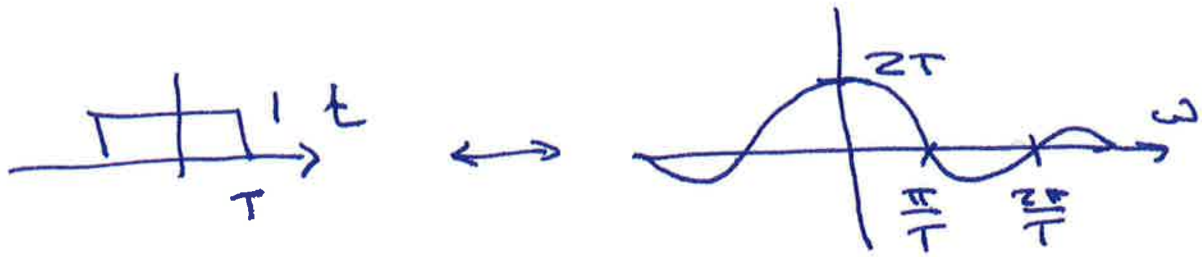
6.003 11/30/2011



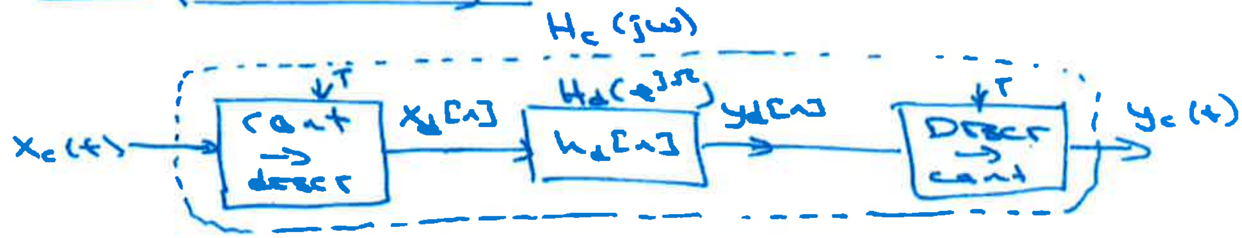
$$\sum_n \delta(t-nT) \Leftrightarrow \frac{2\pi}{T}\sum_k \delta\left(\omega-\frac{2\pi}{T}k\right)$$

$$x(t)\cdot p(t) \leftrightarrow \frac{1}{2\pi} X * P(j\omega)$$

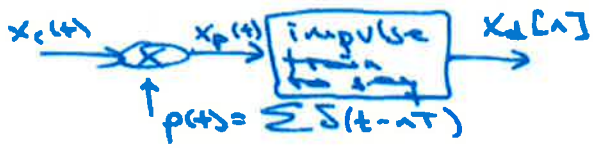
Rec 19

* DT processing of CT signals

DT processing:



Expand C/D box



Expand D/C box

Given $H_d(e^{j\Omega})$, what's the effective $H_c(j\omega)$?

Know
$$X_d(e^{j\Omega}) \overset{\text{def DTFT}}{=} \sum_{n=-\infty}^{\infty} x_d[n]\, e^{-j\Omega n}$$

$$X_d(e^{j\Omega}) \overset{x_d \text{ samples of } x_c}{=} \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\Omega n} \qquad (1)$$

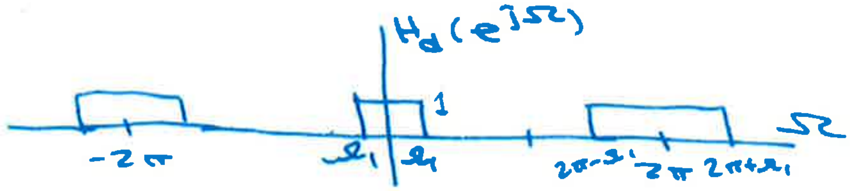
Also:

$$X_p(j\omega) = \text{CTFT}\left\{ \sum_{n=-\infty}^{\infty} x_c(nT)\, \delta(t-nT) \right\}$$
$$= \sum_n x_c(nT)\, e^{-j\omega nT}$$

