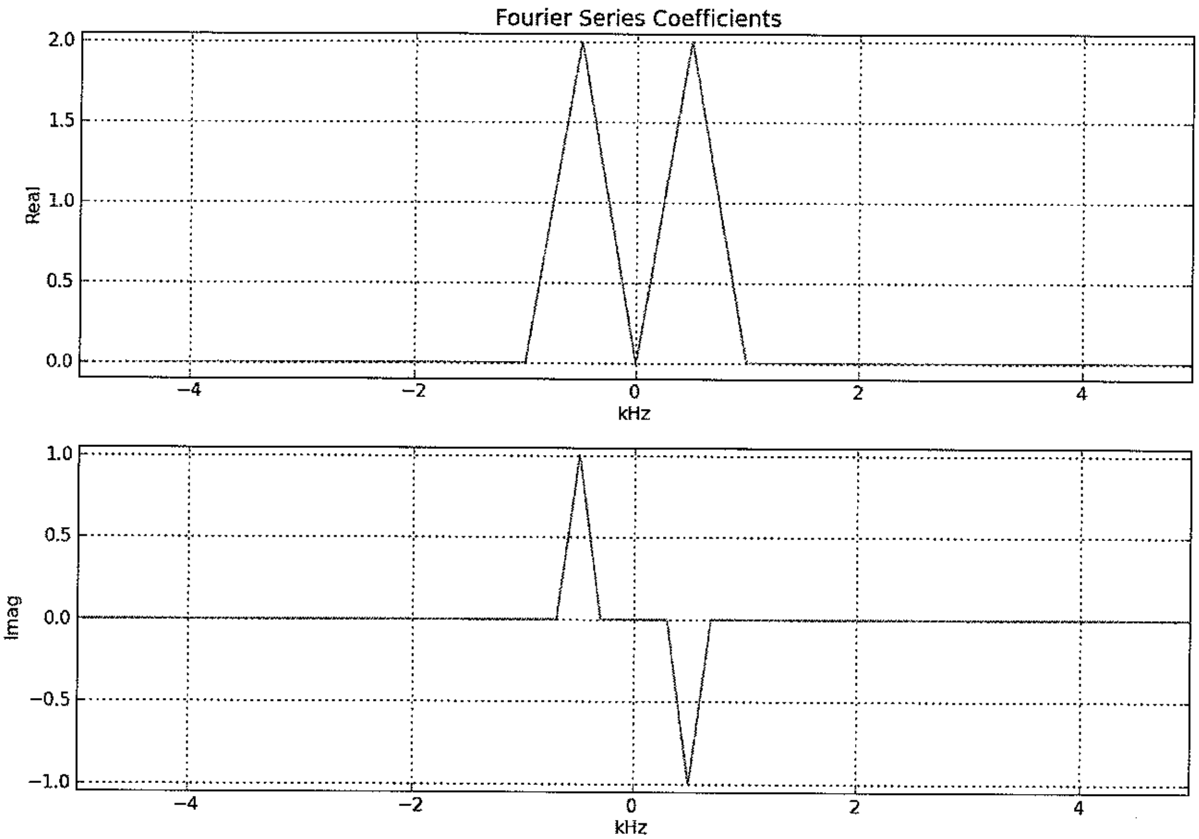
where N is the period of the signal, $-K \leq k < K$ with $K = \frac{N}{2}$.

All parts of this question pertain to the following modulation-demodulation system, where all signals are periodic with period $N = 10000$ and therefore $K = 5000$. Please also assume that the sample rate associated with this system is 10000 samples per second, so that k is both a coefficient index and a frequency. In the diagram, the modulation frequency, k_m, is 500.



