6.003: Signals and Systems

Modulation

December 6, 2011

Subject Evaluations

Your feedback is important to us!

Please give feedback to the staff and future 6.003 students:
http://web.mit.edu/subjectevaluation

Evaluations are open until Friday, December 16, at noon.

You will be able to view quantitative results at
http://web.mit.edu/subjectevaluation/results.html

and student-written summaries at
http://hkn.mit.edu/ug_sel.php

Communications Systems

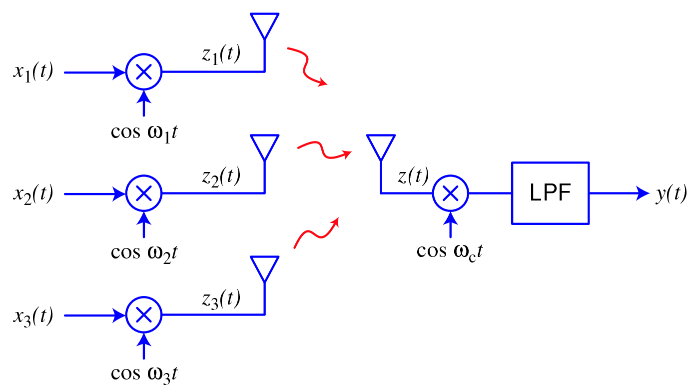
Signals are not always well matched to the media through which we wish to transmit them.

signal	applications
audio	telephone, radio, phonograph, CD, cell phone, MP3
video	television, cinema, HDTV, DVD
internet	coax, twisted pair, cable TV, DSL, optical fiber, E/M

Modulation can improve match based on **frequency**.

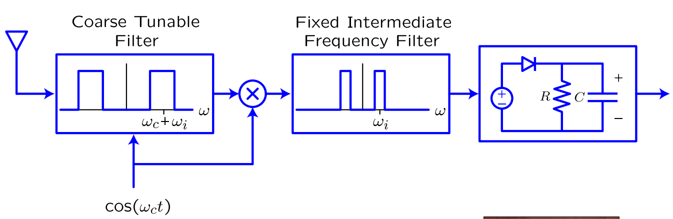
Amplitude Modulation

Amplitude modulation can be used to match audio frequencies to radio frequencies. It allows parallel transmission of multiple channels.



Superheterodyne Receiver

Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.



Edwin Howard Armstrong also invented and patented the "regenerative" (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wide-band FM.

Amplitude, Phase, and Frequency Modulation

There are many ways to embed a "message" in a carrier.

Amplitude Modulation (AM) + carrier: $y_1(t) = \big(x(t) + C\big)\cos(\omega_c t)$

Phase Modulation (PM): $y_2(t) = \cos(\omega_c t + kx(t))$

Frequency Modulation (FM): $y_3(t) = \cos\left(\omega_c t + k\int_{-\infty}^{t} x(\tau)d\tau\right)$

PM: signal modulates instantaneous phase of the carrier.

$$y_2(t) = \cos(\omega_c t + kx(t))$$

FM: signal modulates instantaneous frequency of carrier.

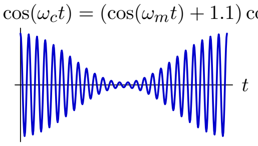
$$y_3(t) = \cos\Big(\omega_c t + \underbrace{k\int_{-\infty}^{t} x(\tau)d\tau}_{\phi(t)}\Big)$$

$$\omega_i(t) = \omega_c + \frac{d}{dt}\phi(t) = \omega_c + kx(t)$$

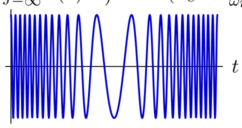
Frequency Modulation

Compare AM to FM for $x(t) = \cos(\omega_m t)$.

AM: $y_1(t) = \big(x(t) + C\big)\cos(\omega_c t) = (\cos(\omega_m t) + 1.1)\cos(\omega_c t)$



FM: $y_3(t) = \cos\left(\omega_c t + k\int_{-\infty}^{t} x(\tau)d\tau\right) = \cos(\omega_c t + \frac{k}{\omega_m}\sin(\omega_m t))$



Advantages of FM:
- constant power
- no need to transmit carrier (unless DC important)
- bandwidth?

