6.003 (Fall 2011)

Quiz #3 *November 16, 2011*

Name:
Kerberos Username:

Please circle your section number:

Section	*Time*
2	11 am
3	1 pm
4	2 pm

Grades will be determined by the correctness of your answers (explanations are not required).

Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use three 8.5×11 sheets of paper (six sides total).

No calculators, computers, cell phones, music players, or other aids.

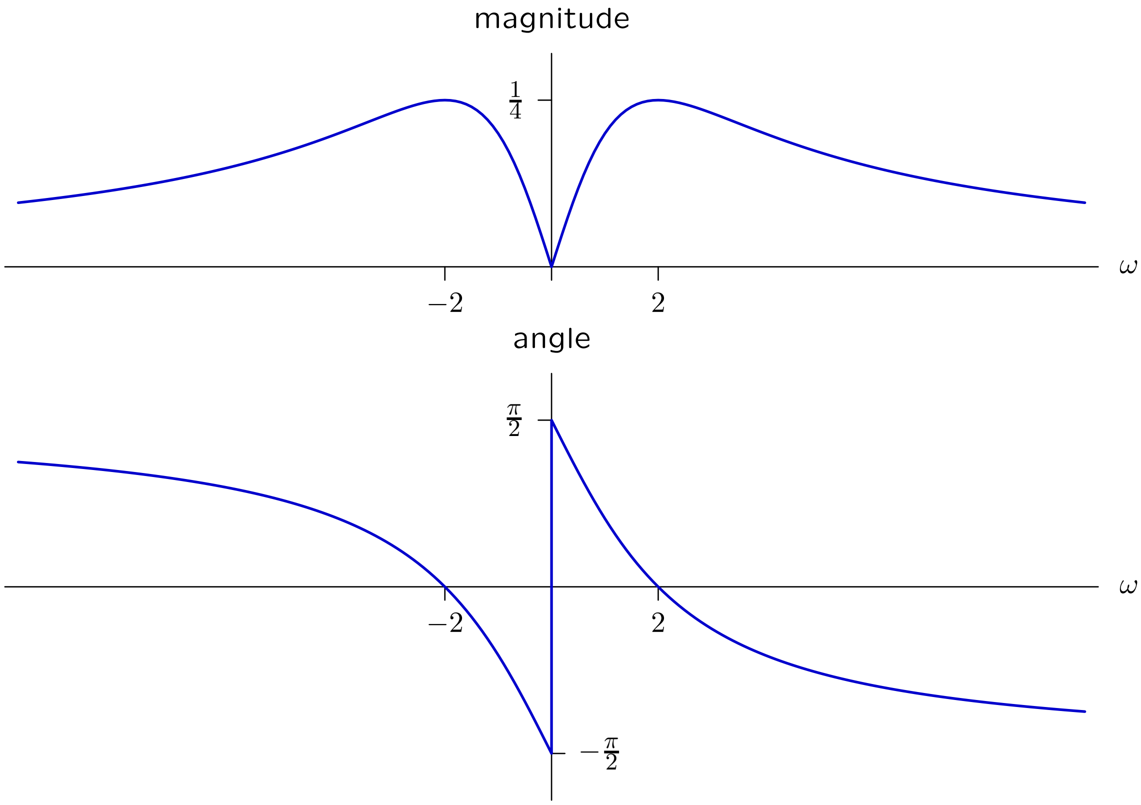
1	/26
2	/24
3	/12
4	/12
5	/26
Total	/100

1. Fourier Transform *[26 points]*

Sketch the magnitude and angle of the Fourier transform of

$$x(t) = (1 - 2t)e^{-2t}u(t)$$

on the (linear, not logarithmic) axes below.



Clearly label the important magnitudes, angles, and frequencies.

First find the Laplace transforms:

$$e^{-2t}u(t) \leftrightarrow \frac{1}{s+2}\,; \quad \mathrm{Re}(s) > -2$$

$$te^{-2t}u(t) \leftrightarrow \frac{1}{(s+2)^2}\,; \quad \mathrm{Re}(s) > -2$$

$$(1-2t)e^{-2t}u(t) \leftrightarrow \frac{1}{s+2} - 2\frac{1}{(s+2)^2} = \frac{s}{(s+2)^2}\,; \quad \mathrm{Re}(s) > -2$$

