6.003 ~17 9/7/2011

Rec #1

Ann: HW #1 Due Wed 14th
Exam #1 Wed Oct 5th

Today: - "6.003 is ..."
- Communications system
- Common transformations
- Even / Odd
- Complex: $e^{j\theta} = \cos\theta + j\sin\theta$

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HW: Due Wed

OH: Open OH @ Stata
M, T, pm

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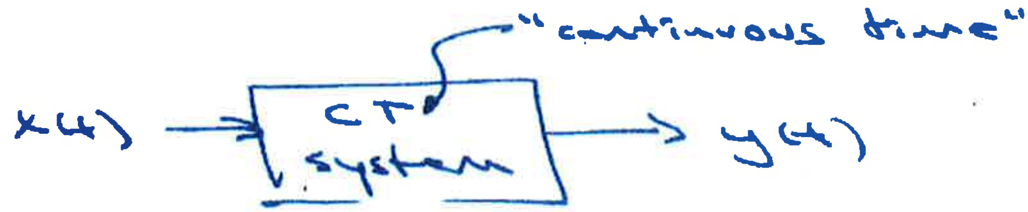
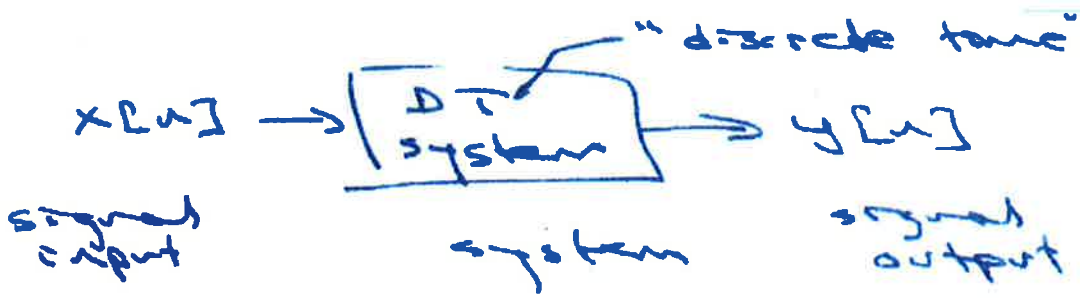
The 6.003 abstraction:

Describes a physical system by the way it transforms $\underbrace{\text{inputs}}_{\text{signal}}$ into $\underbrace{\text{outputs}}_{\text{signal}}$ system



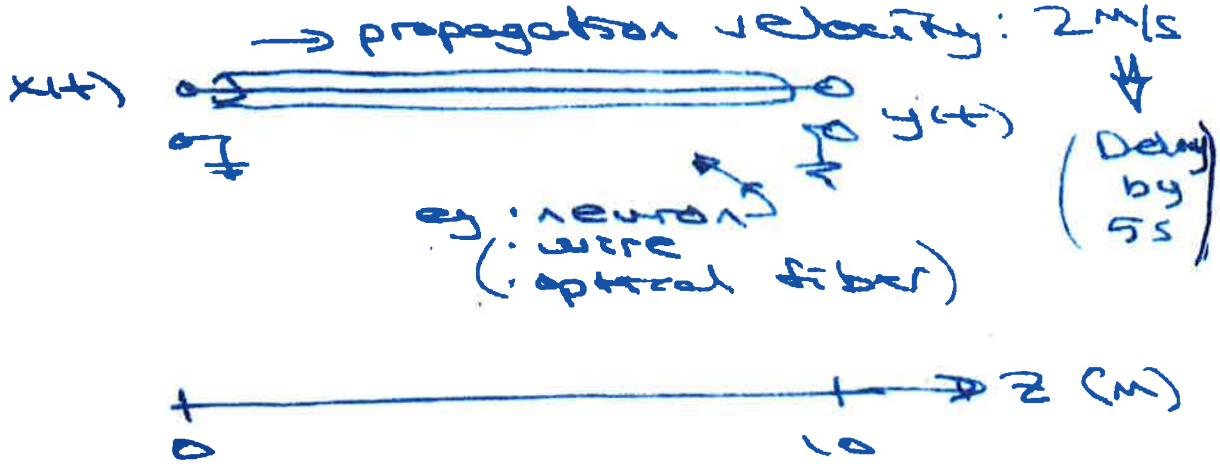


– Others: · hybrids $(x(t) \rightarrow y[n])$
· multi-inp / multi-outp.
· feedback
⋮

