6.003 Recitation 17.

11-18-11

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Today:

- Rehash problems from quiz.
- Causal Filter design.

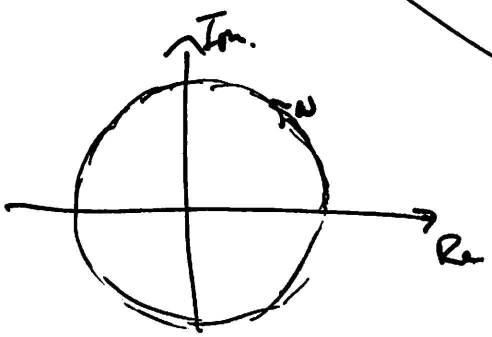
P5 from exam: Echo.

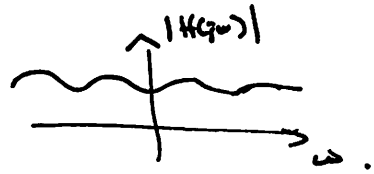
$$h(t) = \delta(t - T_1) + \epsilon\,\delta(t - T_2).$$

$$H(j\omega) = e^{-j\omega T_1} + \epsilon\, e^{-j\omega T_2}.$$

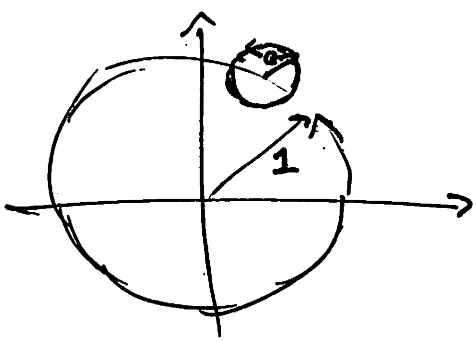
mag = 1.
phase = $-\omega T_1$

mag = ϵ.
phase = $-\omega T_2$





so, why, when they come together, do they have an oscillatory mag response?



amplitude of oscillation = 2ϵ.
& e.g. $-\epsilon$ to $+\epsilon$.

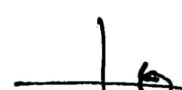
$\Rightarrow \epsilon = .2.$

How fast (in ω) does mag response oscillate?

easier to change coordinates.

$$H(j\omega) = e^{-j\omega T_1} + \epsilon\, e^{-j\omega T_2} = e^{-j\omega T_1}\left(1 + \epsilon\, e^{-j\omega(T_2 - T_1)}\right)$$

rotate into place.



&

do $\angle$ first.

$$|H(j\omega)| = \left|\cancel{e^{-j\omega T_1}}\right|^{\nearrow 1} \cdot \left|1 + \epsilon e^{-j\omega(T_2 - T_1)}\right|$$

$$\epsilon\left(\cos(-\omega(T_2-T_1)) + j\sin(-\omega(T_2-T_1))\right)$$

$$\epsilon\left(\cos(\omega(T_2-T_1)) - j\sin(\omega(T_2-T_1))\right)$$

$$|H(j\omega)| = \sqrt{\left(1+\epsilon\cos(\omega(T_2-T_1))\right)^2 + \left(\epsilon\sin(\omega(T_2-T_1))\right)^2}$$

$$= \sqrt{1 + 2\epsilon\cos(\omega(T_2-T_1)) + \epsilon^2(\cancel{\cos^2 + \sin^2})^{\nearrow 1}}$$

$$= \sqrt{1 + 2\epsilon\cos(\omega(T_2-T_1)) + \epsilon^2}$$

will oscillate in period $\dfrac{2\pi}{T_2 - T_1}$

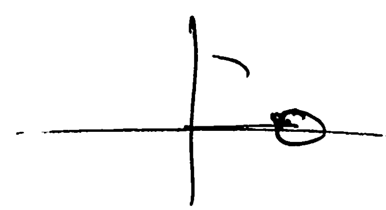
~~period from plot~~ from plot. period = 750. using T_1 below $T_2 = \frac{2\pi}{750} + \frac{\pi}{1500}$

$2\pi = 750\,T_2 - \frac{\pi}{1500}$

$\frac{3001/5}{1500}$

$$\angle H(j\omega) = -\omega T_1 + \angle\left(1 + \epsilon e^{-j\omega(T_2 - T_1)}\right)$$

small compared to first term.



approx $\angle H(j\omega) = -\omega T_1$.

from plot. $\omega = 1500 \rightarrow \angle = -\pi$

$$T_1 = \frac{\pi}{1500}.$$

Prob 4.

$$H(e^{j\Omega}) = \begin{cases} 1 & |\Omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le |\Omega| < \frac{3\pi}{2} \end{cases}$$ periodic in 2π.