Since the ROC includes the $j\omega$ axis, the Fourier transform is equal to the Laplace transform evaluated on $s = j\omega$:

$$X(j\omega) = \frac{j\omega}{(j\omega+2)^2}$$

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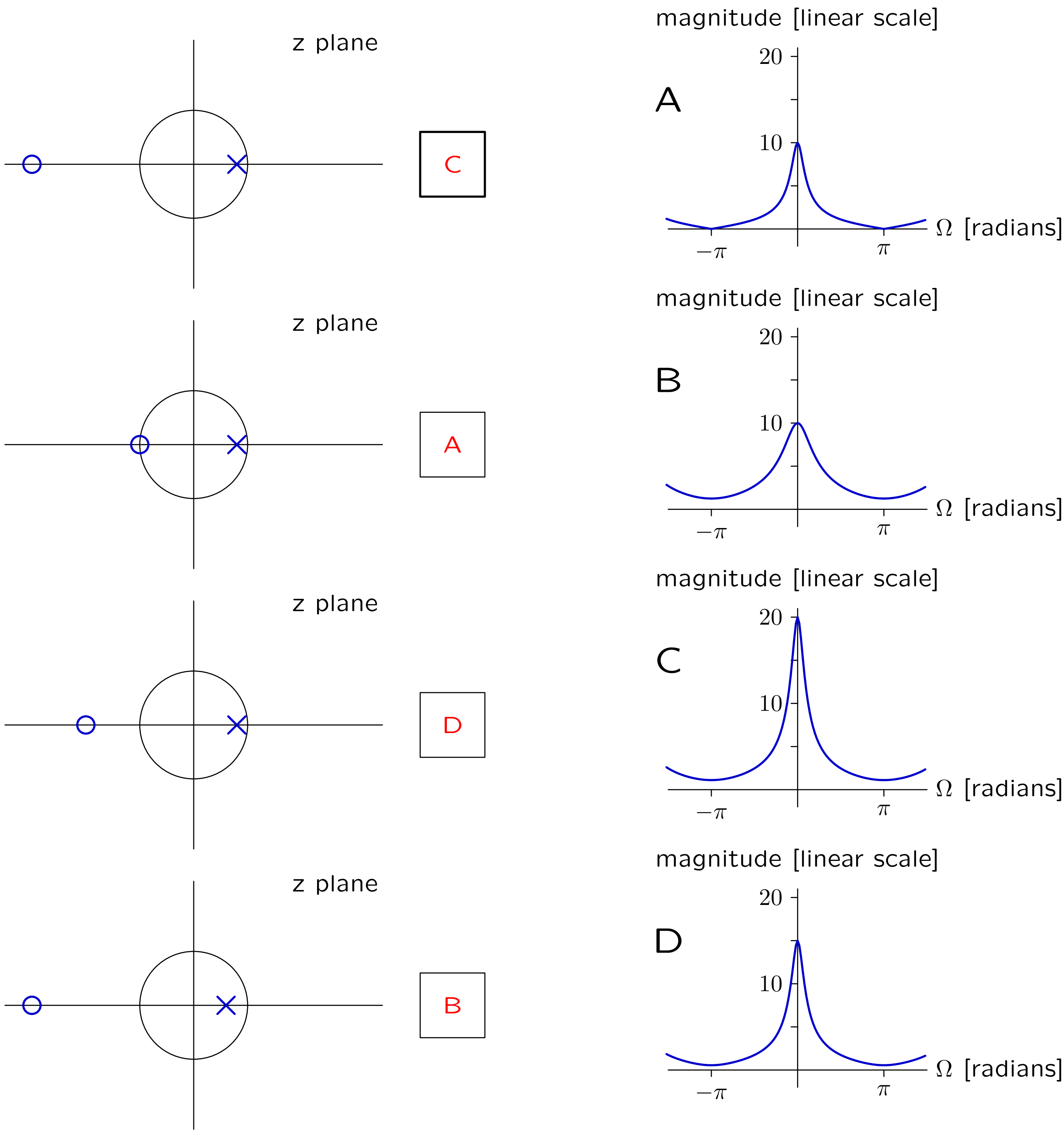
2. Frequency Responses *[24 points]*

Part a. Each pole-zero diagram below shows the unit circle (radius 1) as well as the pole ($\times$) and zero (o) of a **discrete-time**, linear, time-invariant system of the form

$$H(z) = \frac{z-a}{z-b}$$

where a represents the zero and b represents the pole.

Fill in the box to the right of each pole-zero diagram with the letter of the corresponding frequency response **magnitude** shown in the right panels. If none of the frequency response magnitudes match, put **X** in the box.