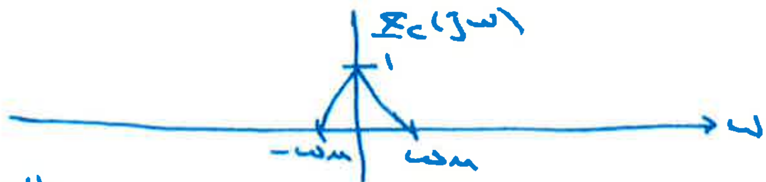
$$X_p\left(j\frac{\Omega}{T}\right) = X_p(j\omega) \overset{\Omega = \omega T}{=} \sum_n x_c(nT)\, e^{-j\Omega n} \qquad (2)$$

(1) & (2): $X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right)$, periodic w/ 2π

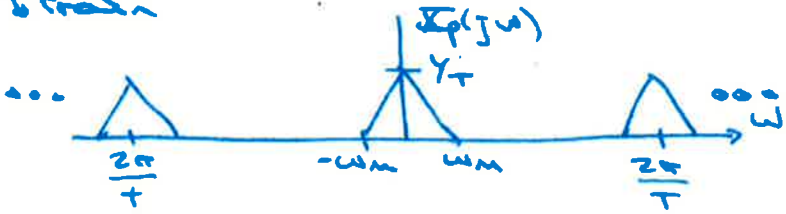
Graphs: Assume



* FT of $x_c(t)$:

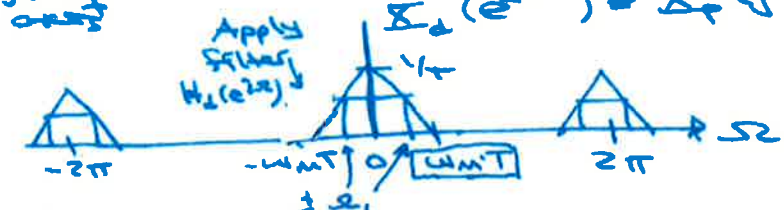


Unit impulse train

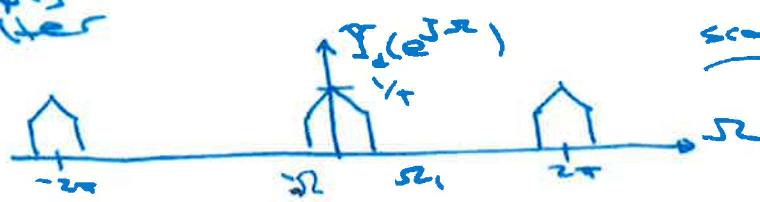


rad/s → rad (scale freq axis)

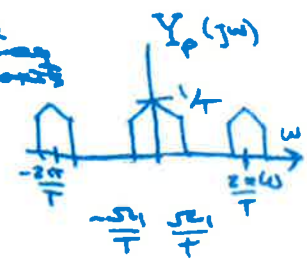
$X_d(e^{j\Omega}) = X_p(j\frac{\Omega}{T})$



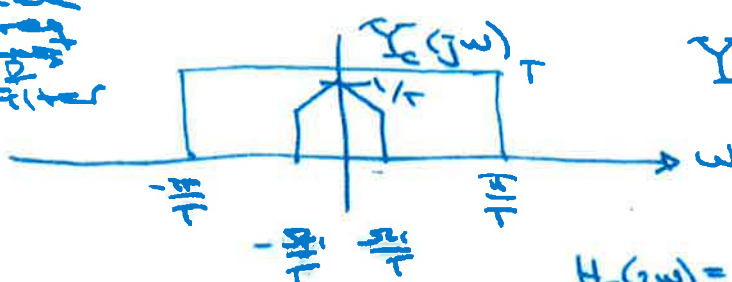
Apply filter



scale freq axis



rad → rad/s (scale freq axis & recon filter)