Frequency Modulation

Early investigators thought that narrowband FM could have arbitrarily narrow bandwidth, allowing more channels than AM.

$$y_3(t) = \cos\left(\omega_c t + \underbrace{k\int_{-\infty}^{t} x(\tau)d\tau}_{\phi(t)}\right)$$

$$\omega_i(t) = \omega_c + \frac{d}{dt}\phi(t) = \omega_c + kx(t)$$

Small $k \to$ small bandwidth. Right?

Frequency Modulation

Early investigators thought that narrowband FM could have arbitrarily narrow bandwidth, allowing more channels than AM. **Wrong!**

$$y_3(t) = \cos\left(\omega_c t + k\int_{-\infty}^{t} x(\tau)d\tau\right)$$
$$= \cos(\omega_c t) \times \cos\left(k\int_{-\infty}^{t} x(\tau)d\tau\right) - \sin(\omega_c t) \times \sin\left(k\int_{-\infty}^{t} x(\tau)d\tau\right)$$

If $k \to 0$ then

$$\cos\left(k\int_{-\infty}^{t} x(\tau)d\tau\right) \to 1$$
$$\sin\left(k\int_{-\infty}^{t} x(\tau)d\tau\right) \to k\int_{-\infty}^{t} x(\tau)d\tau$$
$$y_3(t) \approx \cos(\omega_c t) - \sin(\omega_c t) \times \left(k\int_{-\infty}^{t} x(\tau)d\tau\right)$$

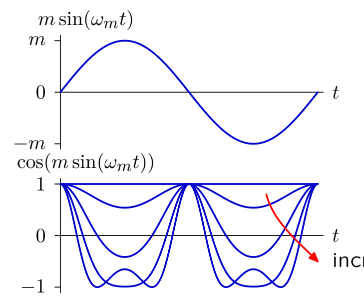
Bandwidth of narrowband FM is the same as that of AM!
(integration does not change the highest frequency in the signal)

Phase/Frequency Modulation

Find the Fourier transform of a PM/FM signal.

$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m\sin(\omega_m t))$$
$$= \cos(\omega_c t)\cos(m\sin(\omega_m t)) - \sin(\omega_c t)\sin(m\sin(\omega_m t))$$

$x(t)$ is periodic in $T = \frac{2\pi}{\omega_m}$, therefore $\cos(m\sin(\omega_m t))$ is periodic in T.



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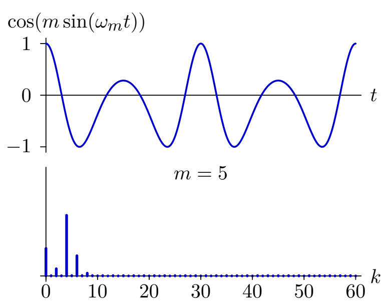

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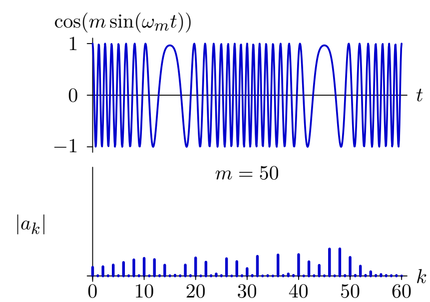
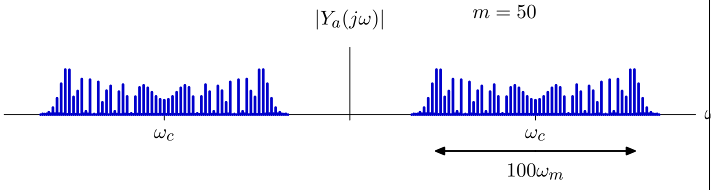


Phase/Frequency Modulation

Fourier transform of first part.

$$x(t) = \sin(\omega_m t)$$
$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m\sin(\omega_m t))$$
$$= \underbrace{\cos(\omega_c t)\cos(m\sin(\omega_m t))}_{y_a(t)} - \sin(\omega_c t)\sin(m\sin(\omega_m t)))$$



Phase/Frequency Modulation

Find the Fourier transform of a PM/FM signal.