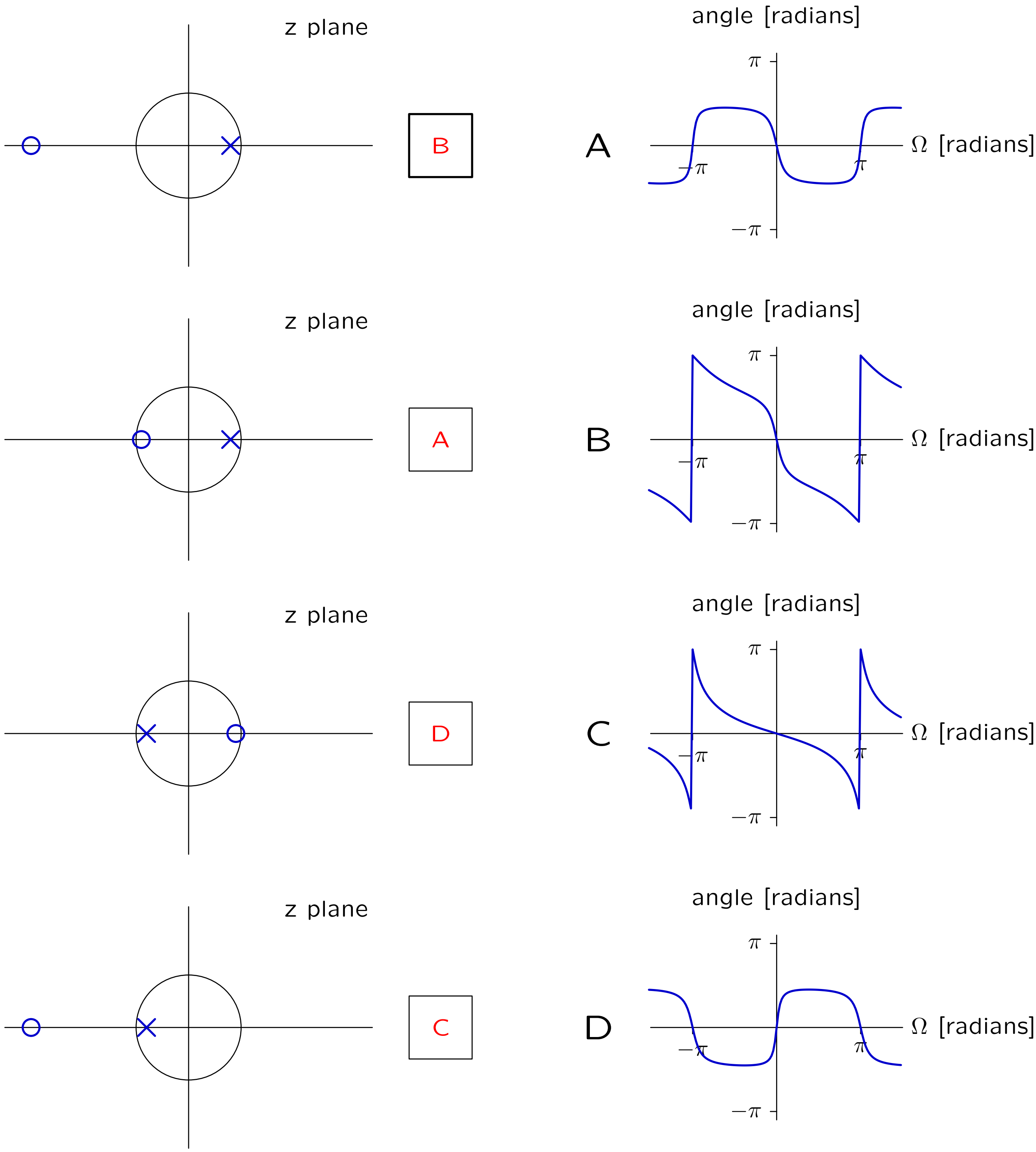


Part b. Each pole-zero diagram below shows the unit circle (radius 1) as well as the pole ($\times$) and zero (o) of a **discrete-time**, linear, time-invariant system of the form

$$H(z) = \frac{z-a}{z-b}$$

where a represents the zero and b represents the pole.

Fill in the box to the right of each pole-zero diagram with the letter of the corresponding frequency response **angle** shown in the right panels. If none of the frequency response angle match, put **X** in the box.