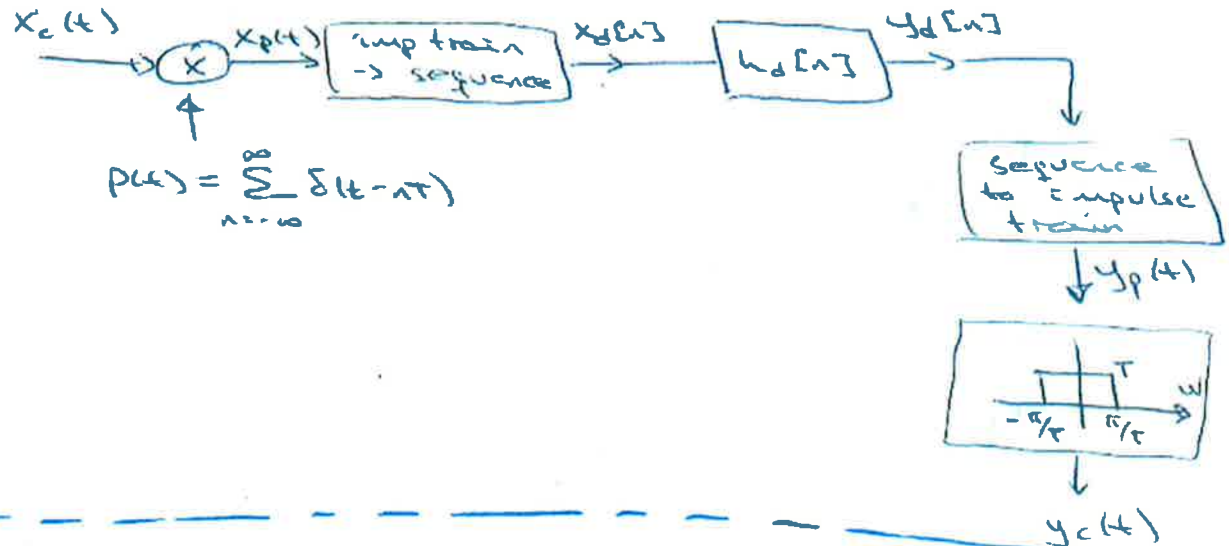


$Y_c(j\omega) = X_c(j\omega)$.

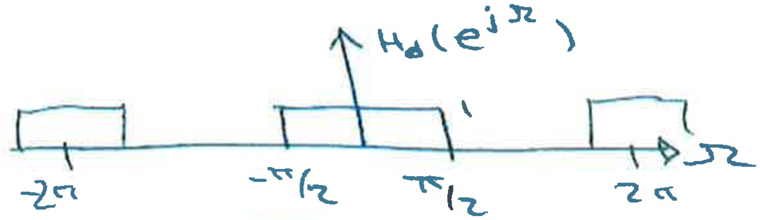
$H_d(e^{j\omega T})$

$$H_c(j\omega) = \begin{cases} H_d(e^{j\omega T}) & |\omega| < \frac{\pi}{T} \\ 0 & \text{o.w.} \end{cases}$$

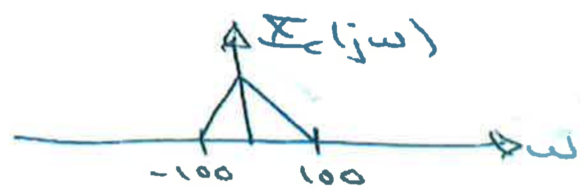
DT Processing of CT signals



Given discrete-time frequency response:



and FT of $x_c(t)$:



Find $Y_p(j\omega)$ for

a) $T = \pi/100$

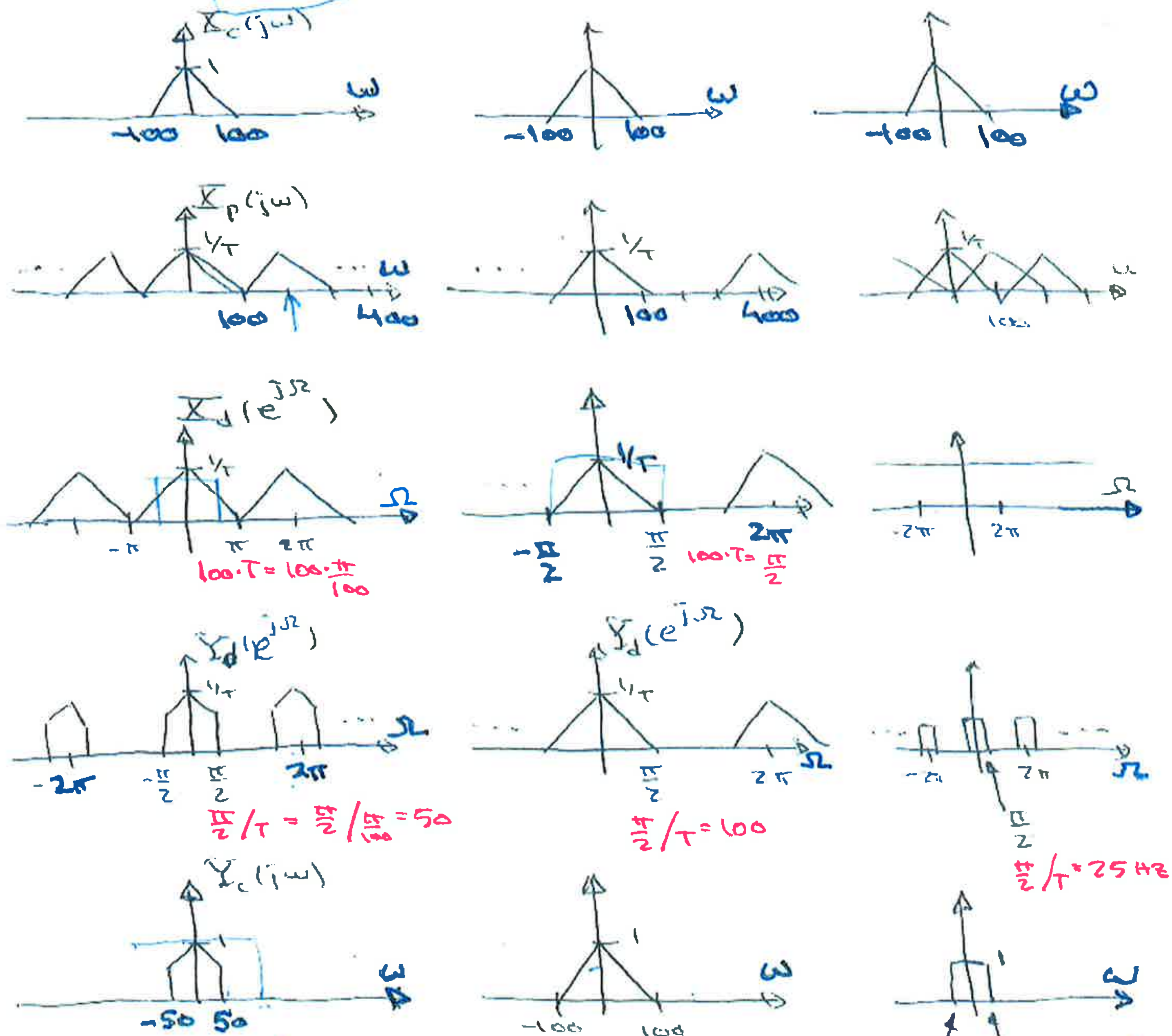
b) $T = \pi/200$

c) $T = \pi/50$

a) $T = \frac{\pi}{100}$ $\boxed{\frac{2\pi}{T} = 200}$

b) $T = \pi/200$; $\frac{2\pi}{T} = 400$

c) $T = \frac{\pi}{50}$; $\frac{2\pi}{T} = 100$



Equivalent to applying scaled DT filter

✓

same as LPF X_c

NOT true b/c aliasing!

$$\boxed{X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right)}$$

$$\boxed{Y_p(j\omega) = Y_d(e^{j\omega T})}$$

Ex: Delay implemented w/ $H_d(e^{j\Omega})$. Q: What's $H_d(e^{j\Omega})$?