– Abstr is Powerful, because
- many different physical systems fit same description
- methods of analysis exist (i.e. 6.003)

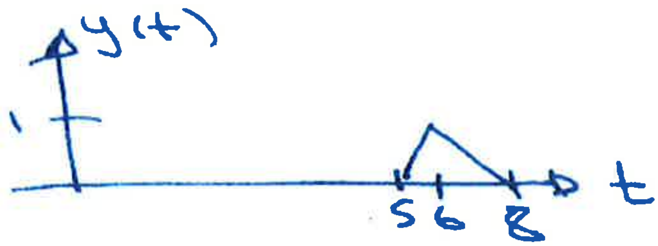
PHYSICAL DESCR

Ex: Communications System



Given input x(t):

Q: Find y(t): Delay of x(t) by $\frac{10\,m}{2\,m/s} = 5\,s$



ABSTRACTION

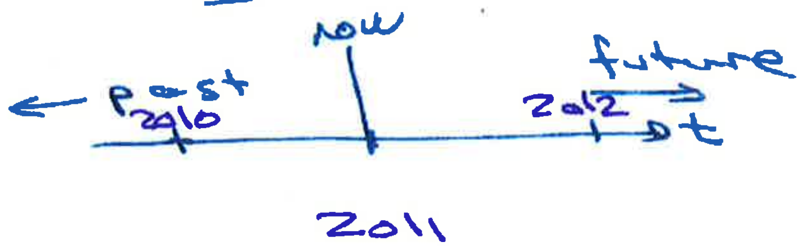
one system description:

x(t) → "neuron" → $y(t) = x(t-5)$

Here, we describe system (neuron) by what it does to signals (shift in time) or delay

Note:

- Delay is simple & common transformation
- the "−" corresponds to a delay



Other recurring transformations

Amplitude scaling — Transforms indep. var

$$x(t) \rightarrow \boxed{} \rightarrow y(t) = A\,x(\alpha t + \beta)$$

modifies time scaling or time reversal

modif. shift

Note could also write as

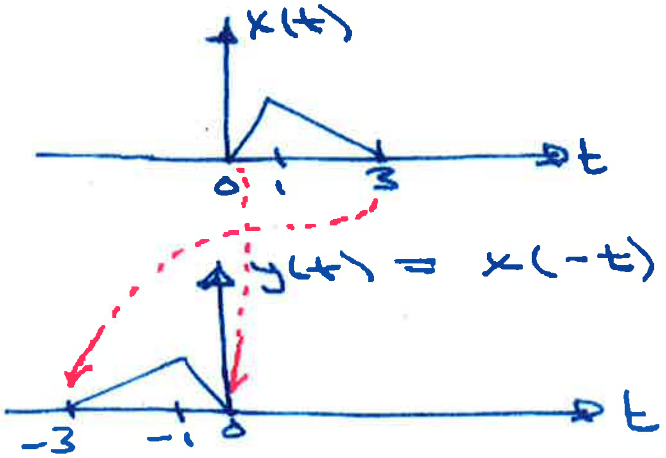
$$y(t) = A\,x\left(\frac{t + \beta/\alpha}{1/\alpha}\right)$$

shift

time scale or time reversal

Ex: Time reversal, $A=1, \alpha=-1, \beta=0$

$y(t) = x(-t)$



"Fiducials"

