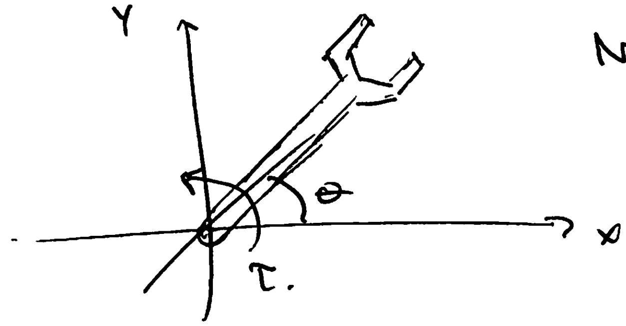
Recitation 11. 10-19-11

CT Feedback + Bode Plots. Russ Tedrake

Consider controlling a simple robot arm.
(in the horizontal plane).



Note: Use robot arm prop in class.

Equations of motion are.

$$I\ddot{\theta} + b\dot{\theta} = \tau.$$

Inertia. (e.g. ml^2) always >0.

damping (e.g from friction) always ≥ 0.

motor torque (motor current ~~∝~~ K · motor torque)

want to think of this as a system.

$$\tau \longrightarrow \boxed{P(s)} \longrightarrow \theta.$$

Q: Is it stable?

$$P(s) = \frac{1}{Is^2 + bs}$$

A: No: pole at zero.

physical demo: (I give a push, doesn't return to zero).

Goal: design a controller to control θ given θ_d. (desired θ)

$$\theta_d \to \boxed{\quad ? \quad} \to \boxed{P(s)} \to \theta.$$

Philosophical aside:

Why not do:

$$\theta_d \to \underbrace{\boxed{\boxed{H_d(s)} \to \boxed{\frac{1}{P(s)}}}}_{\text{e.g. cancel dynamics of the plant, then replace.}} \to \boxed{P(s)} \to$$

which should be equivalent to.

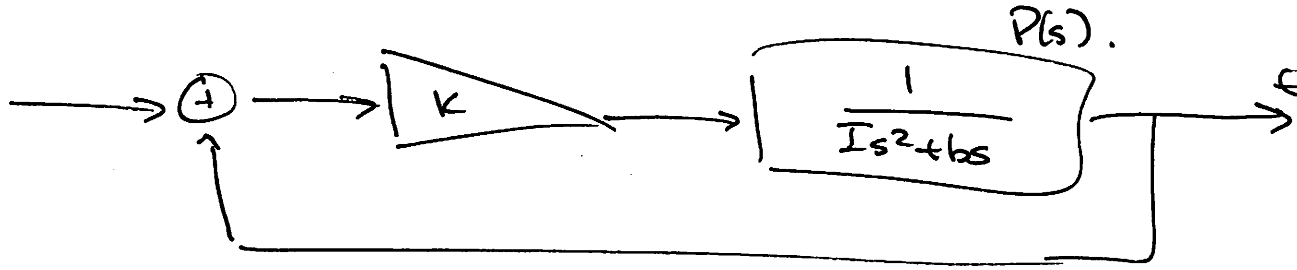
$$\theta_d \to \boxed{H_d(s)} \to \theta.$$

What if my model was a little wrong?

Some delay I didn't consider?

With feedback, relatively simpler controllers can have relatively better performance (e.g. in terms of robustness).

Idea One: Proportional Control.

