6.003

9/23/2011

Rec 5

Ann:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

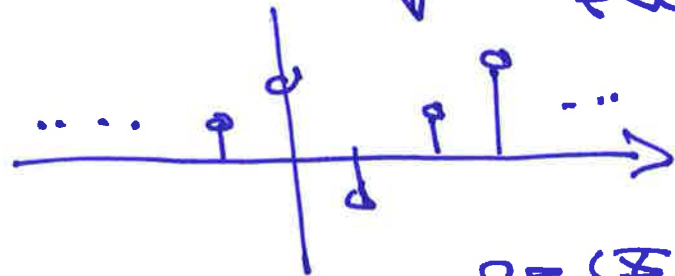
Recap: * Z-transform

* Relations among DT system represent

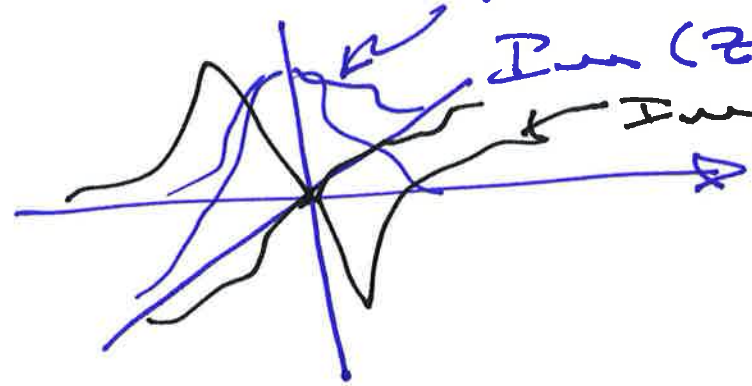
Today: Z-transform

sketch this often

$x[n]$



$X(z)$

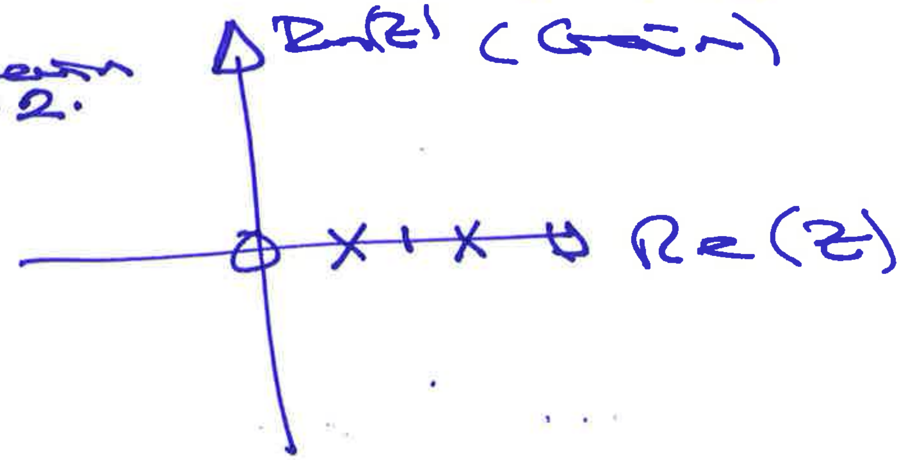


Clearly, this is a mess

... don't sketch $X(z)$!

Rather: sketch param that capture prop of $X(z)$: poles
zeros

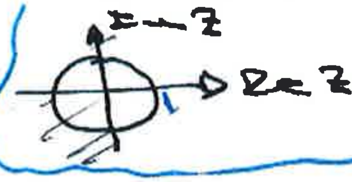
K gain $= 2$.



$$X(z) = \frac{z - z_1}{(z-p_1)(z-p_2)} \cdot K$$

Ex: $\delta[n] = h[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = z^{0} = 1 \quad \forall z$$



Ex: $h[n] = u[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} u[n] z^{-n} = \sum_{n=0}^{\infty} z^{-n}$$

$$= \sum_{n=0}^{\infty} (z^{-1})^{n}$$

$$H(z) = \frac{1}{1-z^{-1}} \quad \text{if } |z^{-1}| < 1 \text{; i.e. } |z| > 1$$

Ex: $h[n] = a^{n} u[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} a^{n} u[n] z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^{n}$$

$$= \frac{1}{1 - a z^{-1}} \text{; } |a z^{-1}| < 1 \text{, i.e. } |z| > |a|$$

Ex: $-a^n u[-n-1] = x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} -a^n u[-n-1] z^{-n} = -\sum_{\ell} a^{-\ell-1} z^{\ell+1}$$