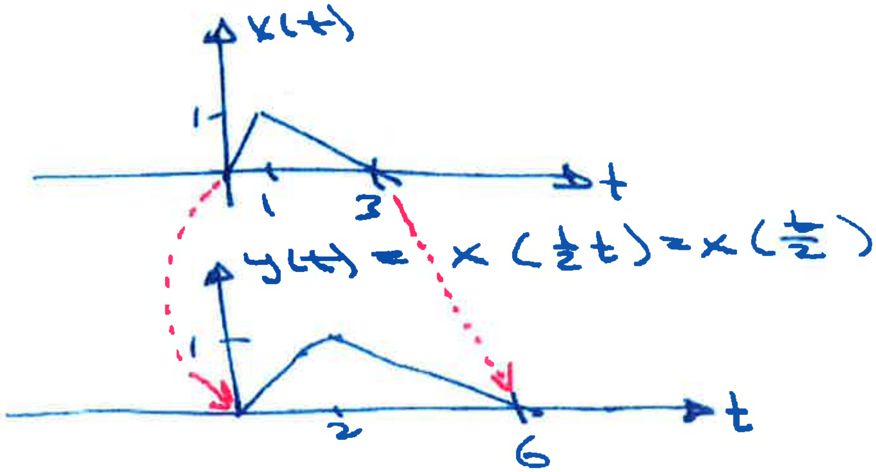
t	$x(-t)$
-3	$x(3)$
-1	$x(1)$
0	$x(0)$
1	$x(-1)$

Ex: Time Scaling, $A=1$, $\alpha = 1/2$, $\beta = 0$

$y(t) = x(\frac{1}{2} \cdot t)$

"where does x(3) map in y(t)?"

$\frac{1}{2}t = 3 \therefore t = 6$



t	$x(\frac{1}{2}t)$
0	$x(0)$
2	$x(1)$
6	$x(3)$

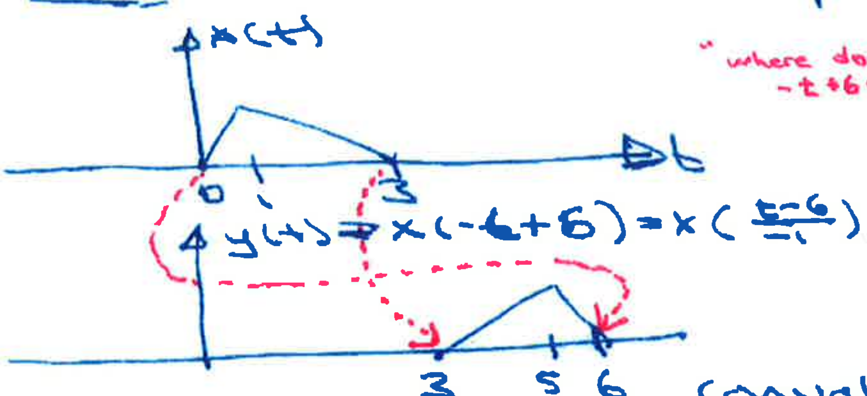
Ex: "Shift & Flip", $A=1$, $\alpha=-1$, $\beta=6$

$\alpha=-1 \quad \beta=6$

$y(t) = x(-t+6)$

"where does x(3) map?"

