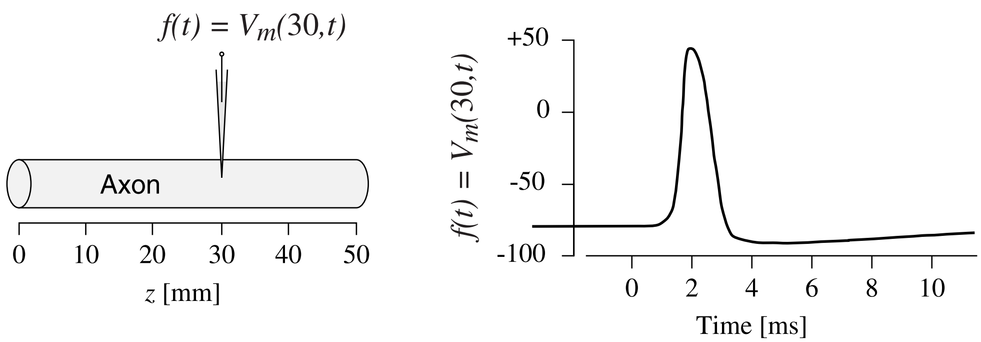
6.003 Homework #14

This homework will not be collected or graded. It is intended to help you practice for the final exam. Solutions will be posted.

Problems

1. Neural signals

The following figure illustrates the measurement of an action potential, which is an electrical pulse that travels along a neuron. Assume that this pulse travels in the positive z direction with constant speed $\nu = 10\,\text{m/s}$ (which is a reasonable assumption for the large unmyelinated fibers found in the squid, where such potentials were first studied). Let $V_m(z,t)$ represent the potential that is measured at position z and time t, where time is measured in milliseconds and distance is measured in millimeters. The right panel illustrates $f(t) = V_m(30,t)$ which is the potential measured as a function of time t at position $z = 30\,\text{mm}$.



Part a. Sketch the dependence of V_m on t at position $z = 40\,\text{mm}$ (i.e., $V_m(40,t)$).

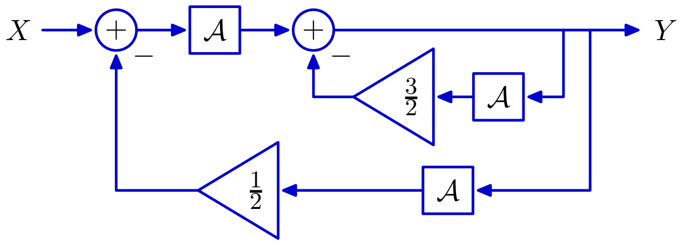
Part b. Sketch the dependence of V_m on z at time $t = 0\,\text{ms}$ (i.e., $V_m(z,0)$).

Part c. Determine an expression for $V_m(z,t)$ in terms of $f(\cdot)$ and ν. Explain the relations between this expression and your results from parts a and b.

$V_m(z,t) =$

2. Characterizing block diagrams

Consider the system defined by the following block diagram:



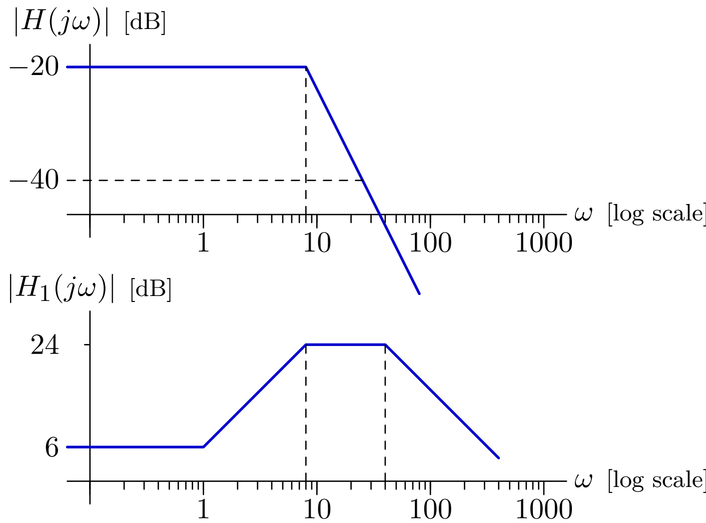
a. Determine the system functional $H = \frac{Y}{X}$.

b. Determine the poles of the system.

c. Determine the impulse response of the system.

