6.003 11/9/11

$$\cos \Omega n \longrightarrow \boxed{H(z)} \longrightarrow |H(e^{j\Omega})| \cos(\Omega n + \angle H(e^{j\Omega}))$$

LTI

Amplitude

Phase

Freq Response

Rec

Ann. Exam 3 in 1 wk.

Walker / No recitations

3pm 36-112 review

+ OH Stata

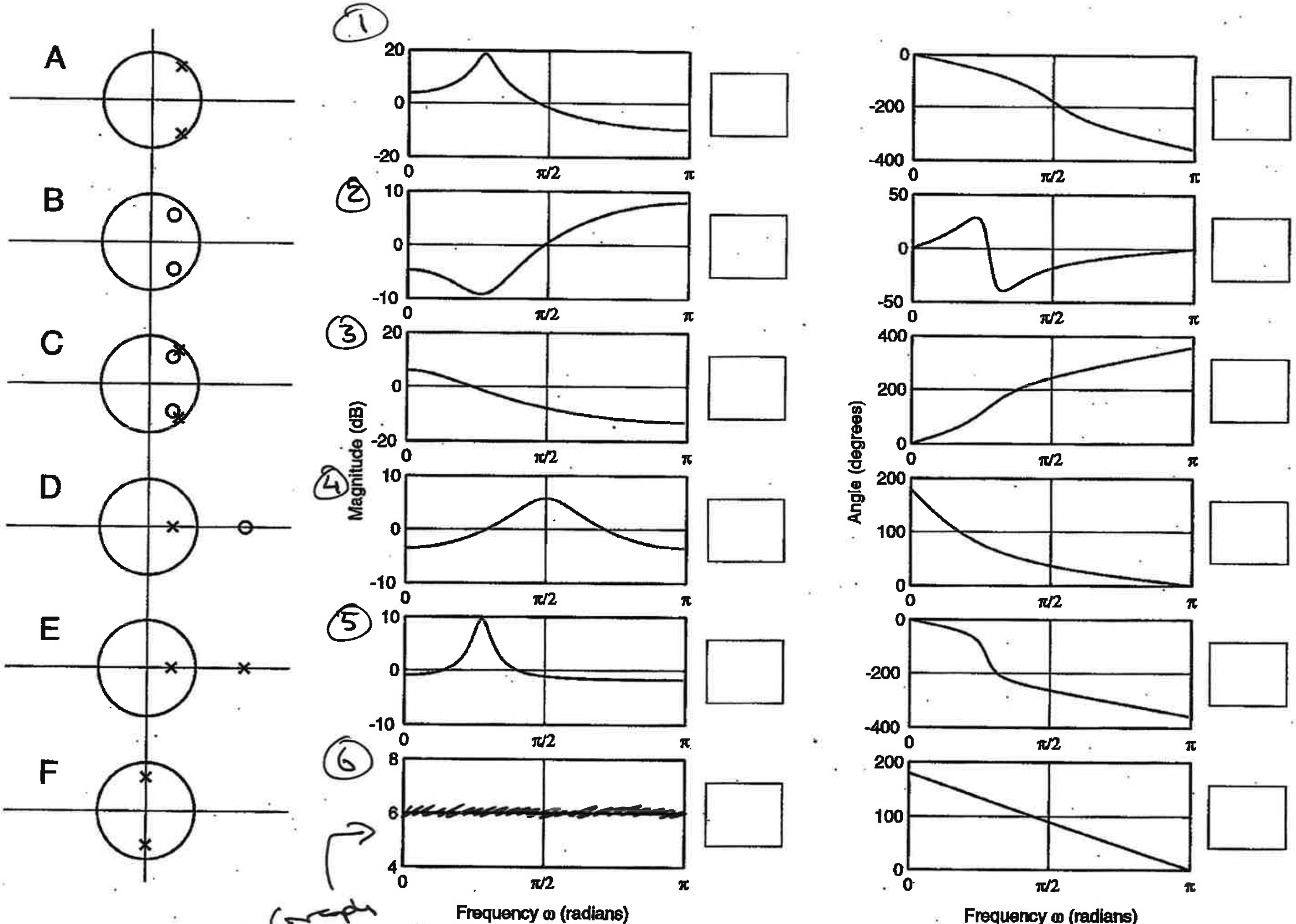
Lecture: * DT freq response

* DT FS

$$x[n] = x[n+N] \xleftrightarrow{DTFS} a_k$$

time

freq



Graph → constant @ 6dB

Given * p/z diagrams,

* $|H(e^{j\Omega})|$, $\measuredangle H(e^{j\Omega})$

Match them up.

Note: D has pole @ $\frac{1}{2}$, zero 2

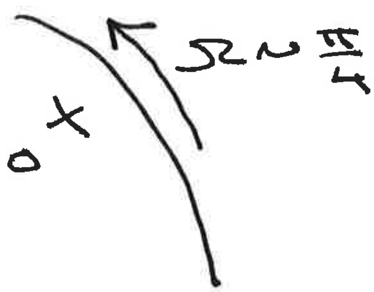
E — — $\frac{1}{2}$, pole 2

<u>Magnitudes:</u> Use graphical evaluation of $|H(e^{j\Omega})|$.

A: peak near $\frac{\pi}{4}$ ∴ either 1, 5

B: notch near $\frac{\pi}{4}$ ∴ 2

C: What happens near $\frac{\pi}{4}$?



- mag from distant vectors ~ same

- mag for x dominates

∴ peak @ $\frac{\pi}{4}$

- Away from $\frac{\pi}{4}$, |zeros| ~ |poles| & response flatter than A

C – 5 ⇐

A – 1

F: Peaked near $\frac{\pi}{2}$ $\therefore$ 4

This leaves D, E vs 3, 6

Can evaluate @ $\Omega = 0, \pi$:

E: $|H(e^{j0})| \propto \frac{1}{\frac{1}{2}\cdot 1} = 2 = 6\,dB$

$|H(e^{j\pi})| \propto \frac{1}{\frac{3}{2}\cdot 3} = \frac{2}{9} < 0\,dB$