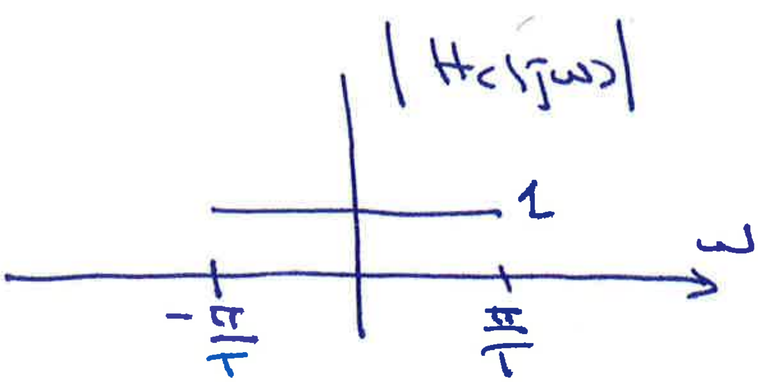
Goal, $y_c(t) = x_c(t-\Delta)$
for band-limited $x_c(t)$ & adeq sampling rate.

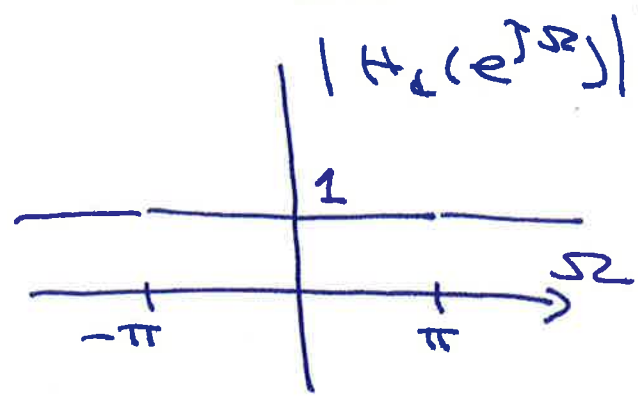
$\Downarrow$ FT

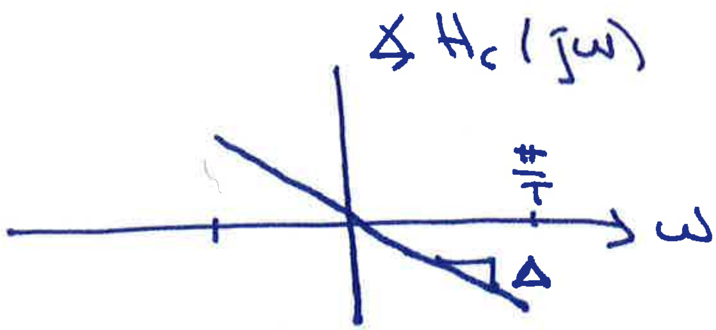
$$Y_c(j\omega) = e^{-j\omega\Delta} X_c(j\omega)$$

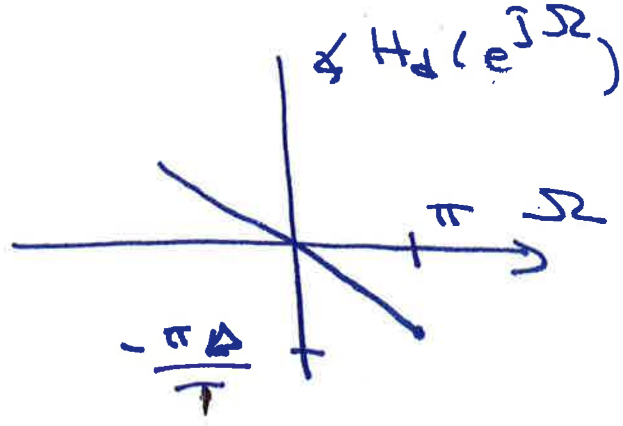
i.e. specs for $H_c(j\omega)$ are

$$H_c(j\omega) = \begin{cases} e^{-j\omega\Delta} & |\omega| < \frac{\pi}{T} \\ 0 & ow \end{cases}$$