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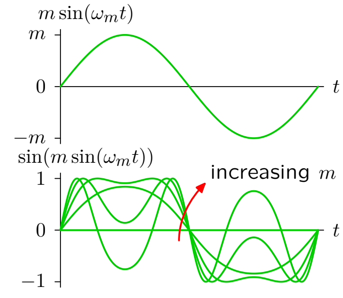

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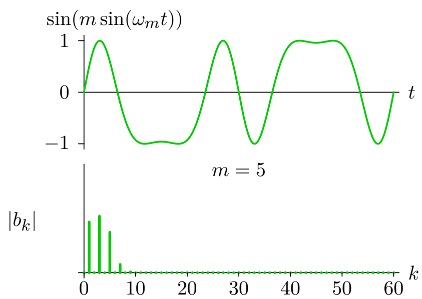

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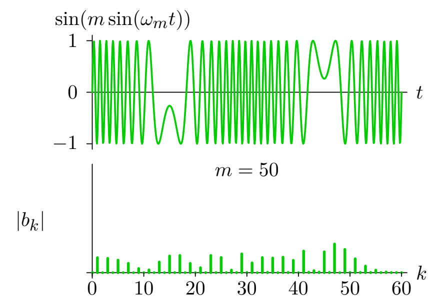
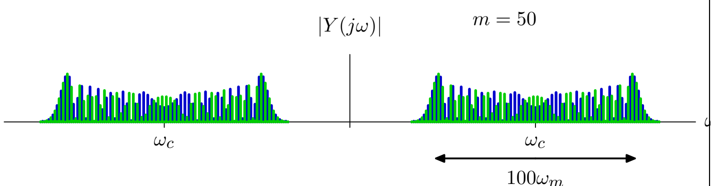


Phase/Frequency Modulation

Fourier transform.

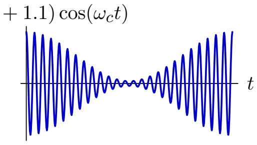
$$x(t) = \sin(\omega_m t)$$
$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m\sin(\omega_m t))$$
$$= \underbrace{\cos(\omega_c t)\cos(m\sin(\omega_m t)))}_{y_a(t)} - \underbrace{\sin(\omega_c t)\sin(m\sin(\omega_m t)))}_{y_b(t)}$$



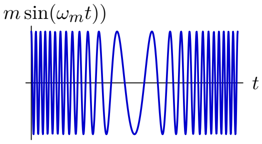
Frequency Modulation

Wideband FM is useful because it is robust to noise.

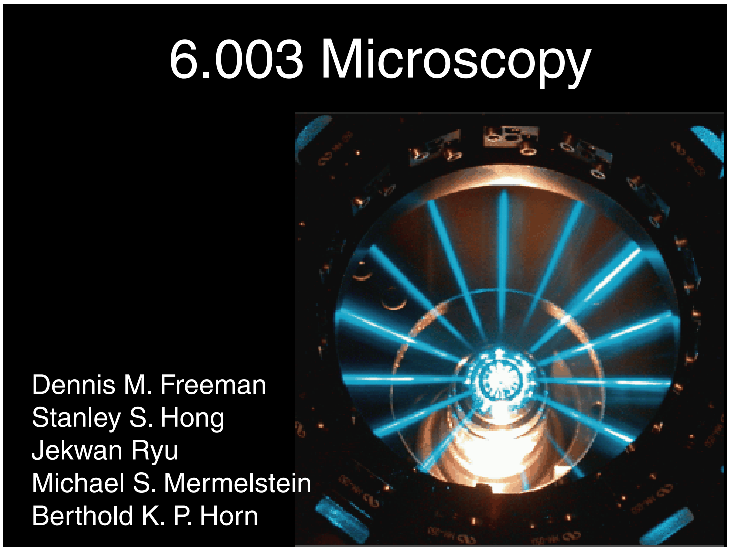
AM: $y_1(t) = (\cos(\omega_m t) + 1.1)\cos(\omega_c t)$