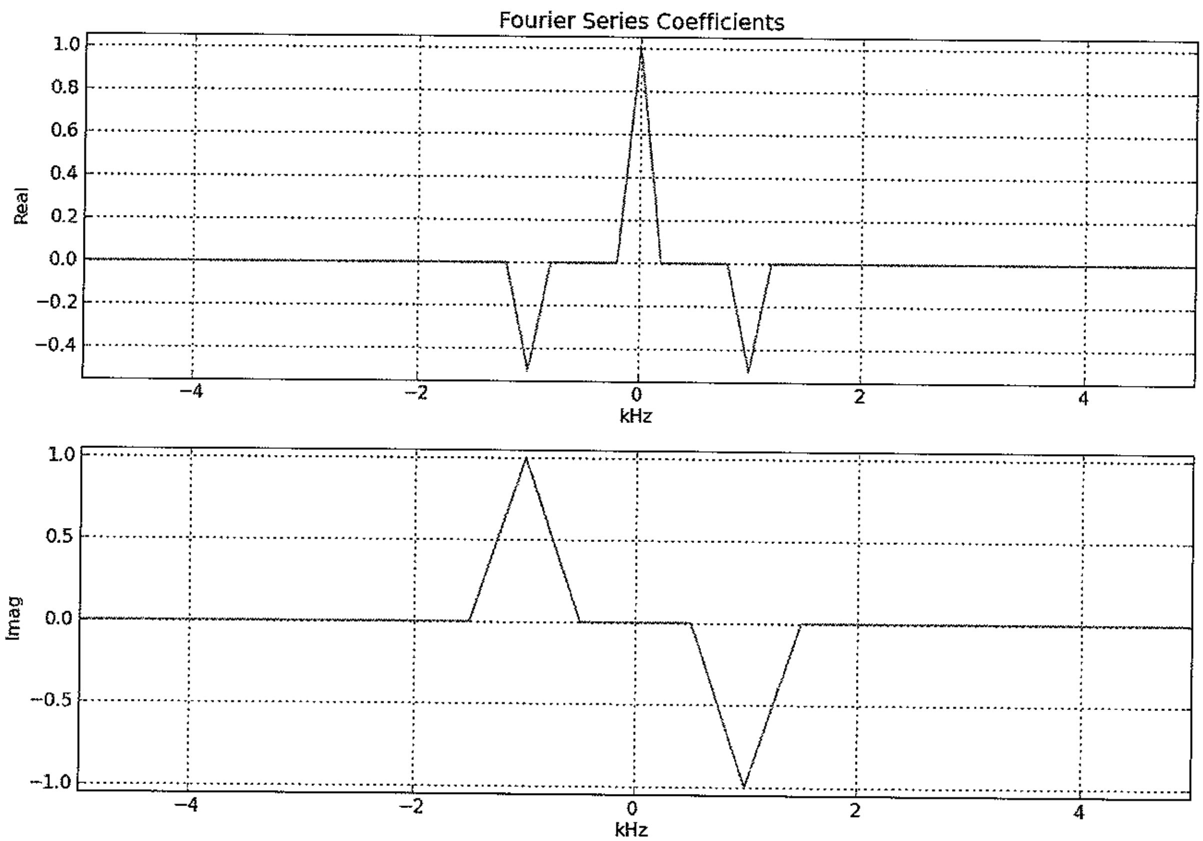
x_1[n]
$\sin k_m \frac{2\pi}{N} n$
x_2[n]
$\cos k_m \frac{2\pi}{N} n$
y[n]
M Sample
Delay
w[n]
$\sin k_m \frac{2\pi}{N} n$
v[n]
$\cos k_m \frac{2\pi}{N} n$

(A) Suppose the DFTS coefficients for the signal $y[n]$ in the modulation/demodulation diagram are as plotted below.