i.e. $H_d(e^{j\Omega}) = e^{-j\frac{\Omega}{T}\Delta}$ $|\Omega| < \pi$

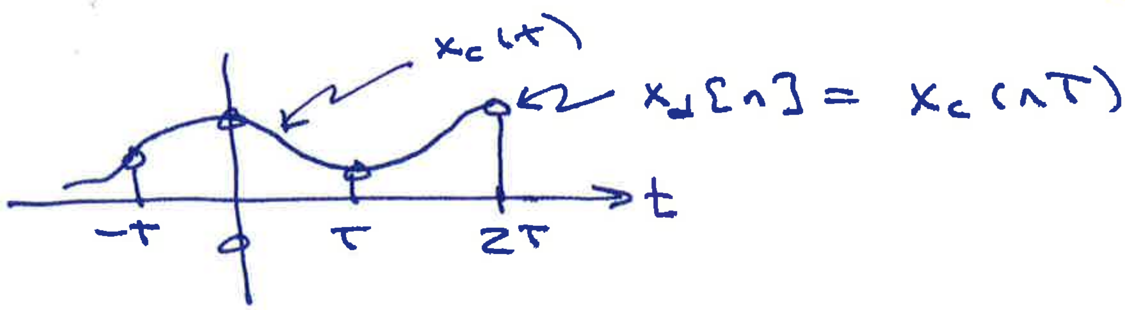
periodic w/ 2π.

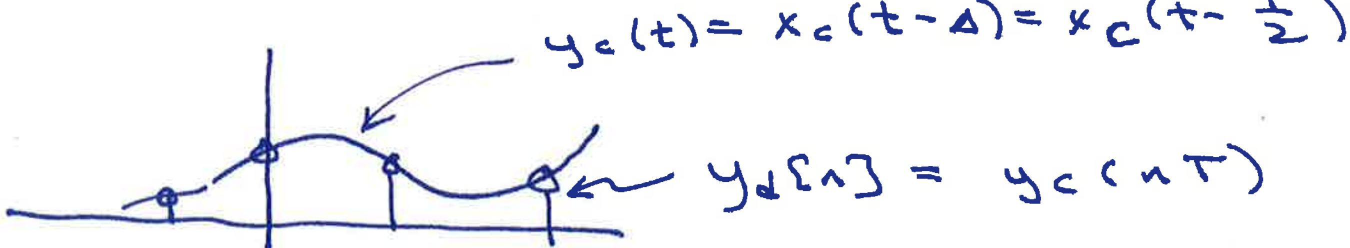
Note: For $\frac{\Delta}{T}$ = integer # of samples,

(1) $y_d[n] = x_d[n - \frac{\Delta}{T}]$ – logical.

For $\frac{\Delta}{T}$, eq (1) does not make sense, but it can be interpret for a bandlimited $x_c(t)$.

e.g. $\frac{\Delta}{T} = \frac{1}{2}$, i.e. $\Delta = T/2$, half-samp delay.





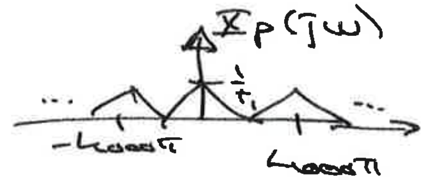
$= x_c(nT - \frac{T}{2})$

if x_c is bandlim & adeq sampled

$= x_c((n-\frac{1}{2})T)$

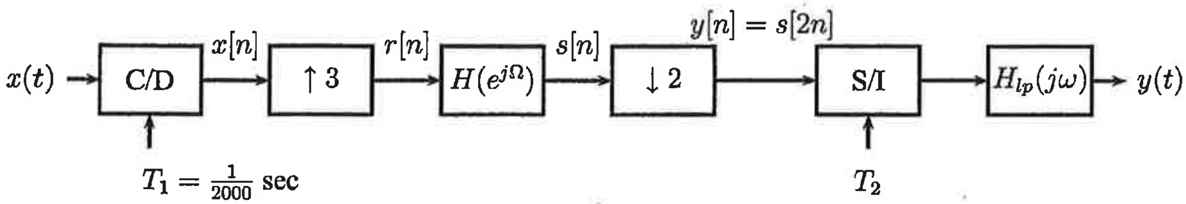
$y_d[n]$: samples of shifted, bandlimited interpolation of $x_d[n]$.

$$\frac{2\pi}{T_1} = \frac{2\pi}{1/2000} = 4000\pi \ \therefore$$

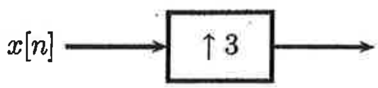


PROBLEM 2 (30 points)