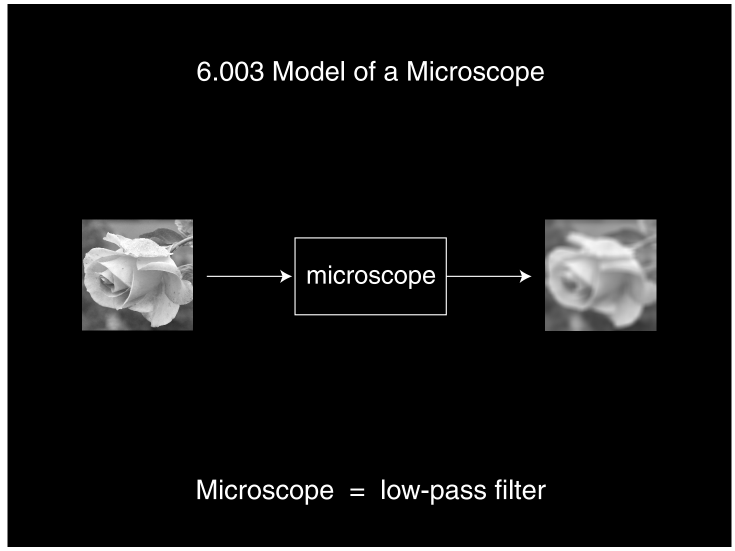


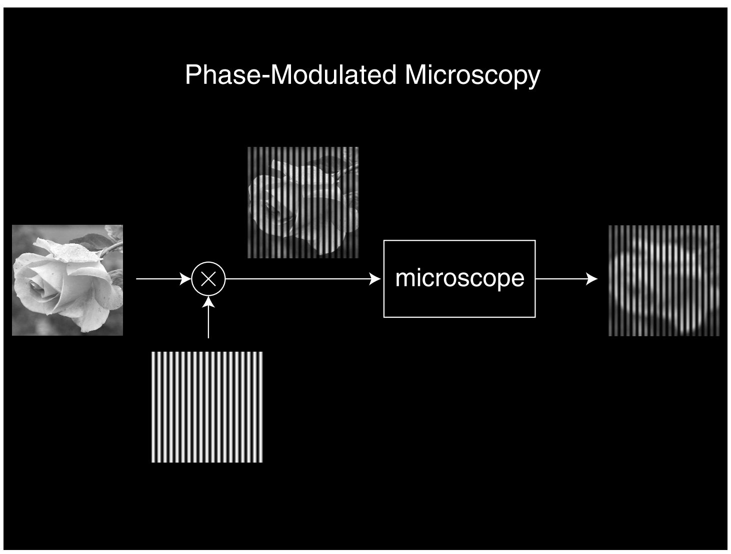
FM: $y_3(t) = \cos(\omega_c t + m\sin(\omega_m t))$