D: $|H(e^{j0})| \propto \frac{1}{\frac{1}{2}} = 2 = 6\,dB$

$|H(e^{j\pi})| \propto \frac{3}{3/2} = 2 = 6\,dB$

$\therefore$ D – 6, E – 3

<u>Note</u>: For D,

$$|H_D(e^{j\Omega})|^2 = \frac{(e^{j\Omega}-2)(e^{-j\Omega}-2)}{(e^{j\Omega}-\frac{1}{2})(e^{-j\Omega}-\frac{1}{2})}$$

$$= \frac{1 - 2(e^{j\Omega}+e^{-j\Omega}) + 4}{1 - \frac{1}{2}(e^{j\Omega}+e^{-j\Omega}) + \frac{1}{4}}$$

$|H_D(e^{j\Omega})|^2 = 4$ for all Ω, ... "Allpass"

Phase Matching

A, C; Rapid variation near $\frac{\pi}{4}$

$\therefore$ A, C — 2 vs. 5

For C, poles & zeros cancel at 0 & π $\therefore$ C - 2

A - 5

B: Phase increases from zero at $\Omega = 0$

$\therefore$ B - 3

F: Zero @ $\Omega = 0$, most rapid variation near $\frac{\pi}{2}$

$\therefore$ F - 1

D vs E — 4 vs 6

D: $\Omega = 0 \quad \measuredangle H_D = \pi - 0 = \pi$

$\Omega = \pi \quad \measuredangle H_D = \pi - \pi = 0$

E: $\Omega = 0 \quad \measuredangle H_E = 0 - \pi = -\pi$

$\Omega = \pi \quad \measuredangle H_E = 0 - 2\pi = -2\pi$

$\Big\}$ $+ n \cdot 2\pi$ $\therefore \pi \to 0$

Need additional info

E.g. For small Ω, $\measuredangle D$ changes faster than $\measuredangle E$ since both pole & zero angles contribute to a decrease in $\measuredangle D$.