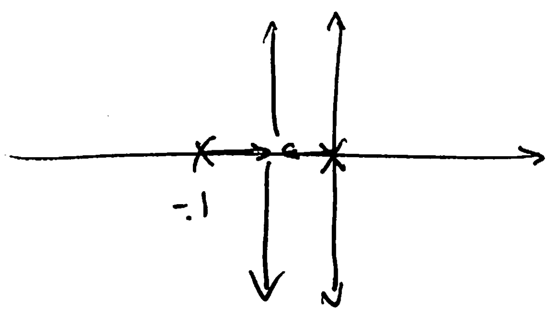


$$H(s) = \frac{\frac{K}{Is^2+bs}}{1+\frac{K}{Is^2+bs}} = \frac{K}{Is^2+bs+K}.$$

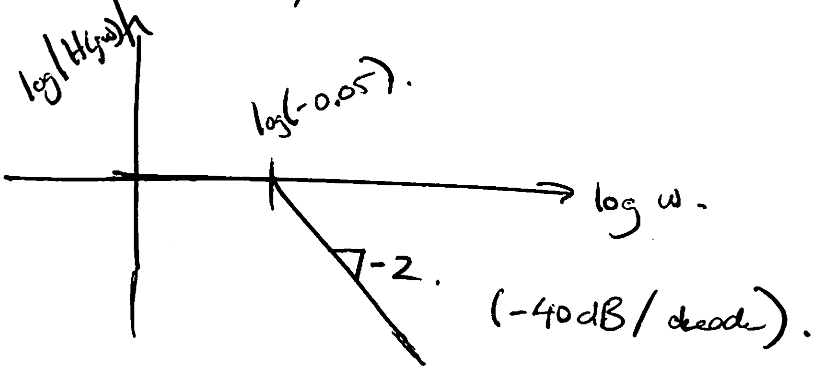
Assume $m=1$, $l=1$, $b=\frac{1}{10}$, $g=10$.

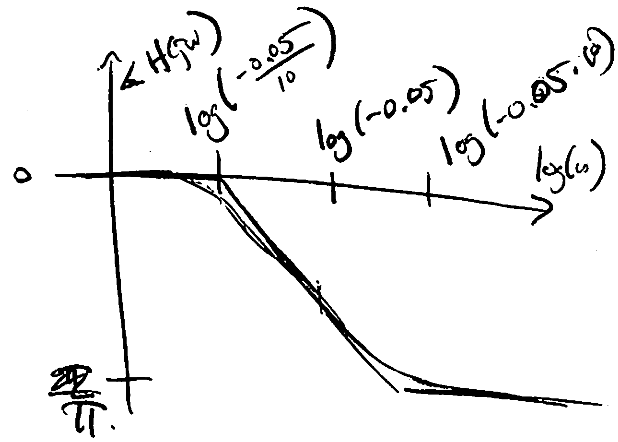
$\Rightarrow I=1$, $b=\frac{1}{10}$.

Root locus.



Bode (say $p_1 = p_2 = -.05$).



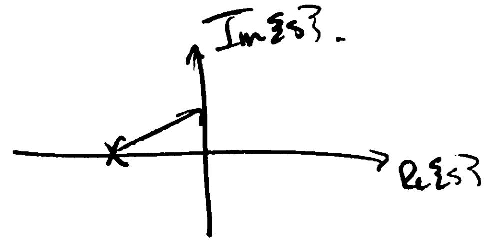


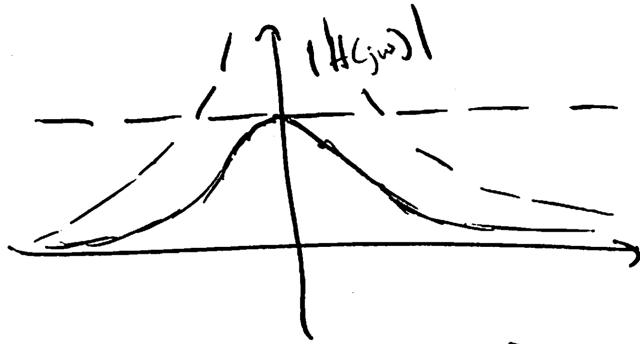
Main Problem: Slow response.

Q: Intuitively, how can we improve it? (A: Add a lead). aka. derivative.

Bode approximations: (Recall from lecture).

Single pole: $H(s) = \frac{a}{s - p_1}$



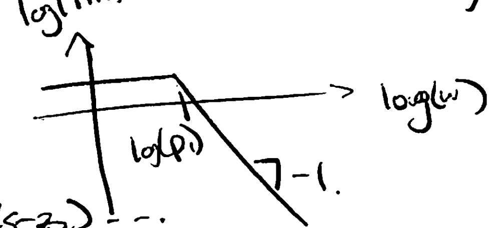


$\frac{a}{|p_1|}$ (when $|s| \ll |p_1|$).