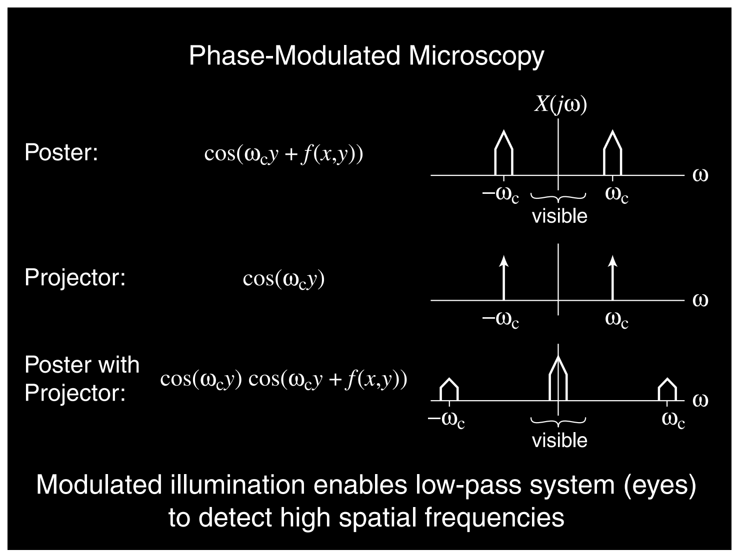


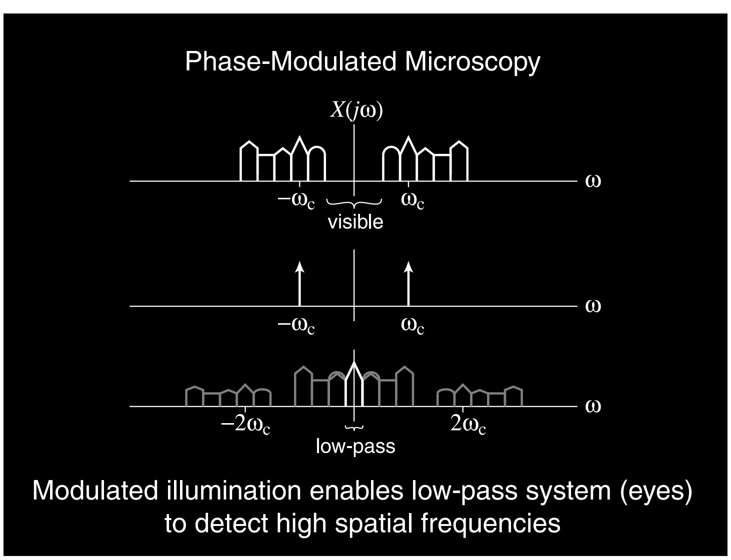
FM generates a redundant signal that is resilient to additive noise.