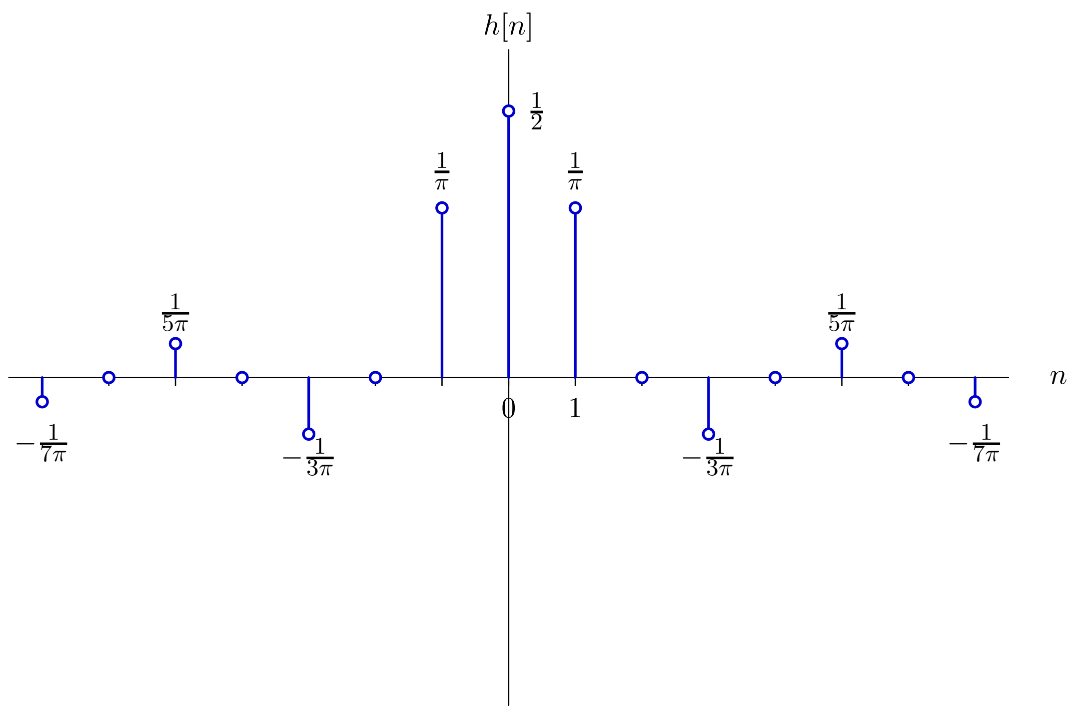
3. Unit-Sample Response *[12 points]*

Let $H(e^{j\Omega})$ represent the frequency response of a discrete-time system, where

$$H(e^{j\Omega}) = \begin{cases} 1 & \text{if } |\Omega| < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < |\Omega| < \frac{3\pi}{2} \end{cases}$$

and $H(e^{j\Omega})$ is periodic in Ω with period 2π.

Determine the unit-sample response $h[n]$ and sketch its values on the following axes.



Clearly label the values for $-7 \leq n \leq 7$.

$$h[n] = \frac{1}{2\pi}\int_{2\pi} H(e^{j\Omega})\, e^{j\Omega n} d\Omega = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\Omega n} d\Omega = \left.\frac{e^{j\Omega n}}{j2\pi n}\right|_{-\pi/2}^{\pi/2} = \frac{e^{j\pi n/2} - e^{-j\pi n/2}}{j2\pi n}$$

$$= \begin{cases} \frac{1}{2} & n = 0 \\ \frac{\sin(\pi n/2)}{\pi n} & n \neq 0 \end{cases}$$

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4. Discrete-Time Response *[12 points]*

Let $H(e^{j\Omega})$ represent the frequency response of a discrete-time system, where

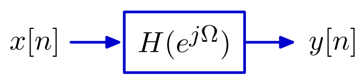
$$H(e^{j\Omega}) = \begin{cases} 1 & \text{if } |\Omega| < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < |\Omega| < \frac{3\pi}{2} \end{cases}$$

and $H(e^{j\Omega})$ is periodic in Ω with period 2π.