$\frac{a}{|\omega|}$ (when $|s| \gg |p_1|$).

where do they cross? $\omega = p_1$.

$\Rightarrow$



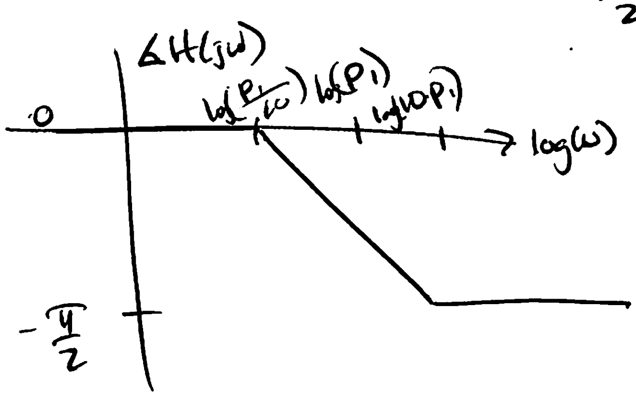
$$H(s) = K \frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots}$$

because $\log|H(j\omega)| = \log|K| + \sum \log|s - z_i| - \sum \log|s - p_i|$

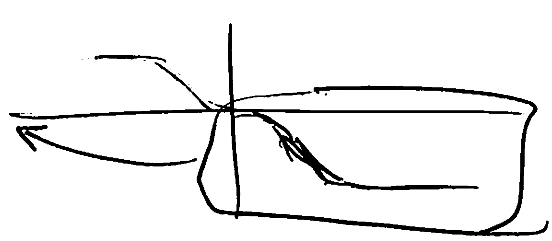
for phase. (don't need log, $\angle$ already sums).

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{p_1}\right)$$

$$\approx \begin{cases} 0 & \omega \ll |p_1| \quad \text{use } \omega \le \frac{|p_1|}{10} \\ \text{linear} & \\ -\frac{\pi}{2} & \omega \gg |p_1|. \quad \text{use } \omega \ge 10|p_1|. \end{cases}$$



Note: this is just for positive ω.



Idea 2: Proportional Derivative Control. (PD).

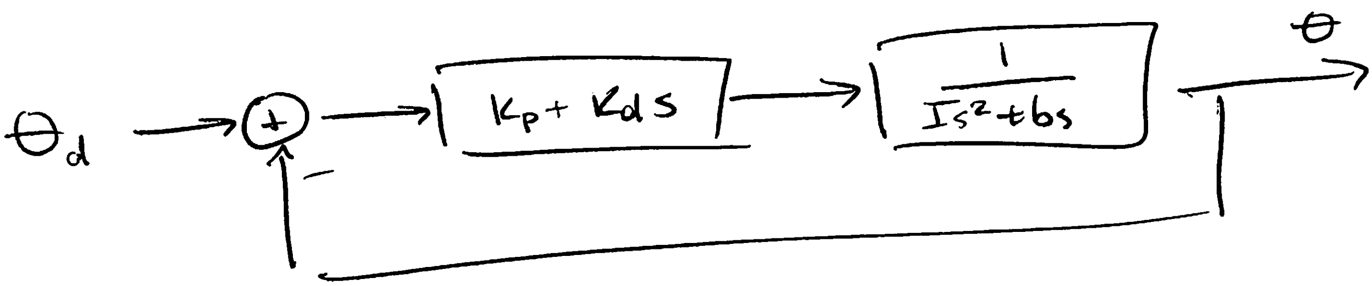
Proportional control was limited (rightmost pole could only be $-\frac{b}{2I}$.)
+ large phase at high freq.

Big idea: for faster response.
need to add a zero.

Derivative is my way to look into the future.

$$\tau = \underbrace{K_p(\theta_d - \theta)}_{\text{P control.}} + K_d(\dot{\theta}_d - \dot{\theta})$$

" look into the future ".