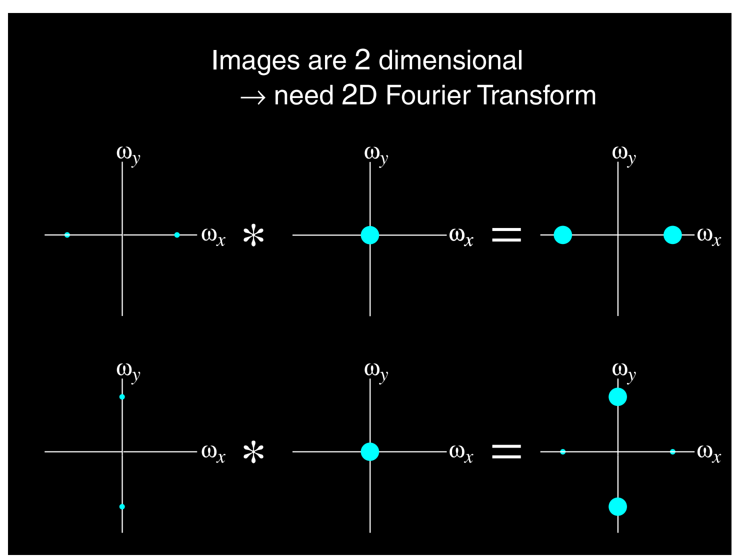

6.003 Microscopy
Dennis M. Freeman
Stanley S. Hong
Jekwan Ryu
Michael S. Mermelstein
Berthold K. P. Horn

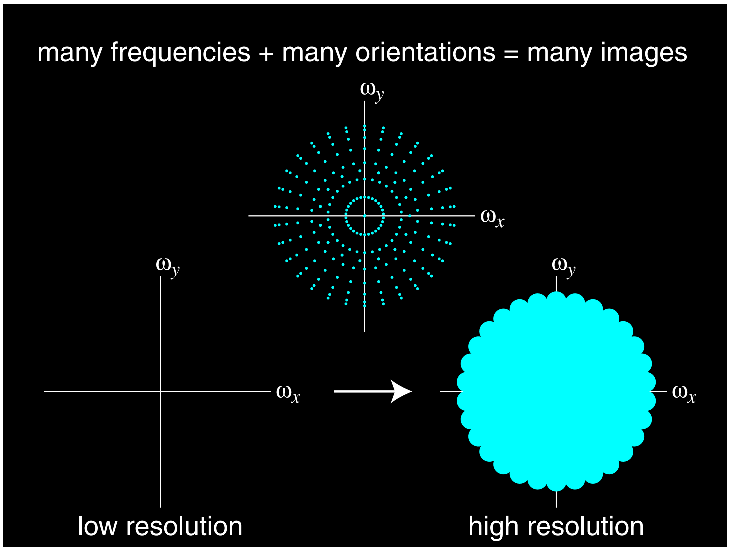

6.003 Model of a Microscope
microscope
Microscope = low-pass filter

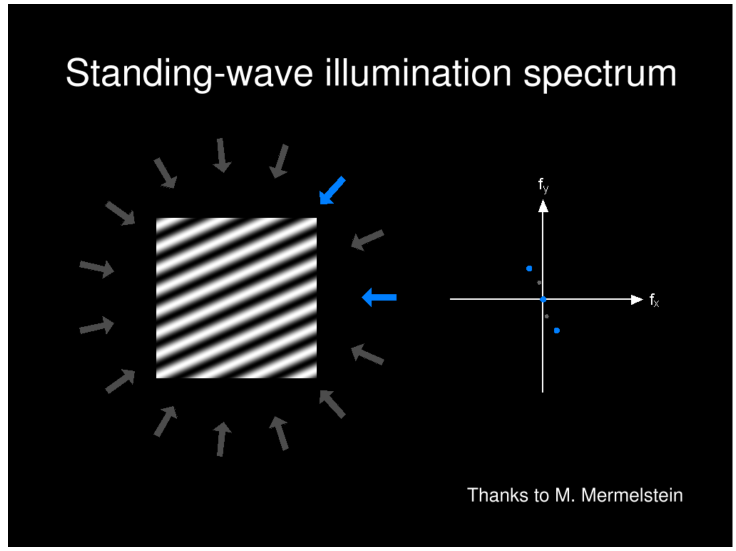

Phase-Modulated Microscopy
microscope

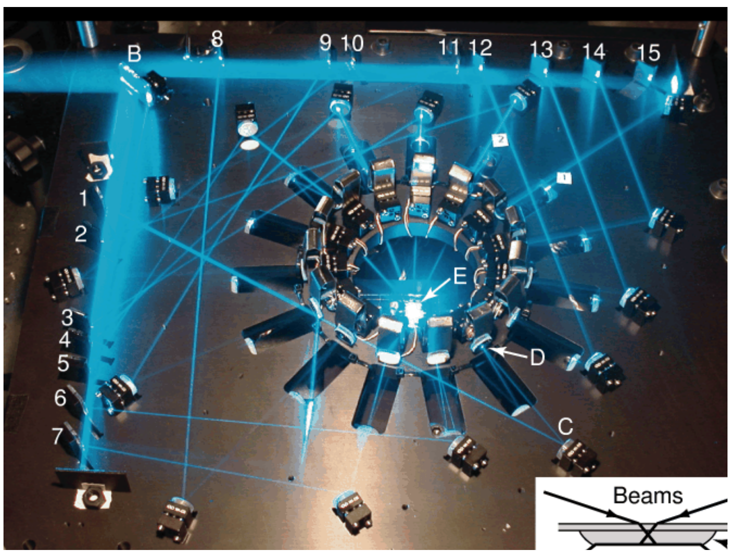

Phase-Modulated Microscopy
X(jω)
Poster:
cos(ω_c y + f(x,y))
−ω_c
ω_c
visible
ω
Projector:
cos(ω_c y)
Poster with Projector:
cos(ω_c y) cos(ω_c y + f(x,y))
Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies

