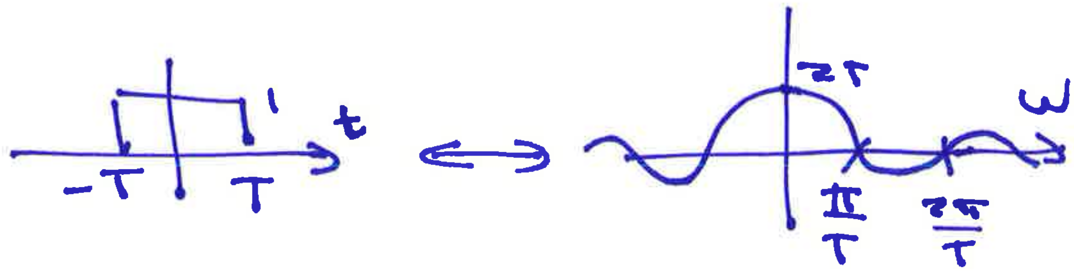
6.003 11/23/2011

Tools: 

$$\sum_n \delta(t-nT) \longleftrightarrow \frac{2\pi}{T}\sum_k \delta\left(\omega-\frac{2\pi}{T}\right)$$

Rec 18

* Sampling
* Aliasing
* Reconstruction

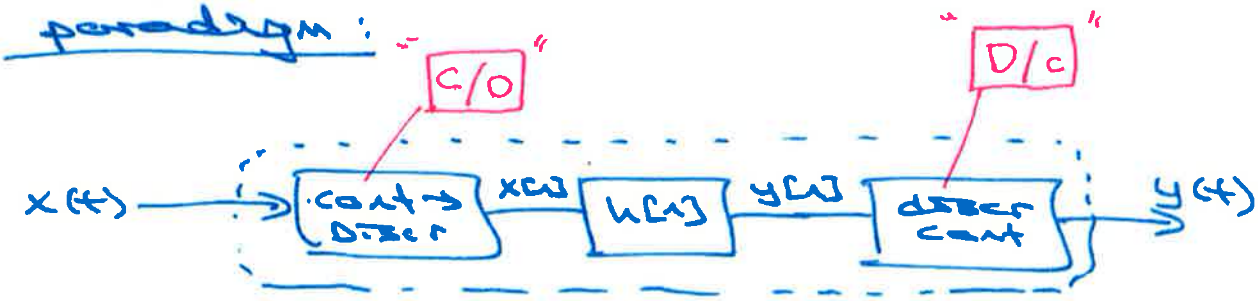
W	F
23	Thx
30	2
7	9
14	

$\boxed{19}$ Final

From Lecture – Recap

Sampling: Fundamental to processing of signals.

Common paradigm:

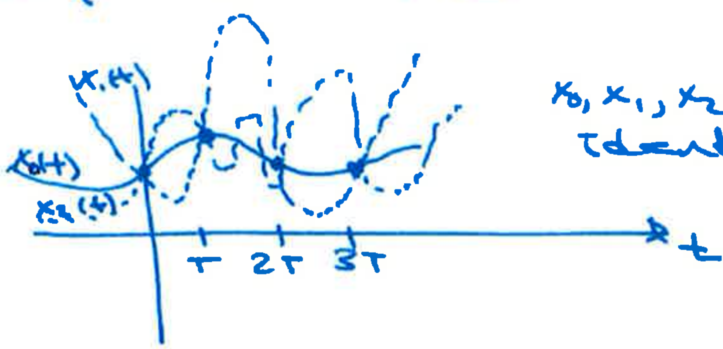


* implements a CT filter w/ DT system

* know about CT, & DT.

* need to understand CT→DT & DT→CT.

What is Adequate Sampling?



x_0, x_1, x_2 all have identical samples @ $t = nT$

Q: Under what conditions can we reconstruct the original CT signal from its samples?

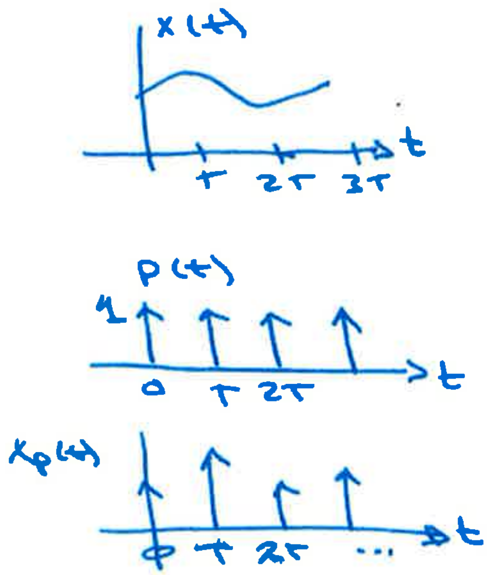
Assume
Impulse sampling:

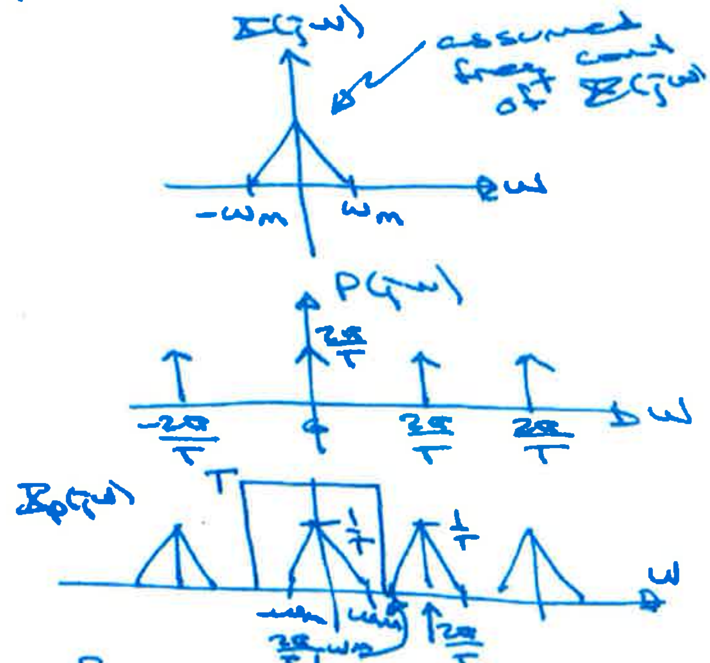
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

then

$$x_p(t) = x(t)\cdot p(t) = \sum_n x(t)\,\delta(t-nT) = \sum_n x(nT)\cdot\delta(t-nT)$$

model: $x(t) \longrightarrow \overset{p(t)\,\downarrow}{\otimes} \longrightarrow x_p(t)$





Analysis in the freq domain

$$P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right) = \frac{2\pi}{T}\sum_k \delta(\omega-\omega_s k)$$

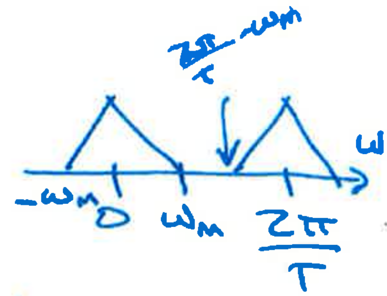
& $$X_p(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * P(j\omega)\right)$$

$$= \frac{1}{2\pi}\cdot\frac{2\pi}{T}\sum_k X(j\omega) * \delta(\omega-\omega_s)$$

$$= \frac{1}{T}\sum_k X(j(\omega-k\omega_s))$$ ↑ see graph

clearly, if

$$\frac{2\pi}{T} - \omega_m > \omega_m$$



i.e. $\underbrace{\frac{2\pi}{T}}_{\text{sampling freq } \omega_s} > \underbrace{2\omega_m}_{\text{2X highest freq in } x(t)}$

then $x(t)$ can be extracted from $x_p(t)$. "reconstructed"

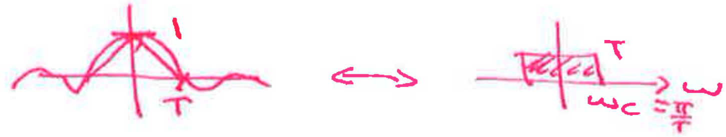
How? Use a LPF @ cutoff freq $\frac{\pi}{T}$ & Gain T. (see graph).

This is the sampling theorem:

If $x(t)$ is bandlimited s.t.
$X(j\omega) = 0$ for $|\omega| > \omega_m$
then $x(t)$ is uniquely determined by its samples, $x(nT)$, if
$\frac{2\pi}{T} = \omega_s > 2\omega_m$ = The "Nyquist" rate

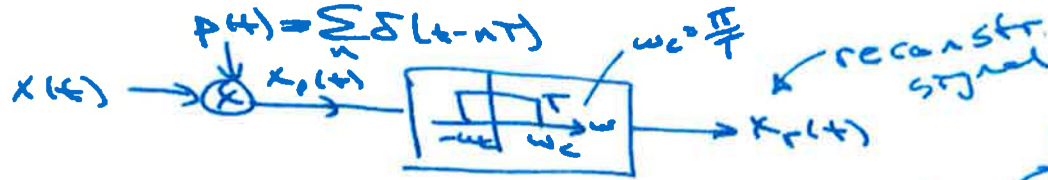
* "sampling freq greater than twice the highest signal freq"

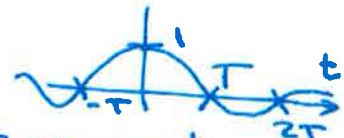
sinusoid: * "Two samples per cycle, at least"