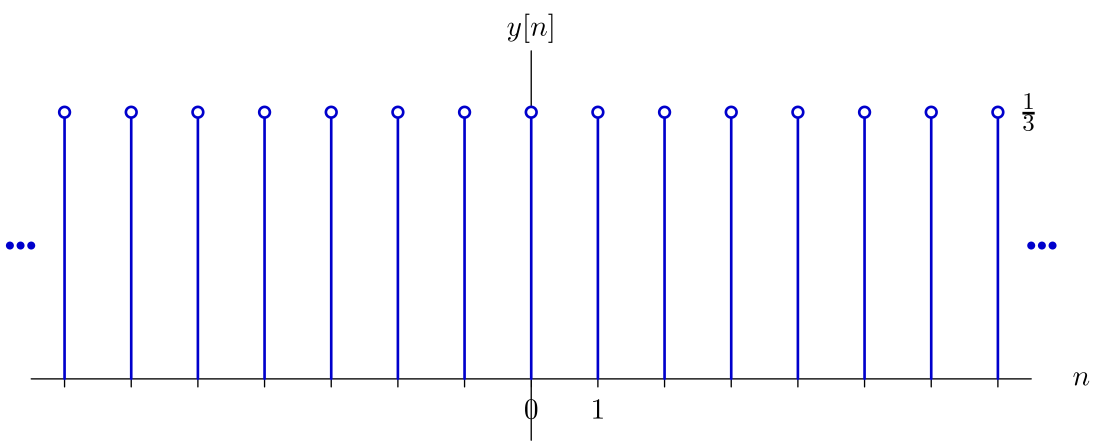
Determine the response $y[n]$ when the input $x[n]$ is

$$x[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n = 1 \text{ or } n = 2 \end{cases}$$

and $x[n]$ is periodic so that $x[n] = x[n+3]$ for all n.



Sketch the values of $y[n]$ for $-7 \le n \le 7$



and label your results.

Since $x[n]$ is periodic, we can represent it with a Fourier series:

$$a_k = \frac{1}{3}\sum_{n=0}^{2} x[n]e^{-j(2\pi kn/3)} = \frac{1}{3}$$

Since the Fourier series coefficients are periodic in $N = 3$, we only need to consider three terms: $k = 0$, $k = 1$, and $k = -1$. The a_0 term corresponds to $\Omega = 0$ and passes through the system with a gain of 1. The a_1 term corresponds to $\Omega = \frac{2}{3}\pi$ and passes through the system with a gain of 0. The a_{-1} term corresponds to $\Omega = -\frac{2}{3}\pi$ and passes through the system with a gain of 0. Thus, only the DC term appears at the output, and $y[n] = 1/3$ for all n.

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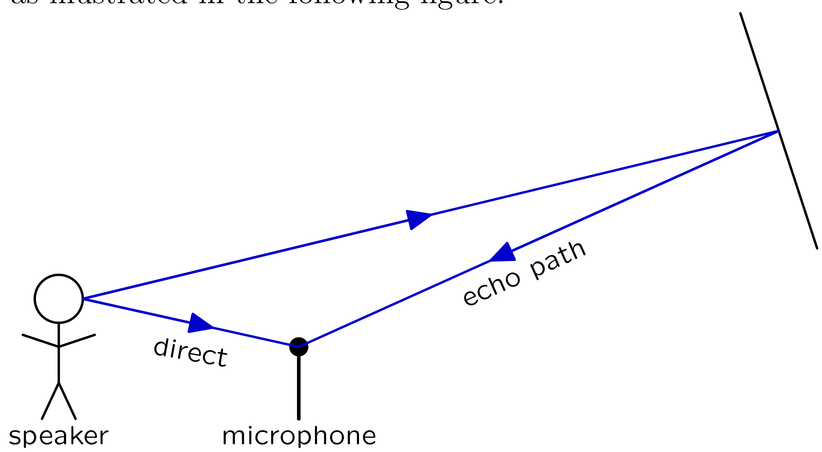
5. Echo *[26 points]*

Assume that a single echo interferes with a speaker's voice that is being recorded by a microphone as illustrated in the following figure.