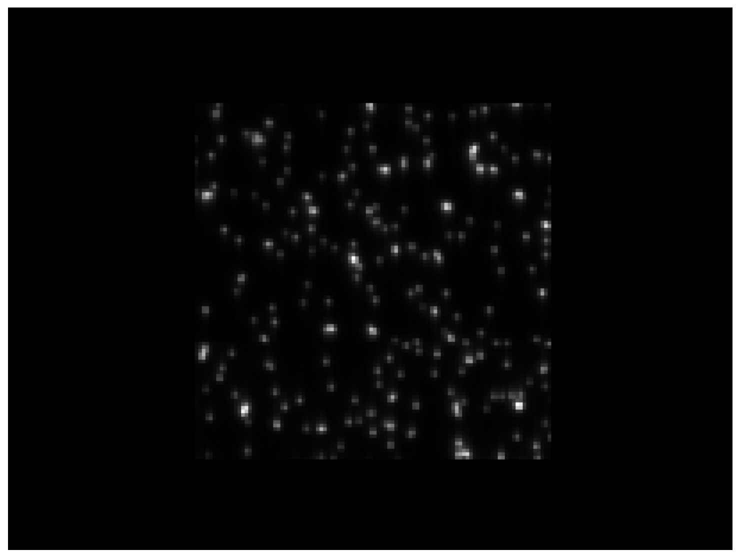

Phase-Modulated Microscopy
X(jω)
−ω_c
ω_c
visible
ω
−2ω_c
2ω_c
low-pass
Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies

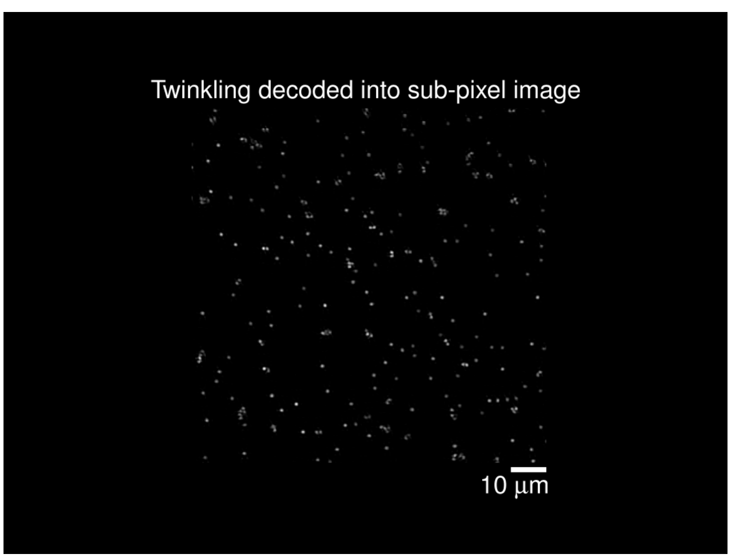

Images are 2 dimensional
→ need 2D Fourier Transform
ω_y
ω_x

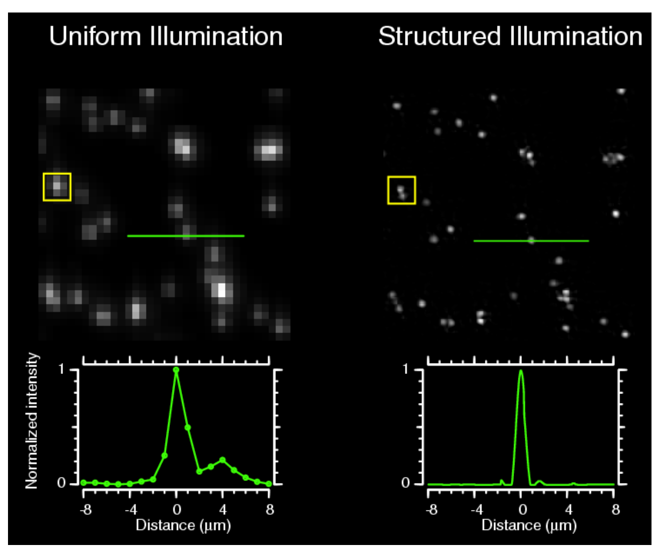

many frequencies + many orientations = many images
ω_y
ω_x
low resolution
high resolution

