6.003 Recitation 3 9.14-11

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Today: Discrete Time Systems.

Let's start w/ an example:

Ex: Fibonacci sequence.

0 1 1 2 3 5 8 13 21 ...

Write this as a difference equation:

$$y[n] = y[n-1] + y[n-2].$$

w/ initial conditions

$$y[0] = 0.$$

$$y[1] = 1.$$

Before drawing block diagram, can we think of it as the response of a system w/ zero initial cond?

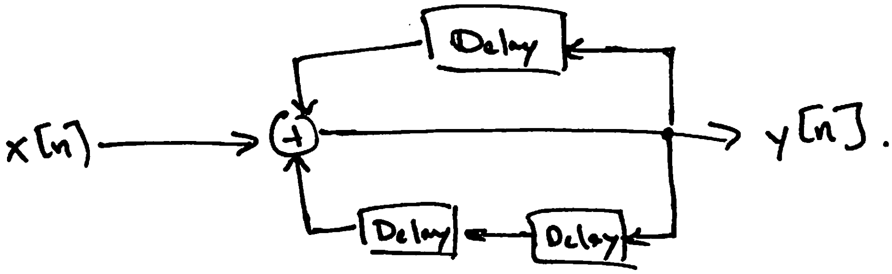
$$y[n] = y[n-1] + y[n-2] + x[n].$$

~~y[0] = x[1]~~ $y[n] = 0 \quad n < 0.$

$$x[n] = ?$$

$$x[n] = \delta[n-1].$$

Now draw the block diagram.



Now write it as a polynomial.

$y[n] \Rightarrow Y.$

$y[n-1] \Rightarrow RY.$

$y[n-2] \Rightarrow R^2 Y.$

$$Y = RY + R^2 Y + X.$$

$$(1 - R - R^2)\, Y = X.$$

$$\frac{Y}{X} = \frac{1}{(1-R-R^2)}.$$ ← gives unit sample response.

Can we expand that out?

$$
\begin{array}{l}
\quad\quad\quad\quad 1 + R + 2R^2 + 3R^3 + 5R^4 + 8R^5 + \ldots \\
1-R-R^2 \,\overline{)\, 1} \\
\quad\quad\quad\quad 1 - R - R^2 \\
\quad\quad\quad\quad \overline{\quad\quad R + R^2} \\
\quad\quad\quad\quad\quad\quad R - R^2 - R^3 \\
\quad\quad\quad\quad \overline{\quad\quad\quad\quad 2R^2 + R^3} \\
\quad\quad\quad\quad\quad\quad\quad\quad 2R^2 - 2R^3 - 2R^4. \\
\quad\quad\quad\quad \overline{\quad\quad\quad\quad\quad\quad 3R^3 + 2R^4.} \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 3R^3 - 3R^4 - 3R^5 \\
\quad\quad\quad\quad \overline{\quad\quad\quad\quad\quad\quad\quad\quad 5R^4 + 3R^5.}
\end{array}
$$

↑ Is this the Fibonacci seq? (No! shifted by 1) as expected.

Aside: Polynomial division.

Ex $$\frac{3x^3 - 2x^2 + 4x - 3}{x^2 + 3x + 3}.$$

particularly useful when deg denom ≤ deg num

Long-division

$$\begin{array}{r l} & 3x - 11 \\ x^2+3x+3 \,\big)\!\!\! & 3x^3 - 2x^2 + 4x - 3. \\ & 3x^3 + 9x^2 + 9x \\ \hline & -11x^2 - 5x - 3 \\ & -11x^2 - 33x - 33 \\ \hline & 28x + 30. \end{array}$$

← often stop here.

$$\Rightarrow \quad \frac{3x^3 - 2x^2 + 4x - 3}{x^2 + 3x + 3} = 3x - 11 + \frac{28x + 30}{x^2 + 3x + 3}.$$

You try:

$$\frac{x^3 - 3x^2 + 5x - 3}{x - 1}.$$

Answer: $= x^2 - 2x + 3$.

Feedback.

Absolutely essential concept for control design.

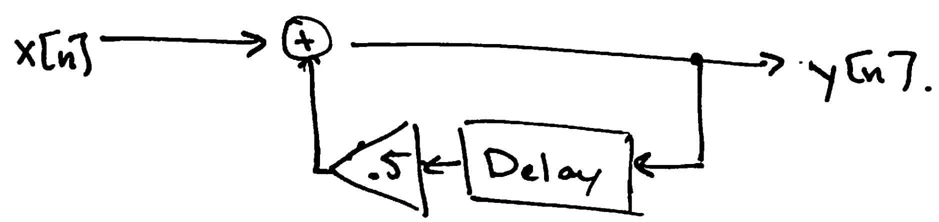
Fibonacci example had feedback.

it lead to an exponential growth. ~~from~~ in unit sample response.

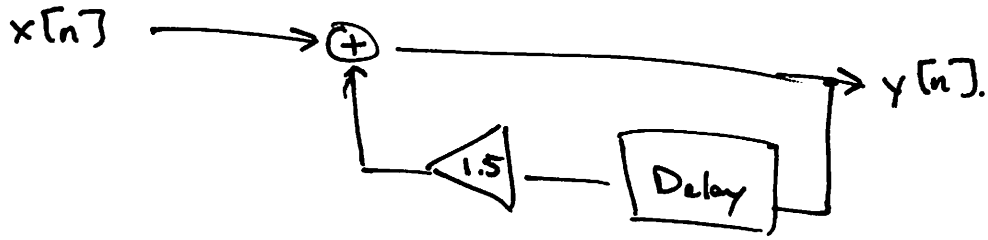
~~Often~~ Feedback often, but not always

Q: ~~How~~ Which of the following systems have divergent unit sample responses? (starting from rest).