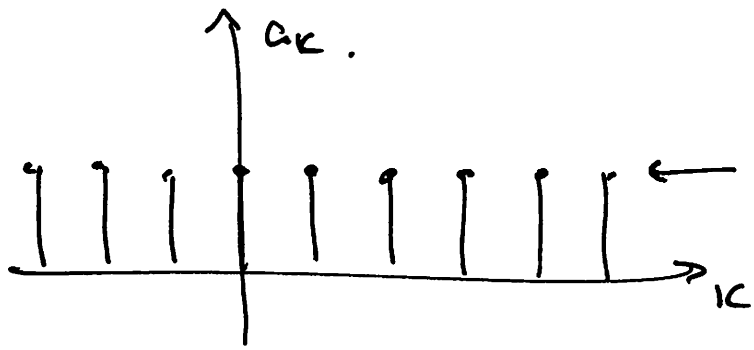
$$x[n] = \begin{cases} 1 & n = 0 \\ 0 & n = 1, 2 \end{cases} \qquad x[n+3] = x[n].$$

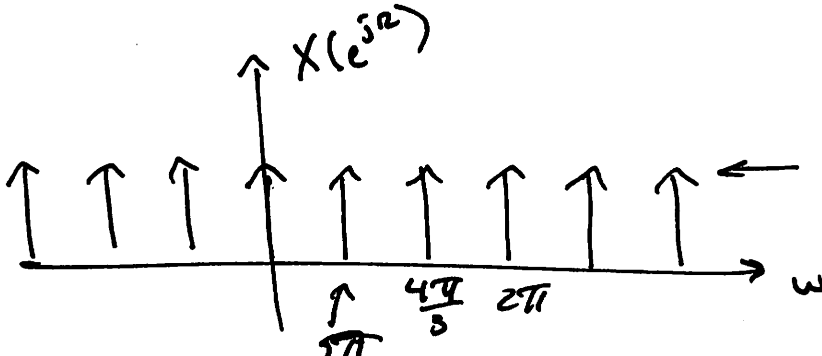
DTFS:

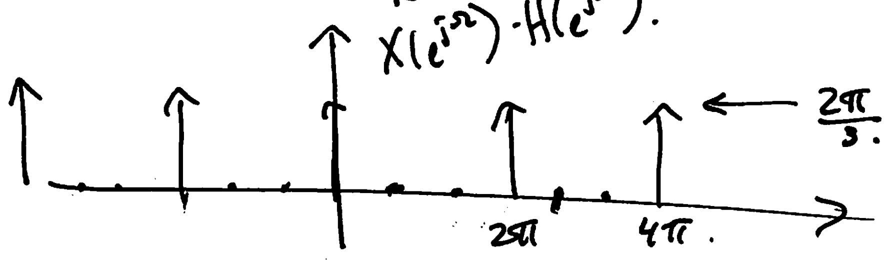
$N = 3$

$$a_k = a_{k+N} = \frac{1}{N} \sum_{n=\langle N\rangle} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{3}.$$

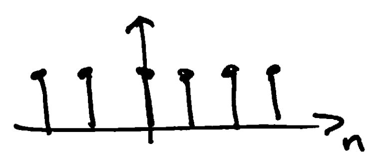






$\frac{2\pi}{3} \to$ opposite of $x[n]$ transform (some form).

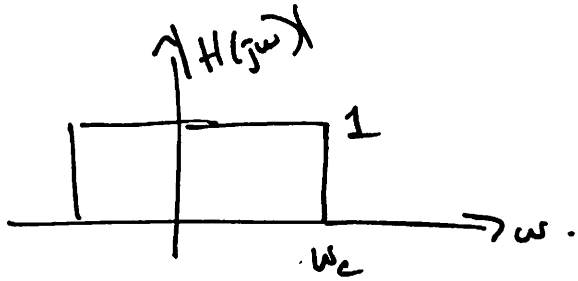
$y[n] = \frac{1}{3}$.



Causal Filter Design. (not on final exam).

Big Idea: use optimization theory to place poles / zeros.

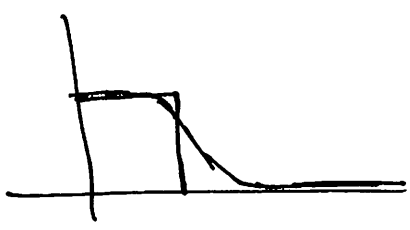
Example. Ideal LPF.



Given N poles, what is "best" causal approximation of ideal LPF?

Define "best".

Idea 1: "maximally flat." in pass & stop bands, but accept slow roll-off.

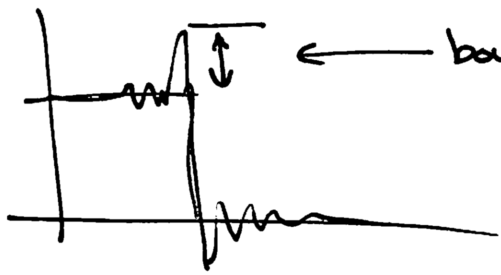


$\Rightarrow$ Butterworth filter

$$H(s) = \frac{w_c^N}{(s-s_1)(s-s_2)\cdots(s-s_N)}$$

Matlab demo of Butterworth w/ $w_c = 10\text{Hz}$. $N = 1, 2, \ldots 7$.

Idea 2: Faster-dropoff, allow some ripple.



bounds on ripple in pass
+ stop bands

⇒ Elliptic filter.

flat pass, ripple in stop ⇒ Chebyshev type I.

ripple in pass, flat stop ⇒ Chebyshev type II.

flat in both → Butterworth !

Matlab demo signal processing for mag field measurements from a powerline. (for perching).

see 60 Hz noise + odd harmonics.

see DC component.

design bandpass for signal at 80 Hz.