6.003 Homework #4

This homework assignment will not be collected. Solutions will be posted.

Problems

1. Laplace Transforms

Determine the Laplace transforms (including the regions of convergence) of each of the following signals:

a. $x_1(t) = e^{-2(t-3)}u(t-3)$

$X_1 =$ [ ]

ROC: [ ]

b. $x_2(t) = (1 - (1 - t)e^{-3t})u(t)$

$X_2 =$

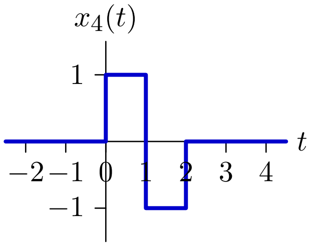
ROC:

c. $x_3(t) = |t|e^{-|t|}$

$X_3 =$

ROC:

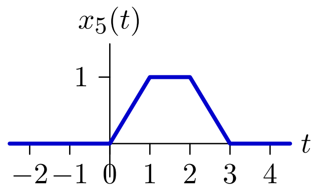
d.



$X_4 =$

ROC:

e.



$X_5 =$

ROC:

2. Inverse Laplace transforms

Determine all possible signals with Laplace transforms of the following forms. For each signal, indicate a closed-form solution as well as the region of time t for which the closed-form solution is valid. Three boxes are given for each part. If fewer than three solutions exist for a given part, indicate **none** in the extra boxes.

a. $X_1(s) = \dfrac{s+2}{(s+1)^2}$

	$t < 0$	$t > 0$
$x_1(t) =$		
or		
or		

b. $X_2(s) = \dfrac{1}{s^2(s-1)}$

	$t < 0$	$t > 0$
$x_2(t) =$		
or		
or		

c. $X_3(s) = \dfrac{s+1}{s^2+2s+2}$

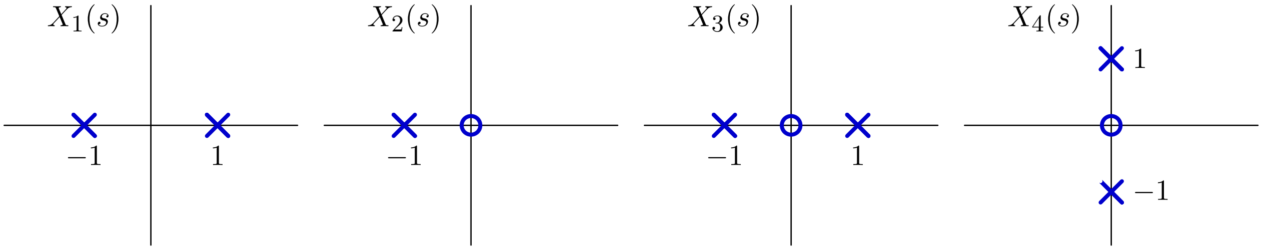
	$t < 0$	$t > 0$
$x_3(t) =$		
or		
or		

d. $X_4(s) = \left(\frac{1-e^{-s}}{s}\right)^2$

	$t<0$	$0<t<1$	$1<t<2$	$t>2$
$x_4(t) =$				
or				
or				

3. Symmetry

Determine which of the following pole-zero diagrams could represent Laplace transforms of even functions of time.



Enter a subset of the numbers 1 through 4 (separated by spaces) to represent $X_1(s)$ through $X_4(s)$ in the answer box below. If none of $X_1(s)$ through $X_4(s)$ apply, enter **none**.

1 and/or 2 and/or 3 and/or 4 or **none**:

Explain.

4. Initial and final values (from "Circuits, Signals, and Systems" by Siebert)

a. Use the initial and final value theorems (where applicable) to find $x(0)$ and $x(\infty)$ for the signals with the following Laplace transforms:

1. $\dfrac{1-e^{-sT}}{s}$
2. $\dfrac{1}{s}e^{-sT}$
3. $\dfrac{1}{s(s+1)^2}$
4. $\dfrac{1}{s^2(s+1)}$
5. $\dfrac{1}{s^2+1}$
6. $\dfrac{(s+1)^2-1}{[(s+1)^2+1]^2}$

Assume that the regions of convergence include $\text{Re}(s) > 0$.

b. Find the inverse Laplace transforms for each of the previous parts and show that the time waveforms and initial and final values agree.

Enter your answers in the boxes below. If the initial or final value theorem cannot be applied, enter **X**.

	$x(0)$	$x(\infty)$
1.		
2.		
3.		
4.		
5.		
6.		

Engineering Design Problems

5. Impulse response

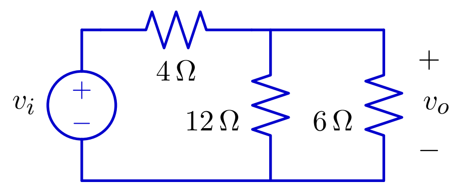
Sketch a block diagram for a CT system with impulse response

$$h(t) = \left(1 - te^{-t}\right) e^{-2t} u(t)\,.$$

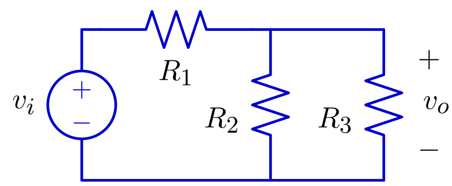
The block diagram should contain only adders, gains, and integrators.

6. Impedance Method

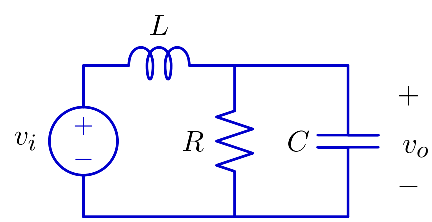
a. Determine the output voltage of the following circuit, using series or parallel resistor combinations and/or voltage or current dividers.



b. Generalize the result from part a for arbitrary resistor values, and determine an expression for the resulting ratio $\frac{v_o}{v_i}$.



c. Consider the following circuit.



Determine a differential equation that relates v_i to v_C as follows. First, determine relations among the element currents (i_R, i_L, and i_C) and element voltages (v_R, v_L, and v_C) using KVL and KCL. Second, relate each element voltage to the corresponding element current using the constitutive relation for the element: i.e., $v_R = Ri_R$, $i_C = C\frac{dv_C}{dt}$, and $v_L = L\frac{di_L}{dt}$. Finally, solve your equations to find a single equation with terms that involve v_i, v_o, and derivatives of v_i and v_o.

d. Determine the system function $H(s) = \frac{V_o(s)}{V_i(s)}$, based on the Laplace transform of your answer to the previous part.

e. Substitute $R_1 \to sL$, $R_2 \to R$, and $R_3 \to \frac{1}{sC}$ into your result from part b. Compare this new expression to your result from part d.

f. The impedance of an electrical element is a function of s that can be analyzed using rules that are quite similar to those for resistances. Explain the basis of this method.