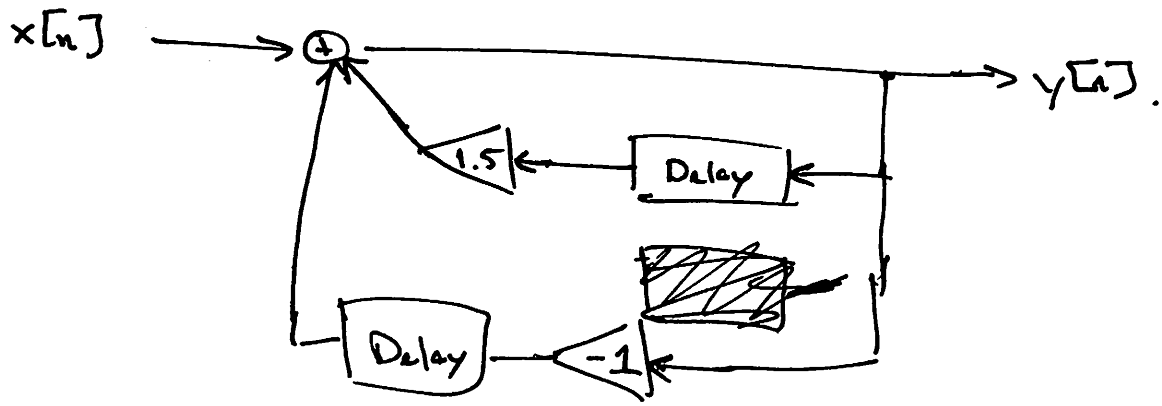
a)



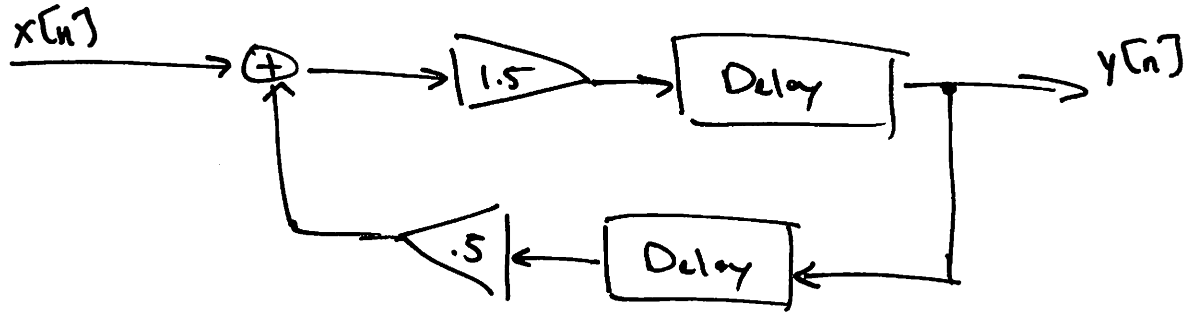
b).



c)



d)



A: a) No.

$y[0] = 1.$

$y[1] = .5$

$.25$

$.125.$

Can always do it by stepping through time.

b) Yes.

$y[0] = 1$

$y[1] = 1.5$

$= \frac{9}{4} = 2.25.$

$= \vdots$

Could also have seen it by writing polynomial.

a) $Y = X + .5RY.$

$$\frac{Y}{X} = \frac{1}{1-.5R}.$$

$$\frac{1}{1-p.R} = 1 + pR + p^2R^2 + p^3R^3 + \dots$$

will converge if $|p| < 1$

p is called a "pole"

b) $\frac{Y}{X} = \frac{1}{1-1.5R}$ diverge.

c) Won't diverge:

It's equivalent to system a!

> * One interpretation: We stabilized an unstable system.

$$Y = X + 1.5RY - 1RY = X + .5RY.$$

won't diverge (check by stepping through)

d) $Y = 1.5R(X + .5RY)$.

$$\left(1 - \frac{3}{4}R^2\right)Y = \frac{3}{2}RX.$$

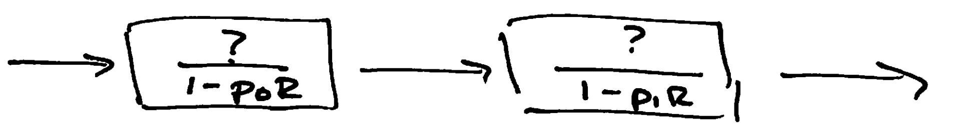
← it's the Y terms that matter for convergence. (X dies out for unit sample)

can factor this into.

$$\left(1 - \frac{\sqrt{3}}{2}R\right)\left(1 + \frac{\sqrt{3}}{2}R\right)Y = 1.5RX.$$

$|p_0| < 1$ $\qquad$ $|p_1| < 1$.

we'll see in lecture tomorrow that this can be written as a cascade of two systems w/ ~~p0 or~~ poles p_0 and p_1



Very powerful idea.