6.003 Homework #7

This homework assignment will not be collected. Solutions will be posted.

Problems

1. Second-order systems

The impulse response of a second-order CT system has the form

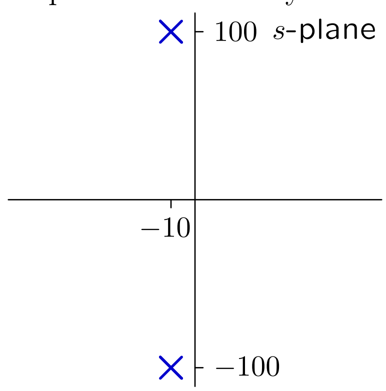
$$h(t) = e^{-\sigma t}\cos(\omega_d t + \phi)u(t)$$

where the parameters σ, ω_d, and ϕ are related to the parameters of the characteristic polynomial for the system: $s^2 + Bs + C$.

a. Determine expressions for σ and ω_d (not ϕ) in terms of B and C.

b. Determine
– the time required for the envelope $e^{-\sigma t}$ of $h(t)$ to diminish by a factor of e,
– the period of the oscillations in $h(t)$, and
– the number of periods of oscillation before $h(t)$ diminishes by a factor of e.
Express your results as functions of B and C only.

c. Estimate the parameters in part b for a CT system with the following poles:



The unit-sample response of a second-order DT system has the form

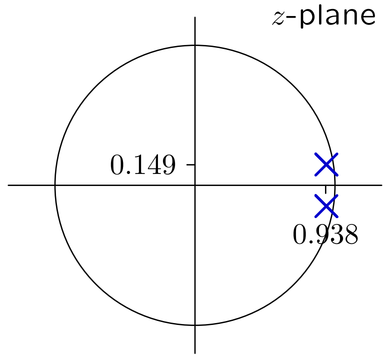
$$h[n] = r_0^n \cos(\Omega_0 n + \Phi)u[n]$$

where the parameters r_0, Ω_0, and Φ are related to the parameters of the characteristic polynomial for the system: $z^2 + Dz + E$.

d. Determine expressions for r_0 and Ω_0 (not Φ) in terms of D and E.

e. Determine
– the length of time required for the envelope r_0^n of $h[n]$ to diminish by a factor of e.
– the period of the oscillations (i.e., $\frac{2\pi}{\Omega_0}$) in $h[n]$, and
– the number of periods of oscillation in $h[n]$ before it diminishes by a factor of e.
Express your results as functions of D and E only.

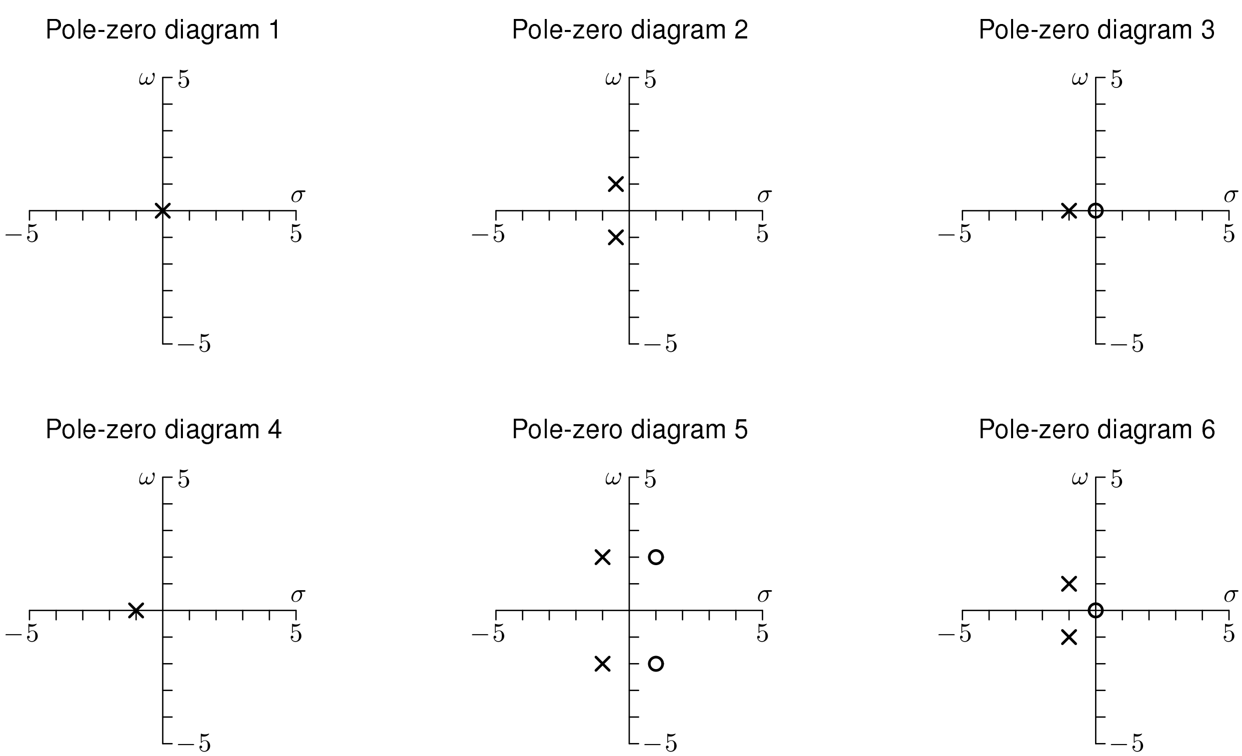
f. Estimate the parameters in part e for a DT system with the following poles:



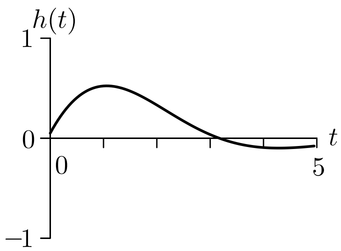
2. Matches

The following plots show pole-zero diagrams, impulse responses, Bode magnitude plots, and Bode angle plots for six causal CT LTI systems. Determine which corresponds to which and fill in the following table.

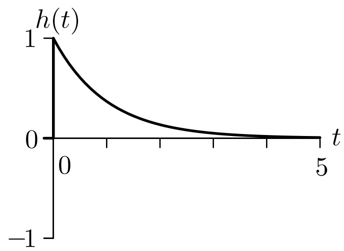
	$h(t)$	Magnitude	Angle
PZ diagram 1:			
PZ diagram 2:			
PZ diagram 3:			
PZ diagram 4:			
PZ diagram 5:			
PZ diagram 6:			



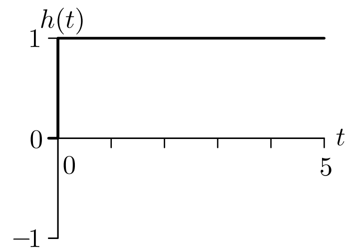
Impulse response 1



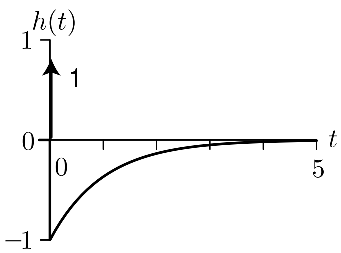
Impulse response 2



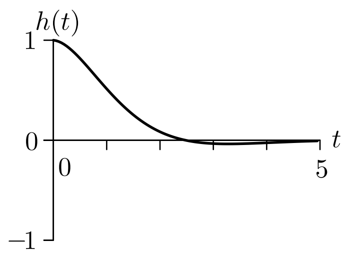
Impulse response 3



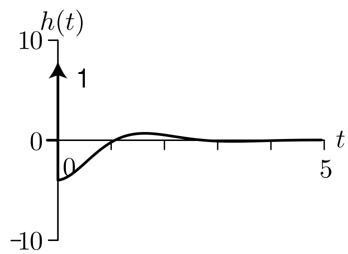
Impulse response 4



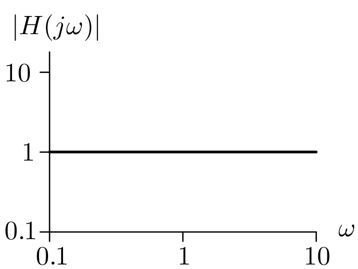
Impulse response 5



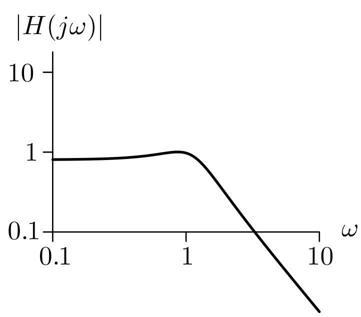
Impulse response 6



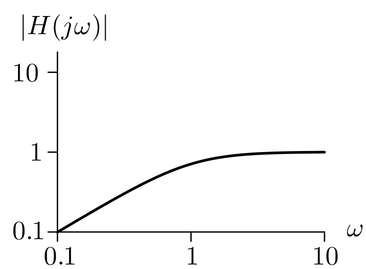
Bode Magnitude 1



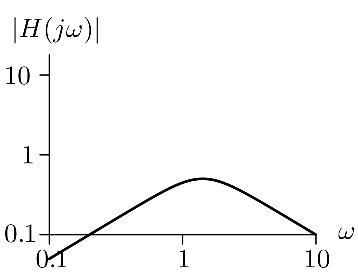
Bode Magnitude 2



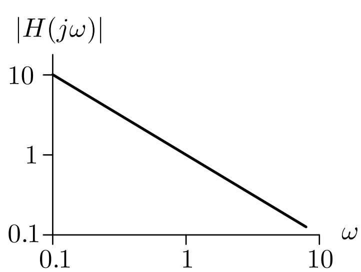
Bode Magnitude 3



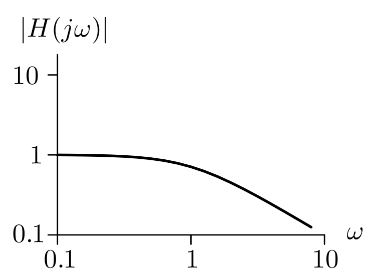
Bode Magnitude 4



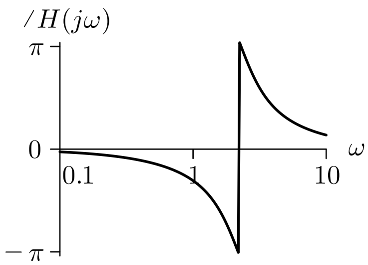
Bode Magnitude 5



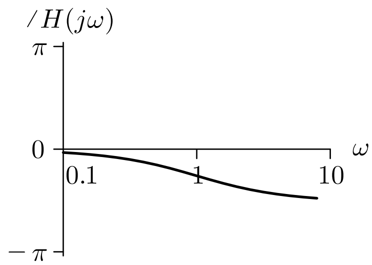
Bode Magnitude 6



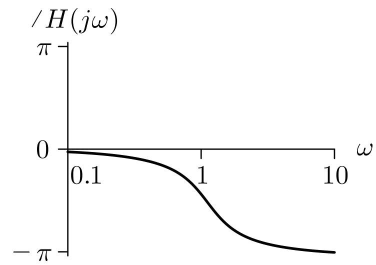
Bode Angle 1



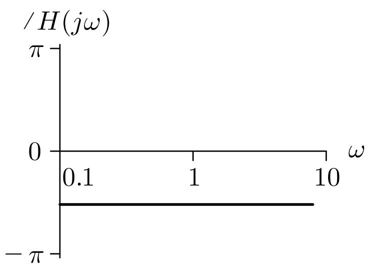
Bode Angle 2



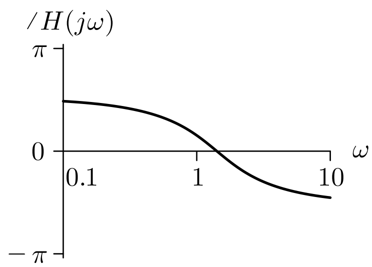
Bode Angle 3



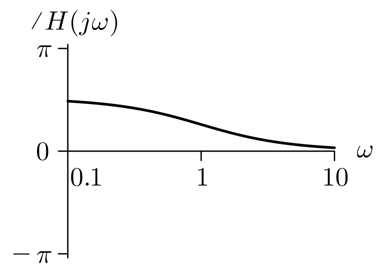
Bode Angle 4



Bode Angle 5



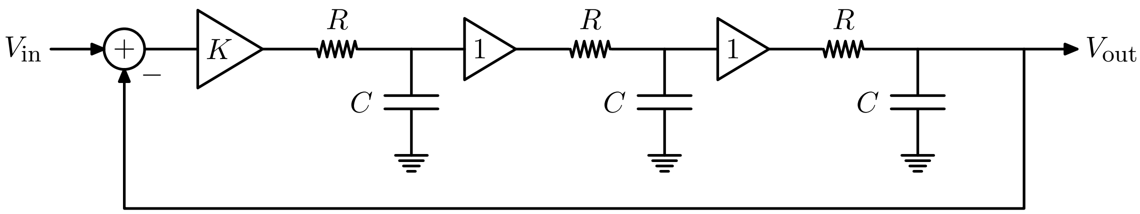
Bode Angle 6



Engineering Design Problems