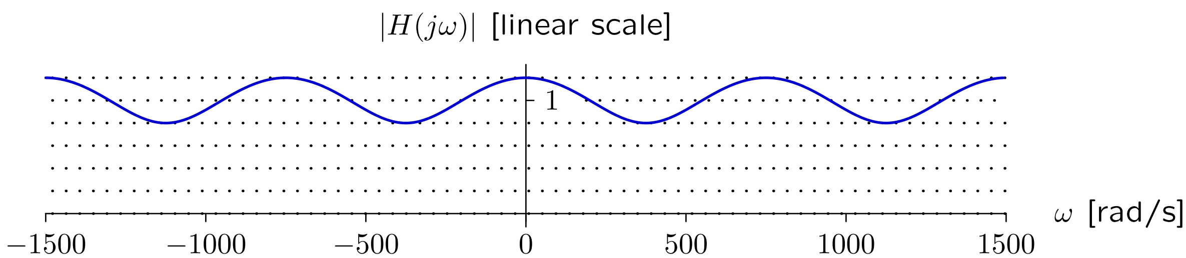


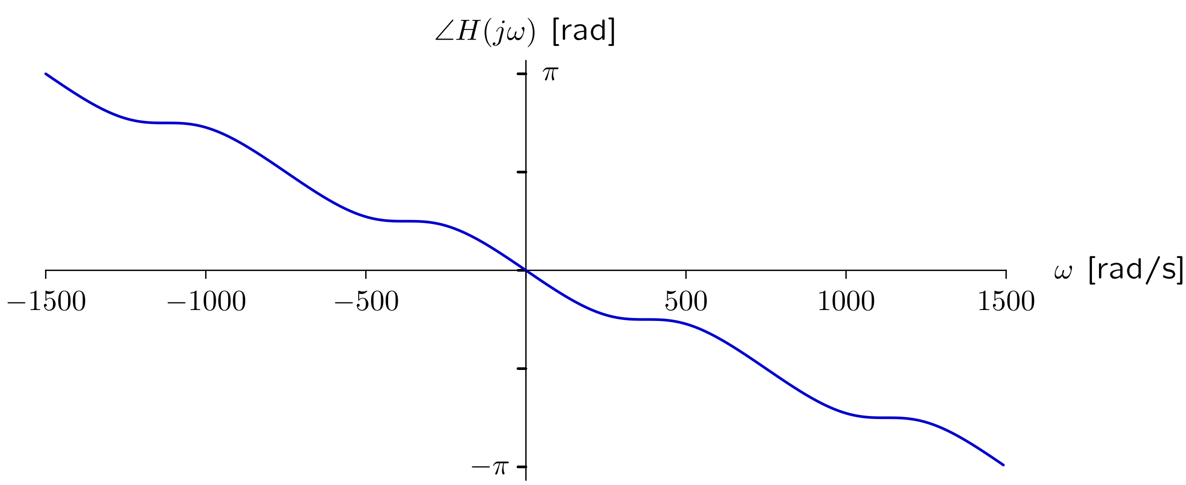
We can represent this recording situation as a linear, time-invariant system, with the speaker's voice as the input and the recorded microphone signal as the output. Assume that the impulse response of this system is

$$h(t) = \delta(t - T_1) + \epsilon\,\delta(t - T_2)$$

where T_1 represents that delay of the direct path from speaker to microphone, T_2 represents that delay through the echo path, and ϵ represents the amplitude of the echo. The amplitude of the direct path is taken as 1.

The following plots show that magnitude and angle of the frequency response of this system, for $|\omega| < 1500$ rad/s.





Determine numerical expressions for T_1, T_2, and ϵ.

$T_1 =$ $\dfrac{\pi}{1500}$

$T_2 =$ $\dfrac{\pi}{300}$

$\epsilon =$ 0.2

Take the Fourier transform of $h(t)$ to obtain the frequency response

$$H(j\omega) = e^{-j\omega T_1} + \epsilon e^{-j\omega T_2} = e^{-j\omega T_1}\left(1 + \epsilon e^{-j\omega(T_2-T_1)}\right).$$

The magnitude function

$$\begin{aligned}|H(j\omega)| &= \left|e^{-j\omega T_1}\right| \left|\left(1 + \epsilon e^{-j\omega(T_2-T_1)}\right)\right| \\ &= 1\sqrt{(1+\epsilon\cos\omega(T_2-T_1))^2 + \epsilon^2\sin^2\omega(T_2-T_1)} \\ &= \sqrt{1 + 2\epsilon\cos\omega(T_2-T_1) + \epsilon^2}\end{aligned}$$

oscillates between $1+\epsilon$ and $1-\epsilon$ with a period (in ω) of $2\pi/(T_2-T_1)$. From the magnitude plot on the previous page, we can see that $\epsilon \approx 0.2$ and $2\pi/(T_2-T_1) \approx 1500/2$, so that $T_2 - T_1 \approx \dfrac{4\pi}{1500}$.

The angle function is

$$\angle H(j\omega) = -\omega T_1 + \angle\left(1 + \epsilon e^{-j\omega(T_2-T_1)}\right)$$

Since ϵ is small compared to 1, the first term dominates the second, which oscillates about an average value near 0. Thus we can estimate T_1 from the average slope of the angle plot on the previous page,

$$T_1 = \frac{\pi}{1500}.$$

Then $T_2 \approx \dfrac{4\pi}{1500} + \dfrac{\pi}{1500} = \dfrac{\pi}{300}$.

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