Consider the following system

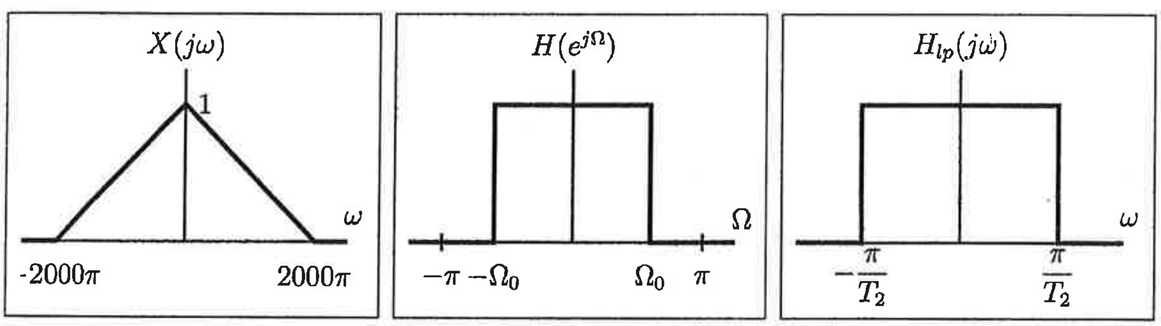


where

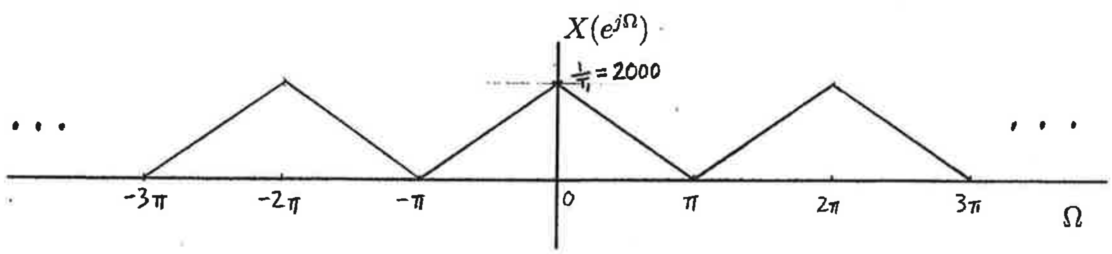


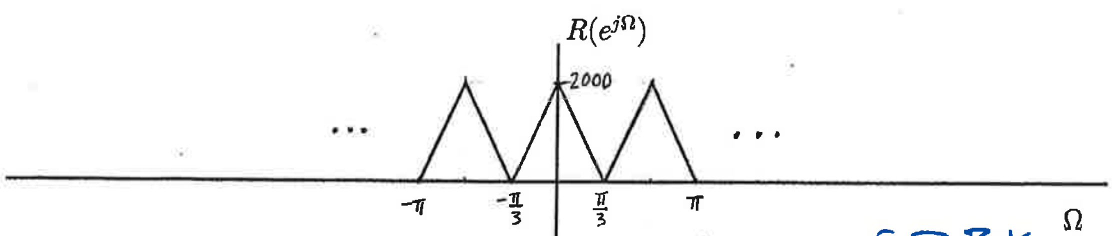
$$r[n] = \begin{cases} x[n/3], & n = 0, \pm 3, \pm 6, \cdots \\ 0, & \text{otherwise} \end{cases}$$

The CTFT $X(j\omega)$ of the input, the DTFT $H(e^{j\Omega})$ of the DT filter, and the CTFT of the low-pass filter $H_{lp}(j\omega)$ are shown below.



Part a. Sketch the DTFT of $x[n]$, $X(e^{j\Omega})$, and the DTFT of $r[n]$, $R(e^{j\Omega})$. Label your axes.





$$R(e^{j\Omega}) = \sum_n r[n] e^{-j\Omega n} = \sum_k r[3k] e^{-j\Omega 3k} = \sum_k x[k] e^{-j\Omega 3k}$$

$$= X(e^{j3\Omega})$$