3. Desired oscillations

The following feedback circuit was the basis of Hewlett and Packard's founding patent.

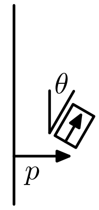


a. With $R = 1\,\text{k}\Omega$ and $C = 1\mu\text{F}$, sketch the pole locations as the gain K varies from 0 to ∞, showing the scale for the real and imaginary axes. Find the K for which the system is barely stable and label your sketch with that information. What is the system's oscillation period for this K?

b. How do your results change if R is increased to $10\,\text{k}\Omega$?

4. Robotic steering

Design a steering controller for a car that is moving forward with constant velocity V.



You can control the steering-wheel angle $w(t)$, which causes the angle $\theta(t)$ of the car to change according to

$$\frac{d\theta(t)}{dt} = \frac{V}{d} w(t)$$

where d is a constant with dimensions of length. As the car moves, the transverse position $p(t)$ of the car changes according to

$$\frac{dp(t)}{dt} = V \sin\big(\theta(t)\big) \approx V\theta(t)\,.$$

Consider three control schemes:

a. $w(t) = Ke(t)$

b. $w(t) = K_v \dot{e}(t)$

c. $w(t) = Ke(t) + K_v \dot{e}(t)$

where $e(t)$ represents the difference between the desired transverse position $x(t) = 0$ and the current transverse position $p(t)$. Describe the behaviors that result for each control scheme when the car starts with a non-zero angle ($\theta(0) = \theta_0$ and $p(0) = 0$). Determine the most acceptable value(s) of K and/or K_v for each control scheme or explain why none are acceptable.