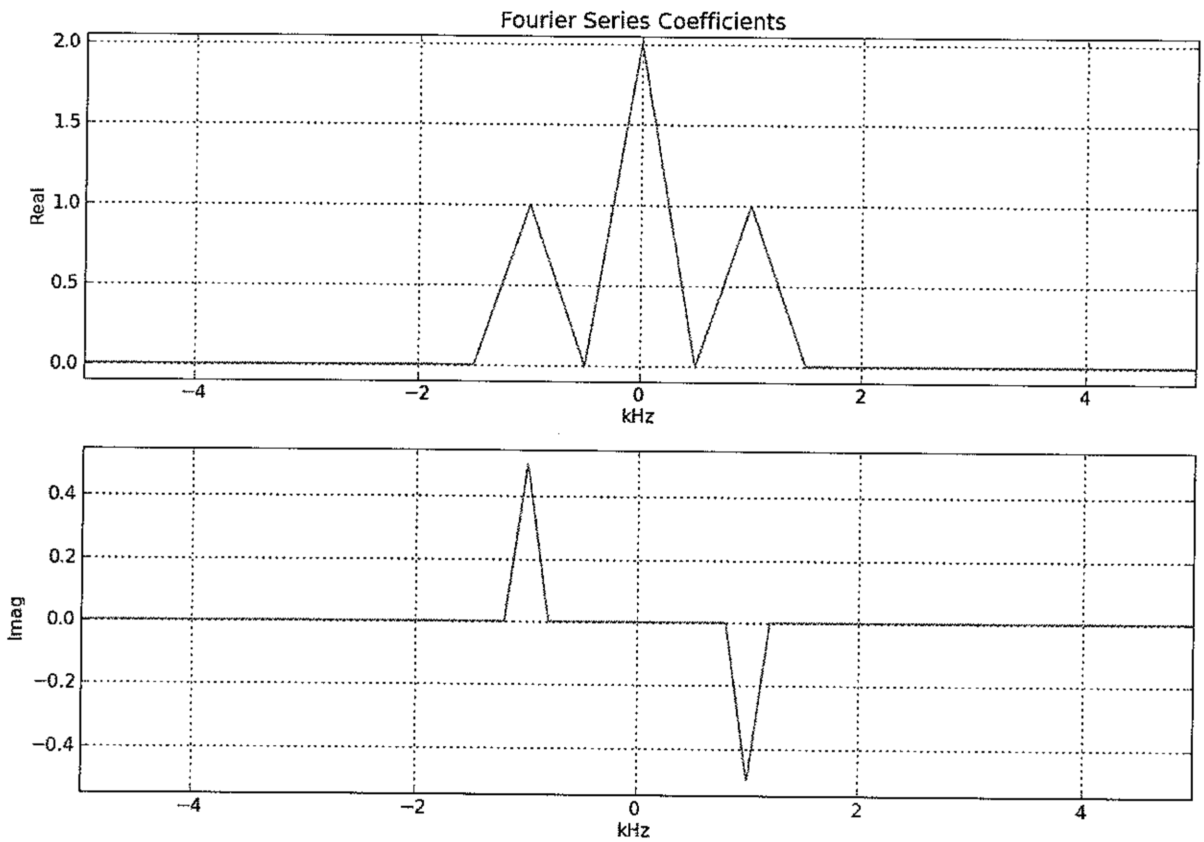


Assuming that $M = 0$ for the M-sample delay (no delay), on the two sets of axes on the next pages, please plot the DFTS coefficients for the signals $\mathbf{w}$ and $\mathbf{v}$ in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for w