$\ell = -n-1 \therefore n = -\ell-1$

$$= -\sum_{n=-1}^{-\infty} a^n z^{-n}$$

$\ell = -n-1 \therefore n = -\ell-1$

$$= -\sum_{\ell=0}^{\infty} a^{-\ell-1} z^{\ell+1}$$

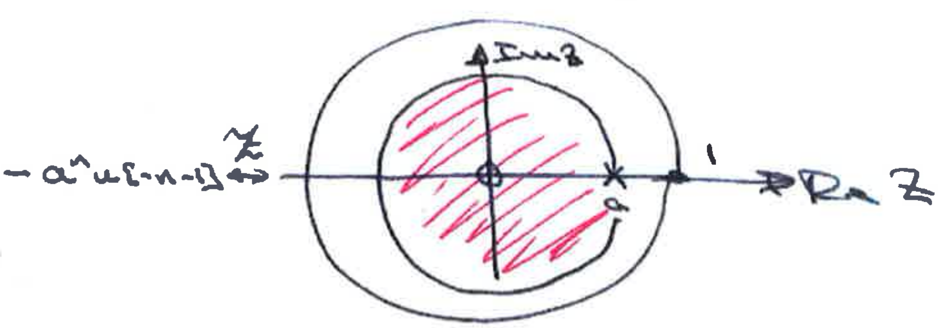
$$= -\frac{z}{a} \sum_{\ell=0}^{\infty} \left(\frac{z}{a}\right)^{\ell}$$

$$X(z) = -\frac{z}{a} \frac{1}{1 - z/a} \qquad \left|\frac{z}{a}\right| < 1, \quad |z| < |a|$$

can rewrite as

$$X(z) = \frac{-z}{a-z} = \underbrace{\frac{z}{z-a}}_{P/Z} = \underbrace{\frac{1}{1-az^{-1}}}_{\text{typical table}} \qquad |z|<|a|$$

Note same z-transform algebraic form as before, diff ROC.