$\frac{\text{area}}{2\pi} = \frac{T \cdot 2\pi/T}{2\pi} = 1$

Time-domain view of reconstr





$$x_r(t) = x_p(t) * h(t), \quad h(t) = \frac{T \sin \omega_c t}{\pi t}, \quad \omega_c = \frac{\pi}{T}$$

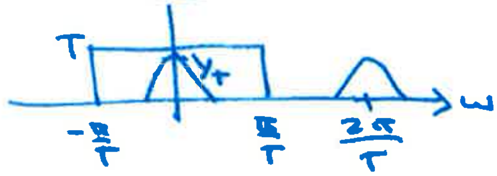
$$= \left(\sum_n x(nT)\, \delta(t - nT) \right) * h(t)$$

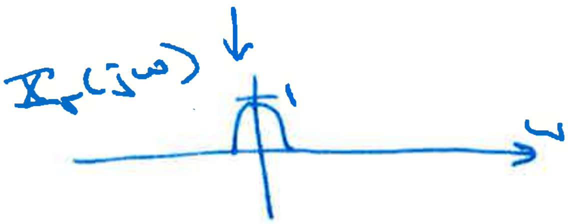
$$= \sum_n x(nT)\, h(t - nT)$$

$$h(t) = \text{sinc}(t/T) = \frac{\sin \pi t/T}{\pi t/T} = \frac{T \sin \omega_c t}{\pi t}$$

$$x_r(t) = \sum_n x(nT) \frac{T \sin(\omega_c (t - nT))}{\pi (t - nT)}$$

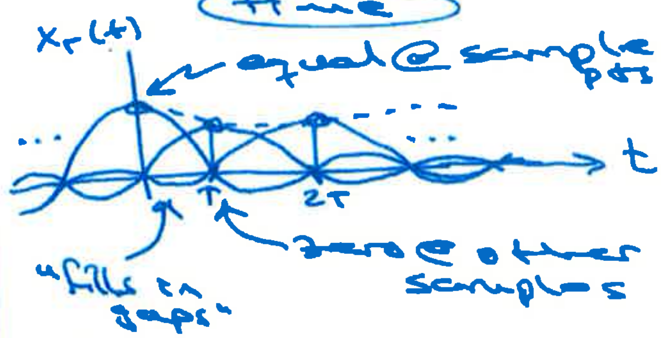
freq





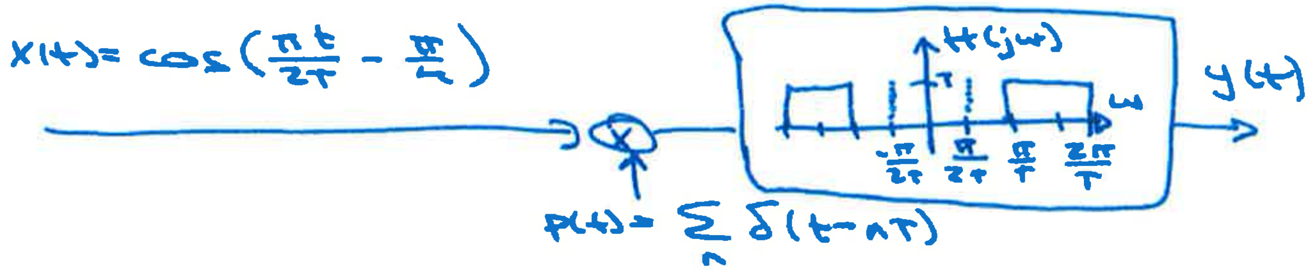
- "Trivial", "obvious" "intuitive", ...

time



- Time-domain interpretation
- Pretty, but not obvious.
- zero @ other samples
- "bandlimited interpolation"
- "sinc interpolation"

Ex: $\omega_0 = \frac{\pi}{2T}$, $\Omega_0 = \frac{\pi}{2}$



$$y(t) = A\cos(\omega_1 t + \phi_1)$$

TRUE ?

$A = 0$ X

$\omega_1 = \frac{\pi}{2T}$ X

$\phi_1 = -\frac{\pi}{4}$ X

$$\boxed{\begin{array}{l}\text{From below} \\ y(t) = \cos\left(\frac{3\pi}{2T} t + \frac{\pi}{4}\right) \\ A = 1 \quad \omega_1 = \frac{3}{2}\frac{\pi}{T} \quad \phi_1 = \frac{\pi}{4}\end{array}}$$

$$\cos\left(\frac{\pi}{2T} t - \frac{\pi}{4}\right) = \frac{1}{2} e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{2T} t} + \frac{1}{2} e^{+j\frac{\pi}{4}} e^{-j\frac{\pi}{2T} t}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t}\, d\omega$$

$$\boxed{e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega - \omega_0)}$$