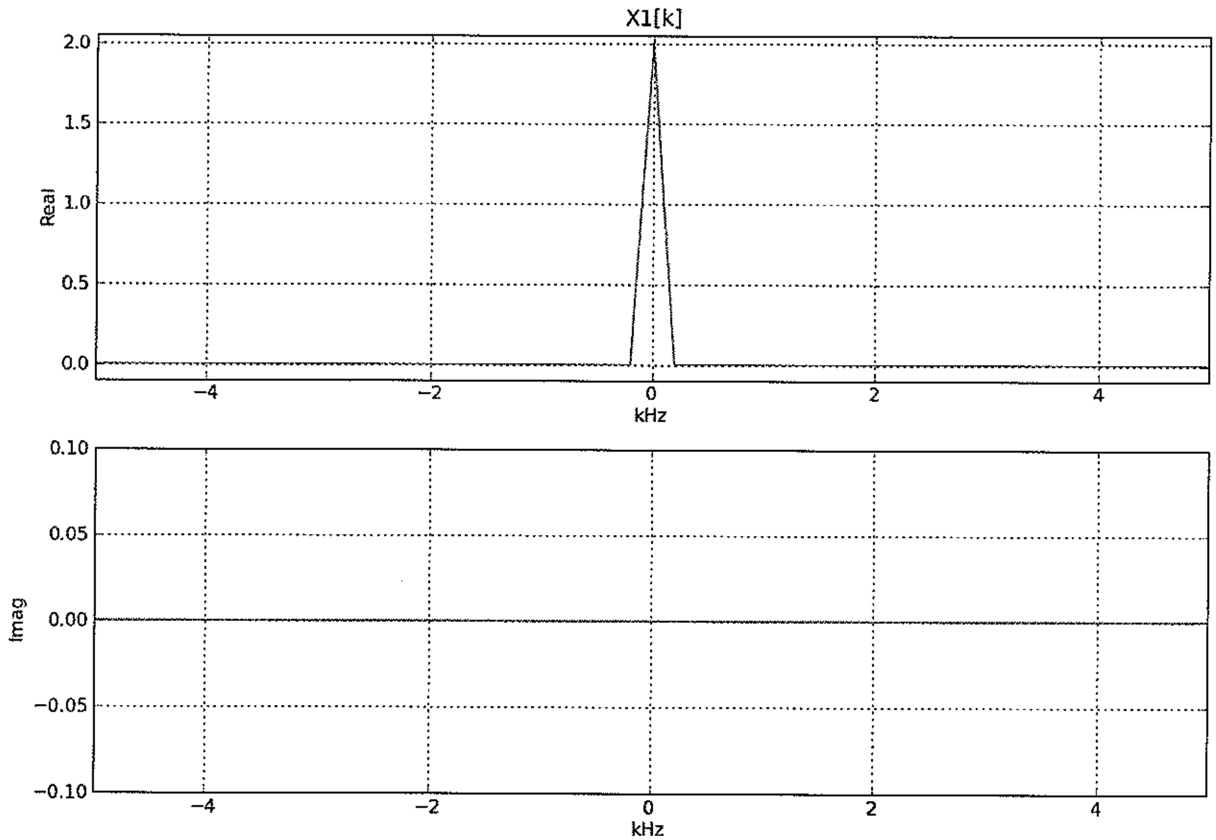


Plot of DFTS coefficients for v

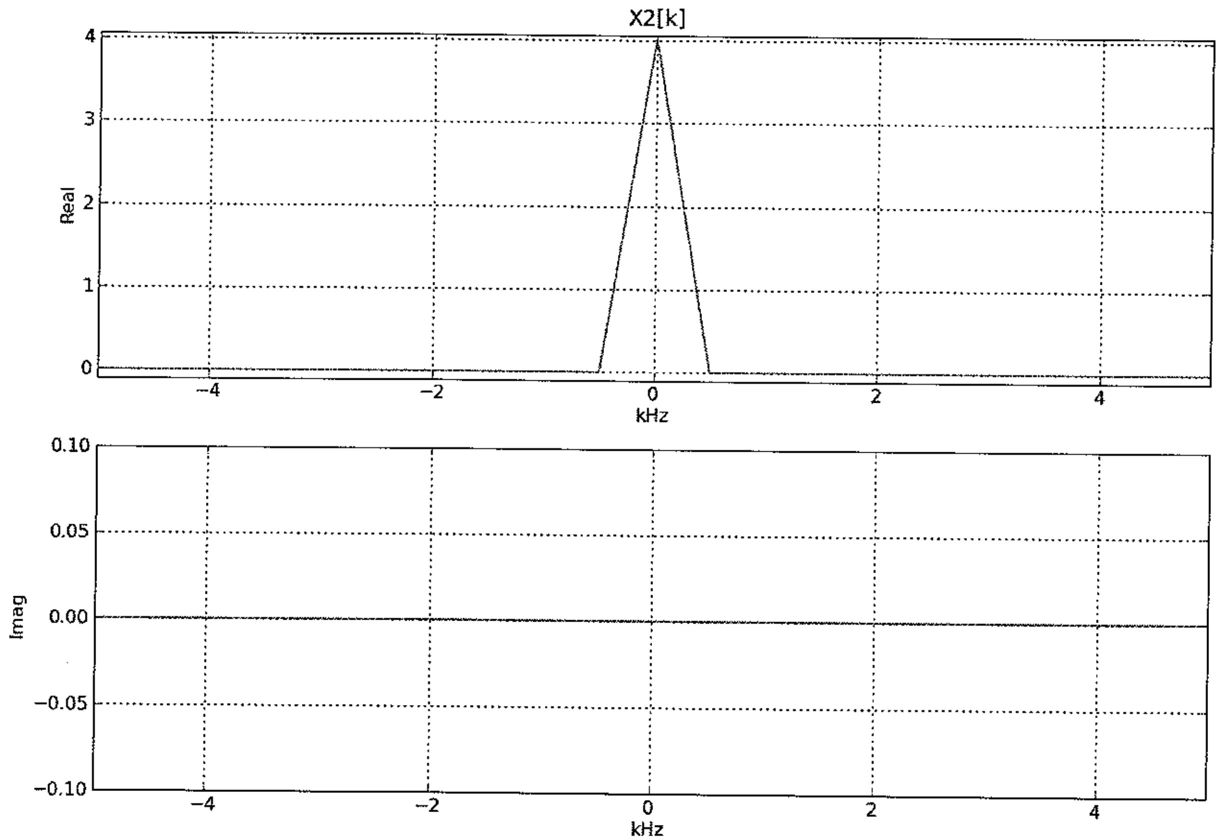


(B) Assuming the DFTS coefficients for the signal $y[n]$ are the same as in part A, on the axes below, please plot the DFT coefficients for the signal x_1 in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for x_1