For $\measuredangle E$, competing effects.

$\therefore$ D — 4

E — 6

Recall: CTFS

$$x_1(t) = x_1(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

periodic

time

↑ all periodic exp. (∞)

Freq index

↑ time

Analog: DTFS

"time"

$$x_2[n] = x_2[n+N] = \sum_{k=\langle N\rangle} b_k e^{j\frac{2\pi}{N}kn}$$

↑ consecutive N indices

only N periodic exp

x synthesized from linear comb of $\underbrace{\text{complex exp.}}_{\text{basis funcs.}}$

Also recall from CTFS, calc of weights:

$$a_k = \frac{1}{T}\int_T x_1(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

interval, T

time

Freq

Freq

Analog: DTFS:

$$b_k = \frac{1}{N}\sum_{n=\langle N\rangle} x_2[n]\, e^{-j\frac{2\pi}{N}k\cdot n}$$

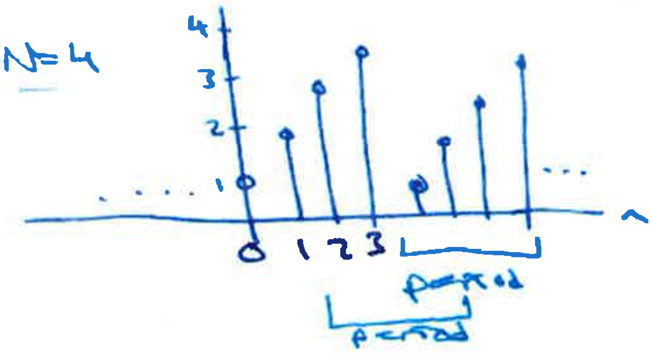
time

N indices.

Analysis: How much does basis k contrib to x.

Ex: Given $x[n]$, $N=4$, calc a_k:

N=4



$\xleftrightarrow{\text{DTFS}}$ $a_k = ?$

$$a_k = \frac{1}{4} \underbrace{\sum_{n=\langle 4\rangle}}_{\text{any 4 consec.}} x[n]\, e^{-j\frac{2\pi}{4}kn}$$

$$= \frac{1}{4}x[0] + \frac{1}{4}x[1]e^{-j\frac{\pi}{2}k} + \frac{1}{4}x[2]e^{-j\pi k} + \frac{1}{4}x[3]e^{-j\frac{3\pi}{2}k}$$

$$a_k = \frac{1}{4} + \frac{1}{2}(-j)^k + \frac{3}{4}(-1)^k + (j)^k$$

k=0: $a_0 = \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 = \frac{10}{4}$

k=1: $a_1 = \frac{1}{4} + \frac{1}{2}(-j) + \frac{3}{4}(-1) + j = -\frac{1}{2} + \frac{1}{2}j$

k=2: $a_2 = \frac{1}{4} + \frac{1}{2}(-1) + \frac{3}{4}\cdot 1 + (-1) = -\frac{1}{2}$

k=3: $a_3 = \frac{1}{4} + \frac{1}{2}\cdot j + \frac{3}{4}(-1) + (-j) = -\frac{1}{2} - \frac{1}{2}j$

i.e.

$$\ldots \overbrace{1,2,3,4}^{\cancel{\text{period}}\ 4=N}, 1,2,3,4 \ldots \xleftrightarrow{\text{DTFS}} \ldots, \overbrace{\frac{10}{4}, -\frac{1}{2}+\frac{1}{2}j, -\frac{1}{2}, -\frac{1}{2}-\frac{1}{2}j}^{\cancel{\text{period}}\ 4=N}$$

N values $x[n]$ $\longleftrightarrow$ N values (complex) a_k

Note conj. symm.

* Two domains
* Transform Pair
*