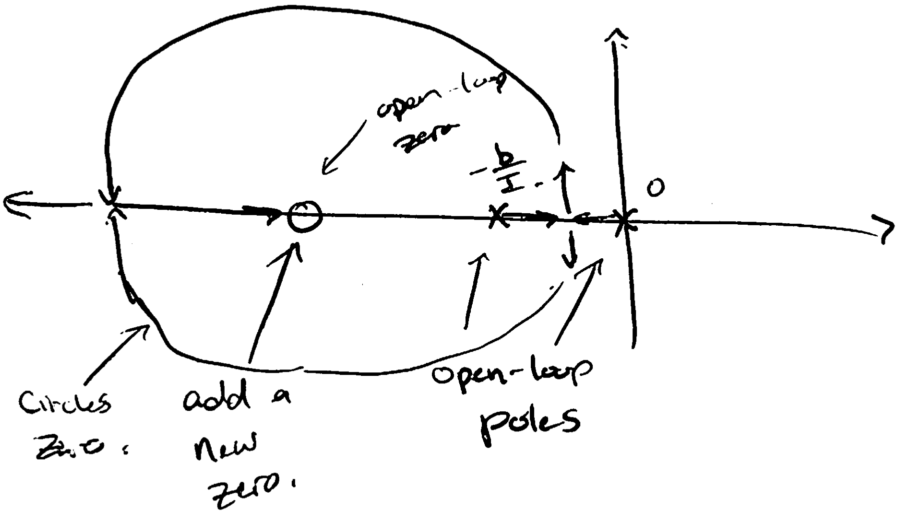


My claim: should get faster response.

$$H(s) = \frac{\frac{K_p + K_d s}{Is^2 + bs}}{1 + \frac{K_p + K_d s}{Is^2 + bs}} = \frac{K_p + K_d s}{Is^2 + (b + K_d)s + K_p}.$$

PD root locus:

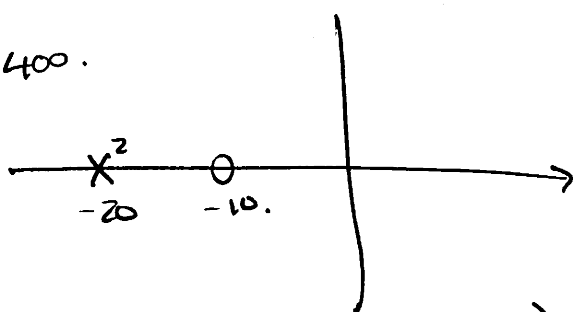
set $K_d = K_p/10$. (only vary one parameter).



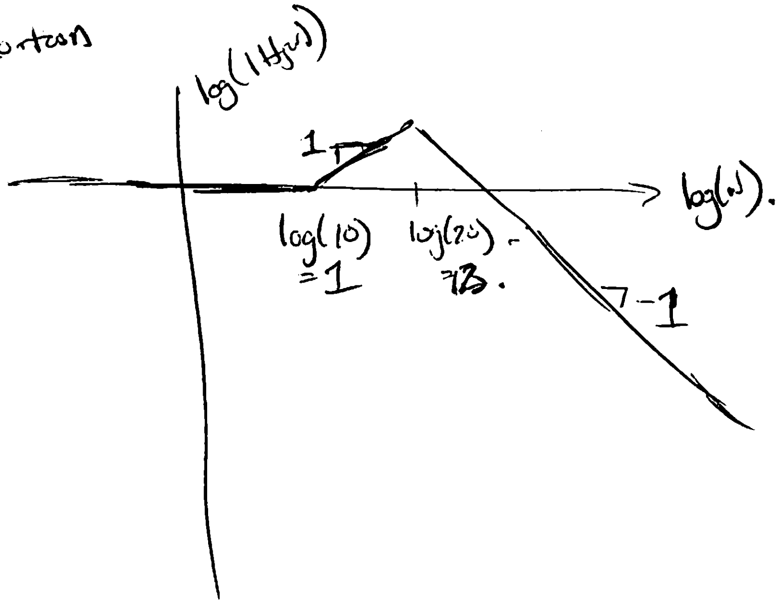
response can be much faster. (rightmost pole more negative).