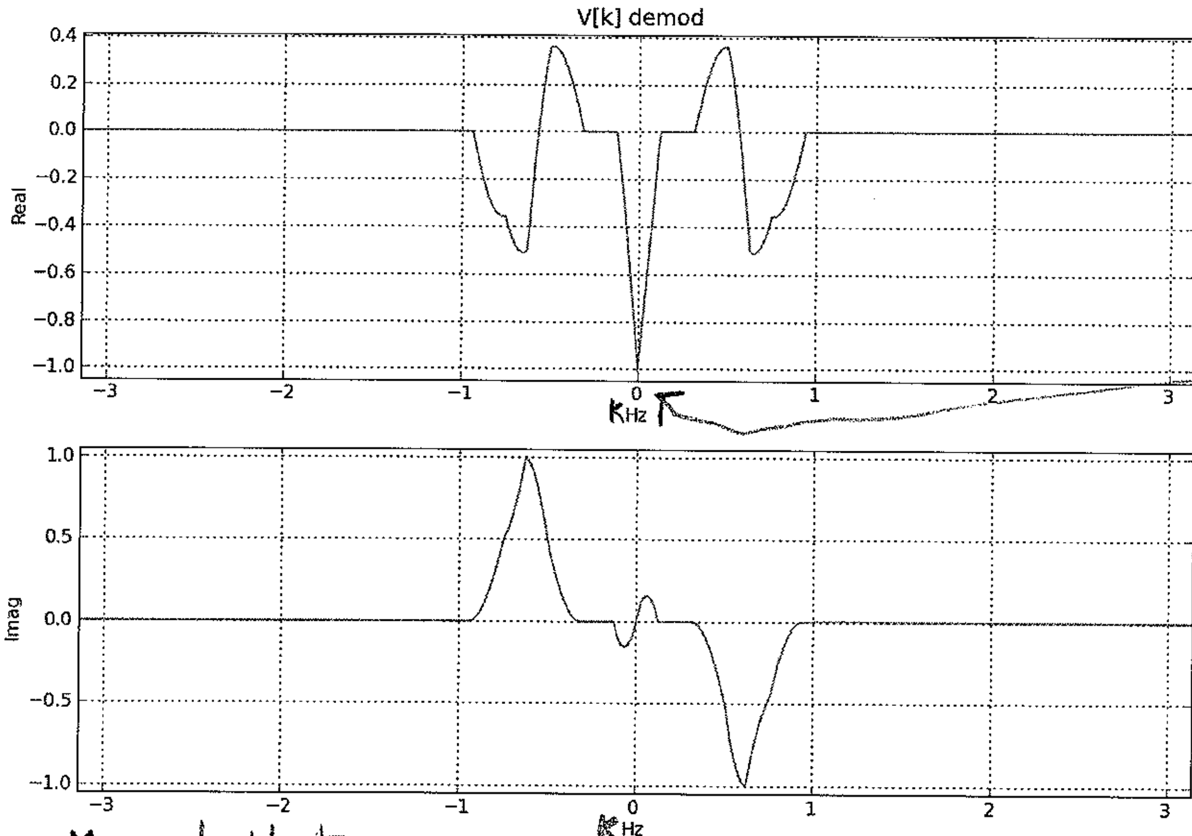


Plot of DFTS coefficients for x_2



(C) If the M-sample delay in the modulation/demodulation diagram has the right number of samples of delay, then it will be possible to nearly perfectly recover $x_1[n]$ by low-pass filtering $v[n]$. Please determine the smallest positive number of samples of delay that are needed and the cut-off frequency for the low-pass filter. Please be sure to justify your answer, using pictures if appropriate.