$-t+6=3 \therefore t=3$

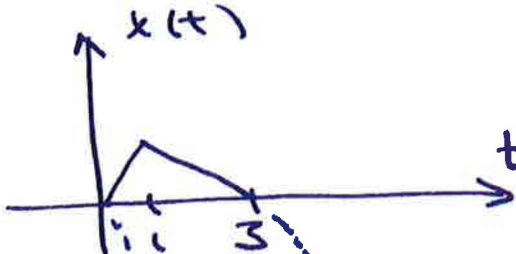


t	$y(t)$
6	$x(0)$
5	$x(1)$
3	$x(3)$

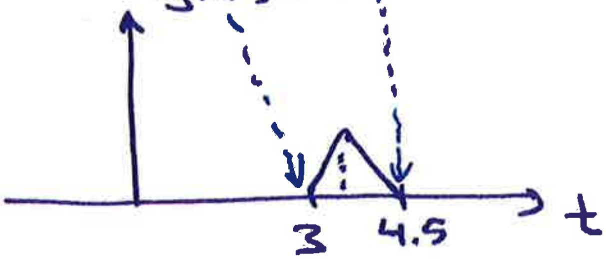
convolution
Lot of this later

Ex: Shift & Scale

$A = 1$
$\alpha = 2$
$\beta = -6$



$$y(t) = x(2t-6) = x\left(\frac{t-3}{1/2}\right)$$

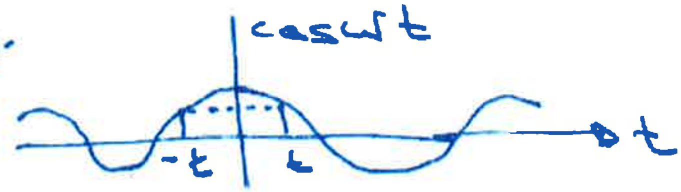


t	$x(2t-6)$
3	$x(0)$
3.5	$x(1)$
4.5	$x(3)$

Even & Odd Signals

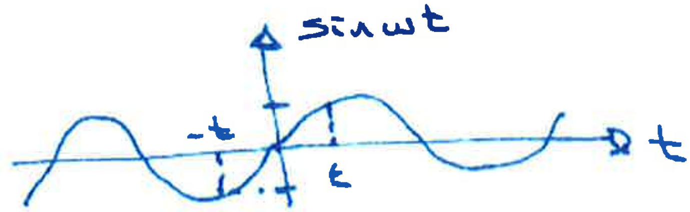
- Define $x(t)$ <u>even</u> if $x(t) = x(-t)$
- symmetric function around y-axis

e.g.



- Define $x(t)$ <u>odd</u> if $x(t) = -x(-t)$
- symmetric w.r.t. origin

e.g.



Any signal can be written as a sum of even & odd signals

Even part:

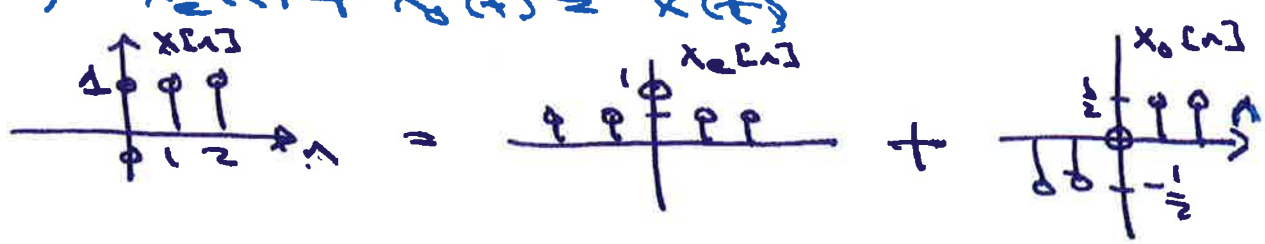
$$x_e(t) = \frac{x(t)+x(-t)}{2}, \quad x_e(t) \text{ is even}$$

odd part

$$x_o(t) = \frac{x(t)-x(-t)}{2}, \quad x_o(t) \text{ is odd}$$

Note, $x_e(t) + x_o(t) = x(t)$

<u>Ex:</u>



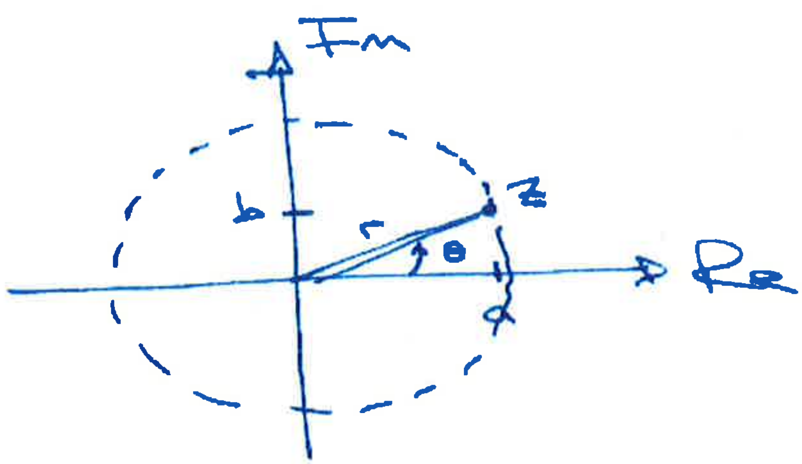
Complex Arithmetic: 6.003 uses $j = \sqrt{-1}$

Standard form

- $z = a + jb$, $a \in \mathbb{R}$, $b \in \mathbb{R}$
- $\mathrm{Re}\{z\} = a$, $\mathrm{Im}\{z\} = b$
- $z^* = a - jb$

Polar form

- $z = re^{j\theta}$ magnitude phase
- $|z| = r = \sqrt{a^2 + b^2}$
- $\measuredangle z = \theta = \tan^{-1}\left(\frac{b}{a}\right)$ (check quadrant!)