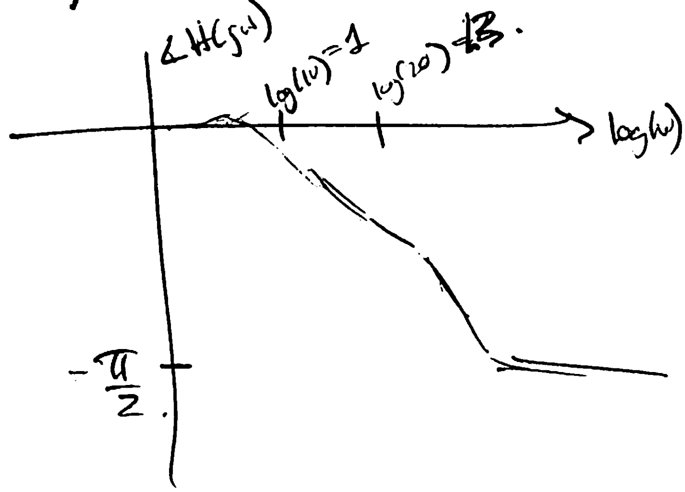
Bode: (say for K_p such that I have ~~both poles~~ best response).

$K_p \approx 400$.



cartoon





Steady-state ~~error~~ error. (for unit-step input).

Final value theorem.

$$\lim_{t\to\infty} h(t) = \lim_{s\to 0} s\,H(s).$$

For step response

$$\lim_{s\to 0} s\,H(s)\,\frac{1}{s} = \lim_{s\to 0} H(s).$$

often just $H(0)$.

Proportional control. steady-state:

$$H(s) = \frac{K}{Is^2+bs+K}. \qquad H(0) = \frac{K}{K} = 1. \ \checkmark.$$

error = 0.

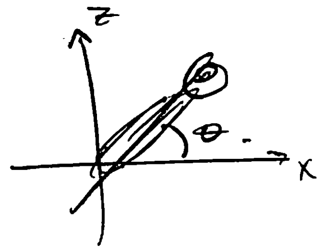
PD control steady-state:

$$H(s) = \frac{K_p + K_d s}{Is^2 + (b+K_d)s + K_p}. \qquad H(0) = \frac{K_p}{K_p} = 1 \ \checkmark.$$

error = 0.

Robot in the vertical plane. (add gravity).

$$I\ddot{\theta} + b\dot{\theta} + mgl\cos\theta = \tau.$$

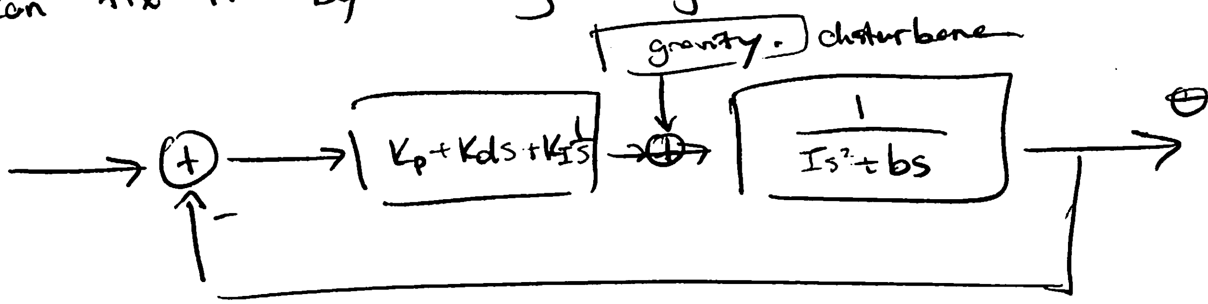


around $\theta = 0$.

$$mgl\cos\theta \approx mgl.$$

will add steady-state error.

Can fix it by adding integral control.



PID control.

integral term will fix steady-state errors, but makes things slower. (pole at zero).

add derivative to speed back up.

Well-tuned P, I, and D gains (K_p, K_d, K_I) can work very well for complex systems.