Plot of DFTS coefficients for v with 5 sample delay



Want M such that

$$\sin\left(K_m \frac{2\pi}{N}(n-M)\right) \simeq \cos\left(K_m \frac{2\pi}{N} n\right)$$

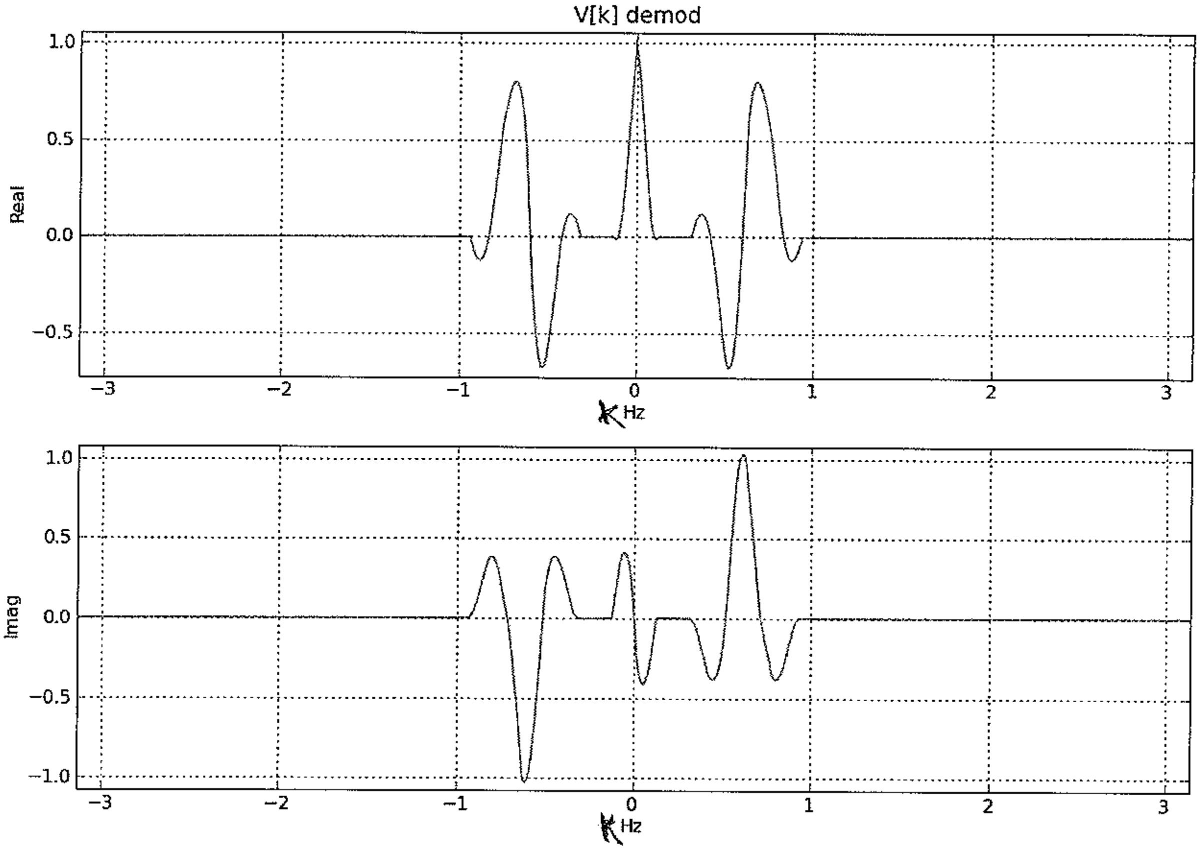
so that cosine demod (V output) will work. With $M = 5$

$$\sin\left(2\pi \frac{500}{10000}(n-5)\right) = \sin\left(\frac{2\pi 500}{10000} n - \frac{\pi}{2}\right)$$

$$= -\cos\left(\frac{2\pi 500}{10000}\right)$$

M = 5 and an LPF with gain −1 and cutoff at 250 Hz

Plot of DFTS coefficients for v with 15 sample delay



Second Soln $\quad M = 15$

$$\sin\left(k_M \frac{2\pi}{N}(n-M)\right) = \sin\left(\frac{2\pi 500}{10000} n - \frac{3}{2}\pi\right)$$

$$= \cos\left(\frac{2\pi 500}{10000} n\right)$$

$M = 15$ and LPF with gain of 1 with cutoff at 500Hz

Solution to part C continued.

Either

M = 5 LPF with gain of -1
and cutoff at 250Hz

or

M = 15 LPF with gain of 1
and cutoff at 250Hz

Smallest M (number of samples of delay) > 0 = _______________
Cutoff Frequency of Low Pass Filter = _______________