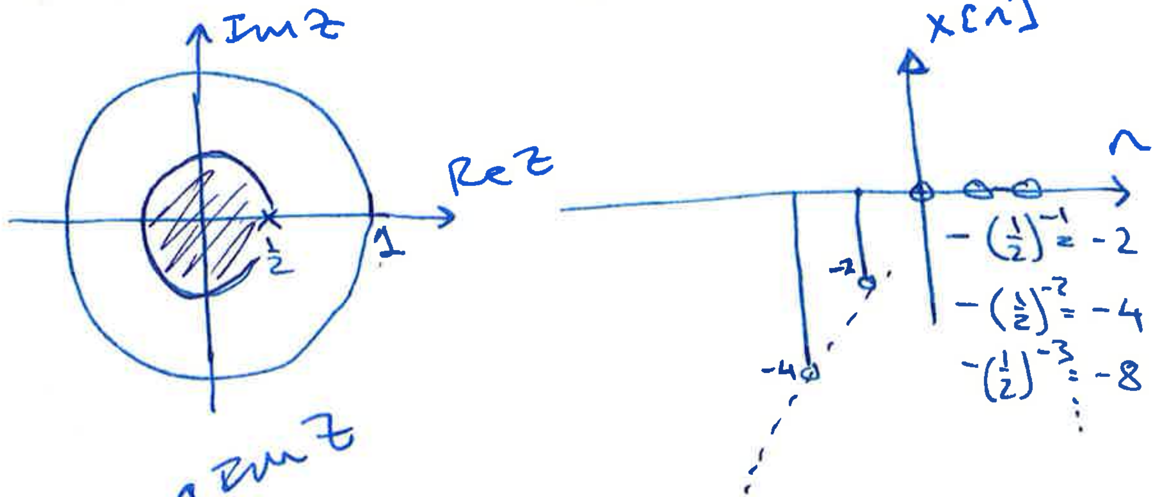


Left-sided

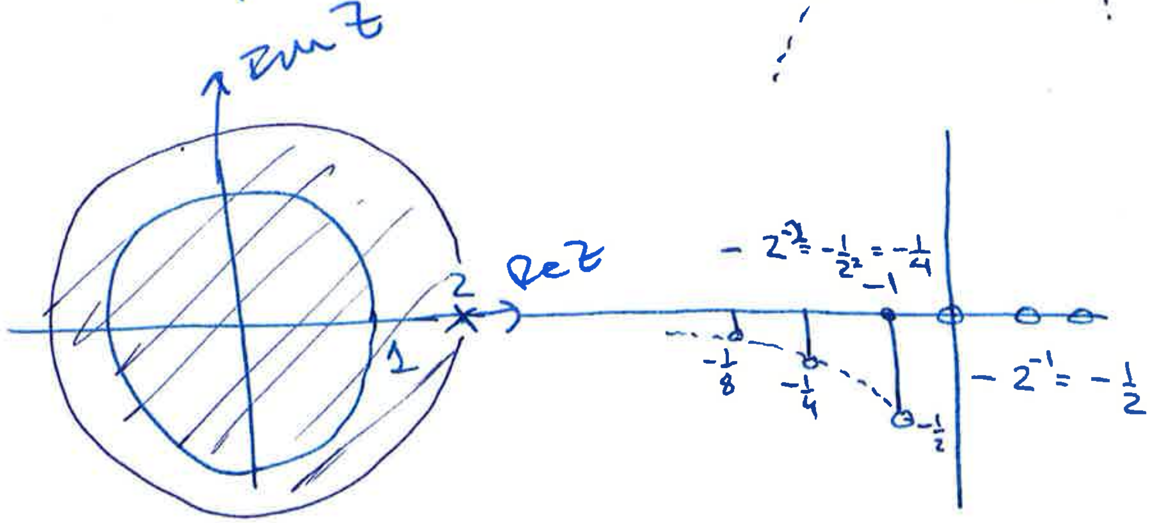
Note: Seq's for various a-values:

$$x[n] = -a^n u[-n-1]$$

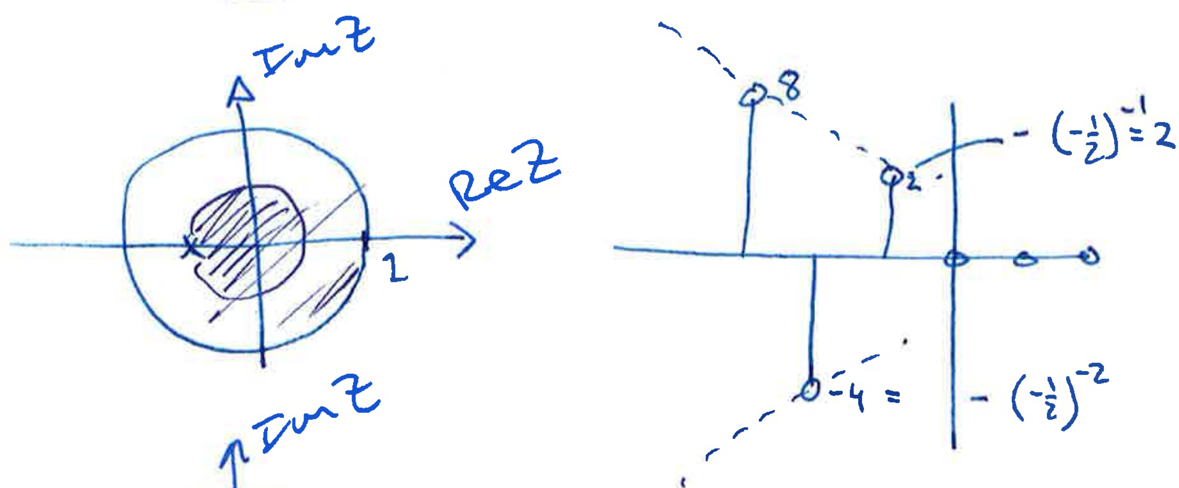
$a = \frac{1}{2}$



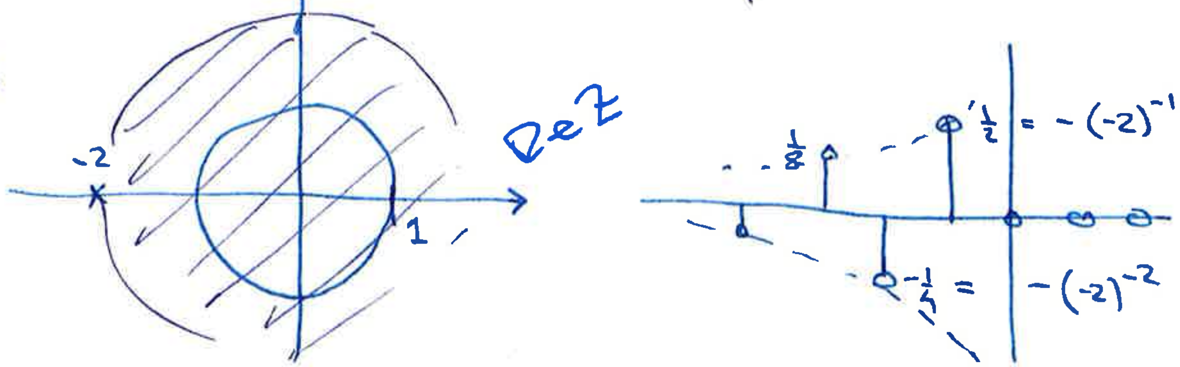
$a = 2$



$a = -\frac{1}{2}$



$a = -2$:



Ex: $H(z) = \dfrac{4z^3 + 2z^2 + z}{z - \frac{1}{2}}$; $|z| > \frac{1}{2}$

Find $h[n]$.

i) PFE: in discrete time, tables work w/ $(z^{-1})^n$

$$\frac{z^{-1}}{z^{-1}} H(z) = \frac{4z^2 + 2z + 1}{1 - \frac{1}{2} z^{-1}}$$

Need long-division to get degree of numerator down:

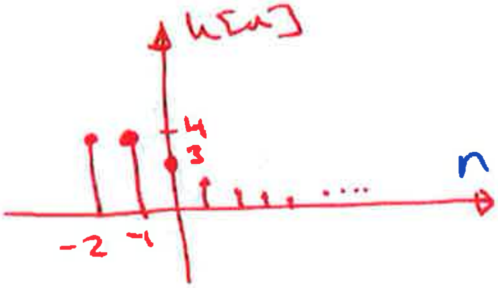
$$\begin{array}{r|l} & 4z^2 + 4z \\ \hline 1 - \frac{1}{2}z^{-1} & 4z^2 + 2z + 1 \\ & 4z^2 - 2z \\ \hline & \quad 4z + 1 \\ & \quad 4z - 2 \\ \hline & \qquad 3 \end{array}$$

$\left(H(z) = \sum h(n) z^{-n} \right)$

i.e.

$$H(z) = 4z^2 + 4z + \frac{3}{1 - \frac{1}{2} z^{-1}}$$

$$h[n] = 4\delta[n+2] + 4\delta[n+1] + 3\left(\tfrac{1}{2}\right)^n u[n]$$



unit sample resp nonzero @ $n<0$!

Note "Non-causal" due to z^2, z terms

Note: Rational, "outside" $H(z)$:
$\deg(N(z)) \le \deg(\text{den}) \Rightarrow$ causal
$\deg(N) > \deg(\text{den}) \Rightarrow$ non causal

or

(ii)

$$\frac{4z^3 + 2z^2 + z}{z - \frac{1}{2}} = \frac{4z^2 + 2z + 1}{1 - \frac{1}{2}z^{-1}}$$

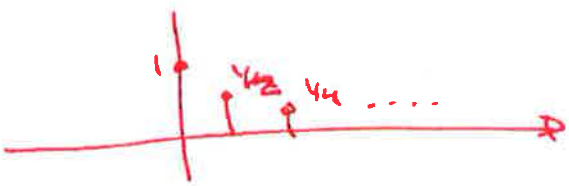
$$= \frac{4z^2}{1-\frac{1}{2}z^{-1}} + \frac{2z}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$$

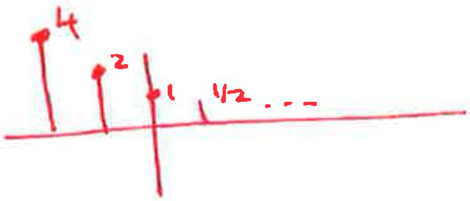
$\downarrow$ $4\cdot\left(\frac{1}{2}\right)^{n+2} u[n+2]$ (a)

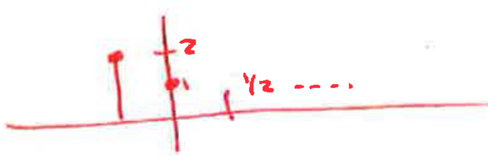
$\downarrow$ $2\cdot\left(\frac{1}{2}\right)^{n+1} u[n+1]$ (b)

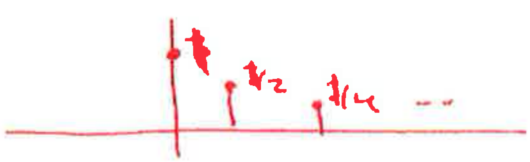
$\downarrow$ $\left(\frac{1}{2}\right)^{n} u[n]$ (c)

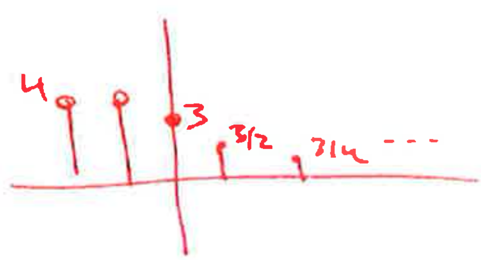
Recall Delay:
$x[n] \leftrightarrow X(z)$
...
$x[n-1] \leftrightarrow z^{-1}X(z)$
generally
$x[n-k] \leftrightarrow z^{-k}X(z)$



(a)


(b)


(c)


$\Sigma =$


OK, same answer!