3. Bode Plots

Our goal is to design a stable CT LTI system H by cascading two causal CT LTI systems: H_1 and H_2. The magnitudes of $H(j\omega)$ and $H_1(j\omega)$ are specified by the following straight-line approximations. We are free to choose other aspects of the systems.



a. Determine all system functions $H_1(s)$ that are consistent with these design specifications, and plot the straight-line approximation to the phase angle of each (as a function of ω).

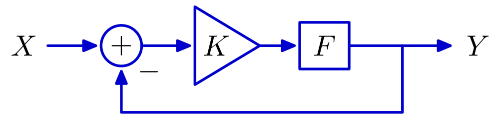
b. Determine all system functions $H_2(s)$ that are consistent with these design specifications, and plot the straight-line approximation to the phase angle of each (as a function of ω).

4. Controlling Systems

Use a proportional controller (gain K) to control a plant whose input and output are related by

$$F = \frac{R^2}{1 + \mathcal{R} - 2\mathcal{R}^2}$$

as shown below.

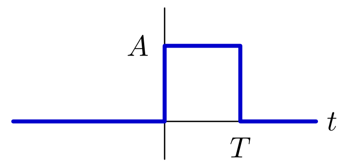


a. Determine the range of K for which the unit-sample response of the closed-loop system converges to zero.

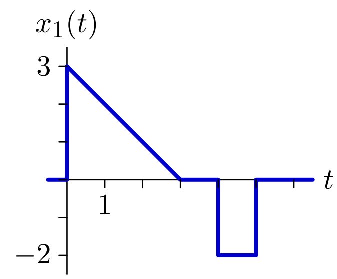
b. Determine the range of K for which the closed-loop poles are real-valued numbers with magnitudes less than 1.

5. CT responses

We are given that the impulse response of a CT LTI system is of the form



where A and T are unknown. When the system is subjected to the input



the output $y_1(t)$ is zero at $t = 5$. When the input is

$$x_2(t) = \sin\left(\frac{\pi t}{3}\right) u(t),$$

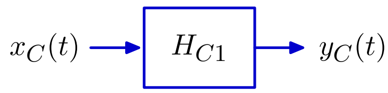
the output $y_2(t)$ is equal to 9 at $t = 9$. Determine A and T. Also determine $y_2(t)$ for all t.

6. DT approximation of a CT system

Let H_{C1} represent a **causal** CT system that is described by

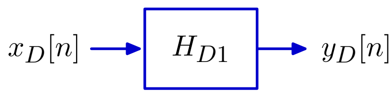
$$\dot{y}_C(t) + 3y_C(t) = x_C(t)$$

where $x_C(t)$ represents the input signal and $y_C(t)$ represents the output signal.



a. Determine the pole(s) of H_{C1}.

Your task is to design a **causal** DT system H_{D1} to approximate the behavior of H_{C1}.



Let $x_D[n] = x_C(nT)$ and $y_D[n] = y_C(nT)$ where T is a constant that represents the time between samples. Then approximate the derivative as

$$\frac{dy_C(t)}{dt} = \frac{y_C(t+T) - y_C(t)}{T}\,.$$

b. Determine an expression for the pole(s) of H_{D1}.

c. Determine the range of values of T for which H_{D1} is stable.

Now consider a second-order **causal** CT system H_{C2}, which is described by

$$\ddot{y}_C(t) + 100y_C(t) = x_C(t)\,.$$

d. Determine the pole(s) of H_{C2}.