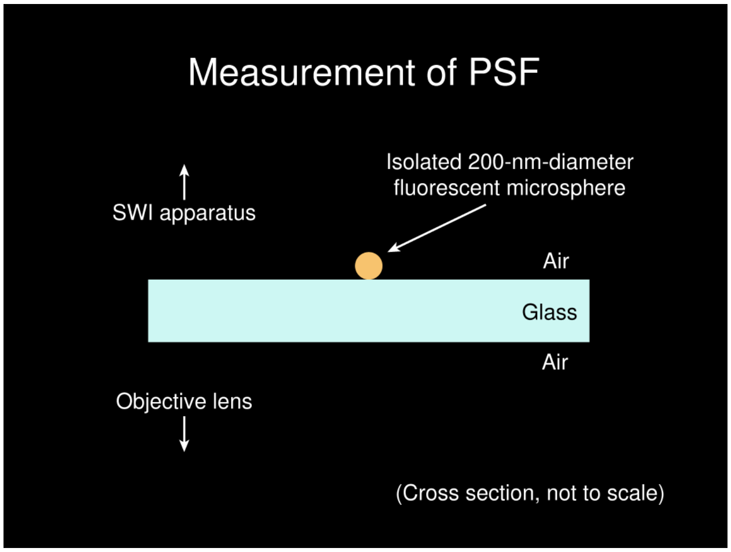

Standing-wave illumination spectrum
f_y
f_x
Thanks to M. Mermelstein

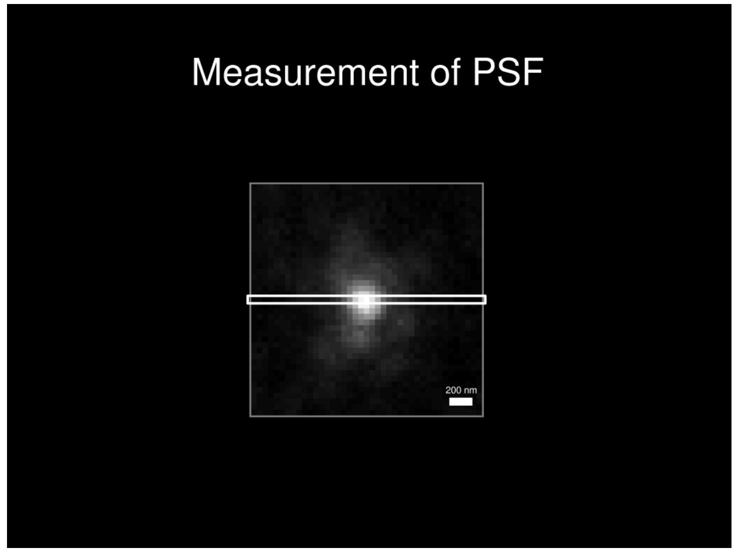

B
8
9 10
11 12
13 14 15
1
2
3
4
5
6
7
E
D
C
Beams

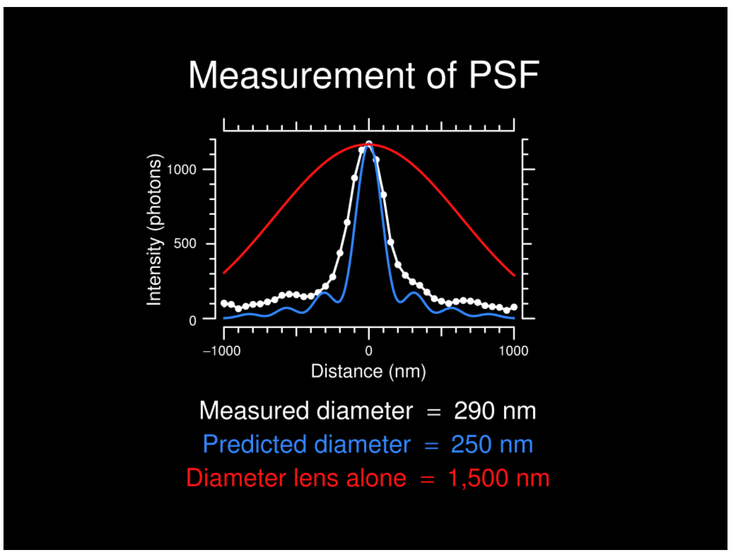



Twinkling decoded into sub-pixel image
10 µm


Uniform Illumination
Structured Illumination
Normalized intensity
Distance (µm)
Distance (µm)


Measurement of PSF
SWI apparatus
Isolated 200-nm-diameter fluorescent microsphere
Air
Glass
Air
Objective lens
(Cross section, not to scale)


Measurement of PSF
200 nm


Measurement of PSF
Intensity (photons)
1000
500
0
–1000
0
1000
Distance (nm)
Measured diameter = 290 nm
Predicted diameter = 250 nm
Diameter lens alone = 1,500 nm