

# 6.003 Homework #11 Solutions

## Engineering Design Problem

### 1. Image reconstruction

The rows and/or columns of the following images have been blurred. Figure out a way to sharpen each image, and identify the building. Here are thumbnails of the images:



a1



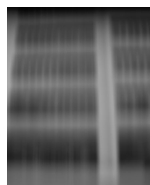
a2



b1



b2



c1



c2

The images are available in machine-readable form (buildings.zip) on the 6.003 website.

In general, blurring could be caused by many different sorts of transformations. We can start by making the simplest 6.003 assumption: that the blurring results from a system with a single pole. Such a system would have a system function of the form

$$H_{\text{blur}}(z) = \frac{1 - p}{1 - pz^{-1}}$$

where  $|p| < 1$  if the system is stable. Such a system would blur the image if it were a lowpass filter, i.e.,  $0 < p < 1$ .

If the image were blurred by a system with a single pole, then it could be deblurred with a system with a single zero:

$$H_{\text{deblur}}(z) = \frac{1 - pz^{-1}}{1 - p}$$

with the corresponding difference equation

$$y[n] = \frac{x[n] - px[n-1]}{1 - p}$$

The blurring appears to be mostly horizontal in the first row of pictures. Thus we should try passing rows of pixels through the deblurring difference equation. We could process the

pixels left to right (i.e., “causally”) or right to left (“anti-causally”). A similar procedure could be used for columns of pixels in the middle row.

Answers:

The first row of pictures were blurred by passing each row of pixels through a system with the pole at 0.985 processing from left to right. The resulting pictures can be unblurred by passing each row through a system with a zero at 0.985 processing from left to right.

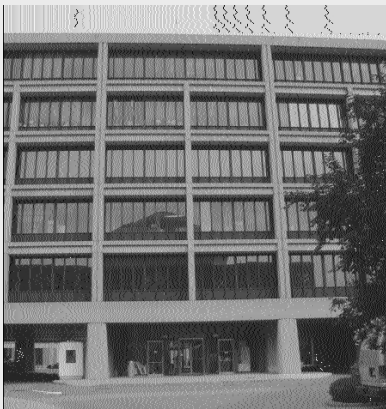
The second row of pictures were blurred by passing each column of pixels through a system with the pole at 0.985 processing from bottom to top. The resulting pictures can be unblurred by passing each column through a system with a zero at 0.985 processing from bottom to top.

The third row of pictures were blurred by passing each column of pixels through a system with the pole at 0.985 processing from top to bottom. The resulting pictures can be unblurred by passing each column through a system with a zero at 0.985 processing from top to bottom.

Results after unblurring are shown below.



a1: biology building



a2: Building 39



b1: Medical Center



b2: Building 13



c1: Building 9



c2: Hayden