Design a **causal** DT system H_{D2} to approximate the behavior of H_{C2}. Approximate derivatives as before:

$$\dot{y_C}(t) = \frac{dy_C(t)}{dt} = \frac{y_C(t+T) - y_C(t)}{T} \quad \text{and}$$

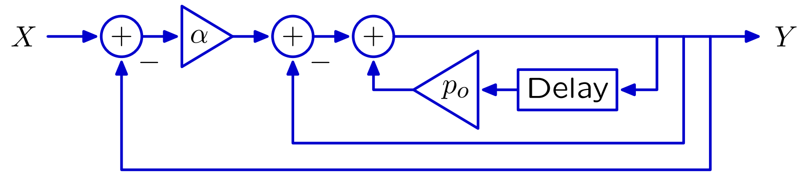
$$\frac{d^2y_C(t)}{dt^2} = \frac{\dot{y_C}(t+T) - \dot{y_C}(t)}{T}\,.$$

e. Determine an expression for the pole(s) of H_{D2}.

f. Determine the range of values of T for which H_{D2} stable.

7. Feedback

Consider the system defined by the following block diagram.



a. Determine the system functional $\frac{Y}{X}$.

b. Determine the number of closed-loop poles.

c. Determine the range of gains (α) for which the closed-loop system is stable.

8. Finding a system

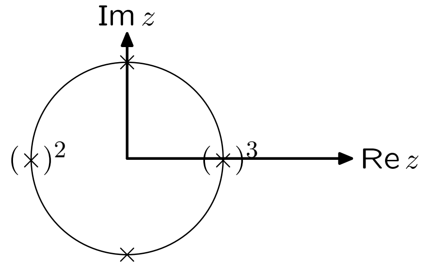
a. Determine the difference equation and block diagram representations for a system whose output is $10, 1, 1, 1, 1, \ldots$ when the input is $1, 1, 1, 1, 1, \ldots$.

b. Determine the difference equation and block diagram representations for a system whose output is $1, 1, 1, 1, 1, \ldots$ when the input is $10, 1, 1, 1, 1, \ldots$.

c. Compare the difference equations in parts a and b. Compare the block diagrams in parts a and b.

9. Lots of poles

All of the poles of a system fall on the unit circle, as shown in the following plot, where the '2' and '3' means that the adjacent pole, marked with parentheses, is a repeated pole of order 2 or 3 respectively.



Which of the following choices represents the order of growth of this system's unit-sample response for large n? Give the letter of your choice plus the information requested.

a. $y[n]$ is periodic. If you choose this option, determine the period.

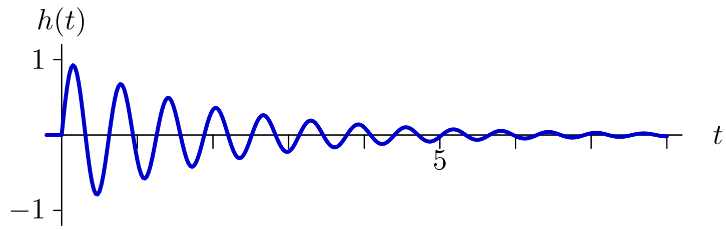
b. $y[n] \sim An^k$ (where A is a constant). If you choose this option, determine k.

c. $y[n] \sim Az^n$ (where A is a constant). If you choose this option, determine z.

d. None of the above. If you choose this option, determine a closed-form asymptotic expression for $y[n]$.

10. Relation between time and frequency responses

The impulse response of an LTI system is shown below.



If the input to the system is an eternal cosine, i.e., $x(t) = \cos(\omega t)$, then the output will have the form

$$y(t) = C\cos(\omega t + \phi)$$

a. Determine ω_m, the frequency ω for which the constant C is greatest. What is the value of C when $\omega = \omega_m$?

b. Determine ω_p, the frequency ω for which the phase angle ϕ is $-\frac{\pi}{4}$. What is the value of C when $\omega = \omega_p$?