Euler's relation unifies exponential & trigonometric functions

$$\underbrace{e^{j\theta}}_{\text{often simple to manipulate.}} = \underbrace{\cos\theta + j\sin\theta}_{\text{often laborous}}$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

Ex: $\cos[(\omega_1+\omega_2)t] = ?$ sum of products of $\sin\omega_i t$ & $\cos\omega_i t$.

i) By memory, lookup table, etc:

$$\cos[(\omega_1+\omega_2)t] = \cos\omega_1 t \cos\omega_2 t - \sin\omega_1 t \cdot \sin\omega_2 t$$

ii) By complex numbers:

$$\cos[(\omega_1+\omega_2)t] = \mathrm{Re}\{e^{j(\omega_1+\omega_2)t}\}$$

$$= \mathrm{Re}\{e^{j\omega_1 t} e^{j\omega_2 t}\}$$

$$= \mathrm{Re}\{(\cos(\omega_1 t) + j\sin\omega_1 t)\cdot(\cos(\omega_2 t) + j\sin\omega_2 t)\}$$

$$= \cos\omega_1 t \cos\omega_2 t - \sin\omega_1 t \cdot \sin\omega_2 t$$

6.003 uses <u>lots</u> of sinusoids,

For instance,

$$A\cos(\omega t+\phi) = \mathrm{Re}\{C\cdot e^{j\omega t}\}$$

with $|C| = A$, $\measuredangle C = \phi$

often harder

often easier

$\mathrm{Re}\{|c|e^{j\phi}e^{j\omega t}\}$

$= A\,\mathrm{Re}\{e^{j(\omega t+\phi)}\}$

Be good at both

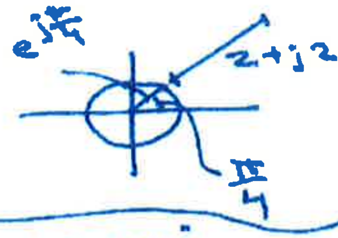
<u>Ex:</u> "How many of the following are true" HMOTFAT

1. $\dfrac{1}{\cos\theta + j\sin\theta} = \cos\theta - j\sin\theta$
2. $(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin(n\theta)$
3. $|2+2j+e^{j\frac{\pi}{4}}| = |2+2j| + |e^{j\frac{\pi}{4}}|$
4. $\mathrm{Im}\{(j)^j\} > \mathrm{Re}\{(j)^j\}$
5. $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1} 1.$

Ex — cont

① $(e^{j\theta})^{-1} = e^{-j\theta} = \cos(-\theta) + j\sin(-\theta)$
$= \cos\theta - j\sin\theta$ ✓

② $(e^{j\theta})^{n} = e^{j\cdot n\theta} = \cos n\theta + j\sin n\theta$ ✓

③ 

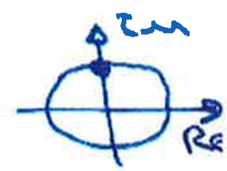
same angle of $e^{j\frac{\pi}{4}}$ & $2+j\cdot 2$. ✓

$\boxed{④ \ (e^{j\frac{\pi}{2}})^{j} = e^{-\frac{\pi}{2}} \ \text{real number} \ \times}$

⑤ $\angle(2+j) + \angle(3+j)$ ✓

$\angle\{(2+j)(3+j)\}$

$\angle\{5+5j\} = \angle(1+j)$

$(j)^{j} = \left(e^{j\frac{\pi}{2}+j2\pi m}\right)^{j}$, m integer 

$= e^{-\frac{\pi}{2}} e^{-2\pi m}$, all real, all > 0