# 6.003 (Fall 2011)

# Final Examination *December 19, 2011*

**Name:**

**Kerberos Username:**

**Please circle your section number:**

| *Section* | *Time* |
|---|---|
| 2 | 11 am |
| 3 | 1 pm |
| 4 | 2 pm |

**Grades will be determined by the correctness of your answers (explanations are not required).**

**Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.**

You have **three hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use four 8.5 × 11 sheets of paper (eight sides total).

No calculators, computers, cell phones, music players, or other aids.

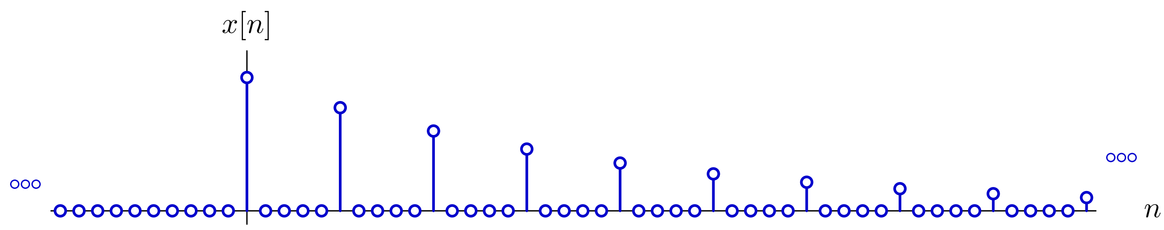
| | |
|---|---|
| 1 | /14 |
| 2 | /14 |
| 3 | /12 |
| 4 | /16 |
| 5 | /16 |
| 6 | /14 |
| 7 | /14 |
| Total | /100 |

## 1. Z Transform *[14 points]*

Determine $X(z)$, the Z transform of $x[n]$, where

$$x[n] \;=\; \sum_{k=0}^{\infty} a^k \delta[n-5k] \;=\; \delta[n] + a\delta[n-5] + a^2\delta[n-10] + a^3\delta[n-15] + \cdots$$

is plotted below.

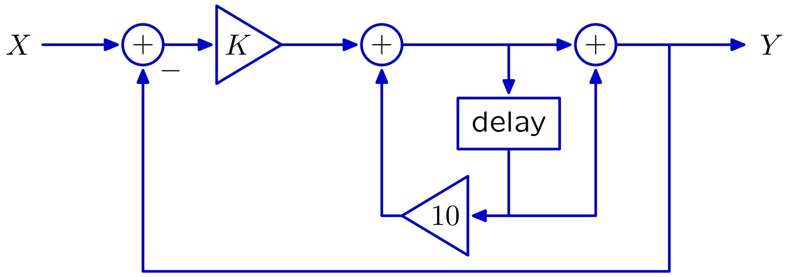



Enter a closed-form expression for $X(z)$ in the box below.

$X(z) =$ [ ]

## 2. DT Stability *[14 points]*

Determine the range of $K$ for which the following discrete-time system is stable (and causal).




range of $K$:

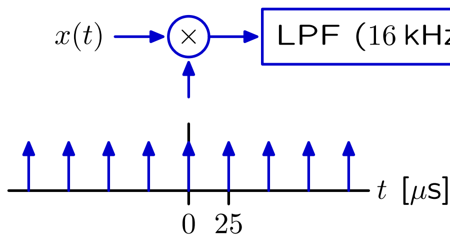
## 3. Harmonic Aliasing *[12 points]*

Let $x(t)$ represent a periodic signal with the following harmonics:

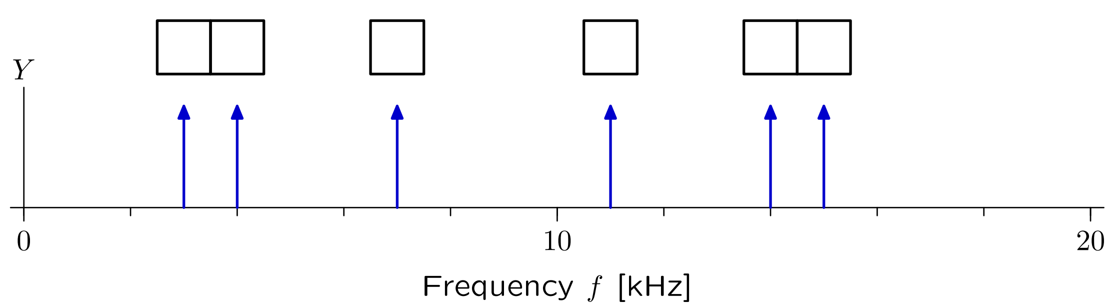
| harmonic number | frequency [kHz] |
|---|---|
| 1 | 11 |
| 2 | 22 |
| 3 | 33 |
| 4 | 44 |
| 5 | 55 |
| 6 | 66 |
| 7 | 77 |

Throughout this problem, frequencies ($f$) are expressed in cycles per second (Hz), which are related to corresponding radian frequencies ($\omega$) by $f = \frac{\omega}{2\pi}$.

The signal $x(t)$ is multiplied by an infinite train of impulses separated by $25 \times 10^{-6}$ seconds, and the result is passed through an ideal lowpass filter with a cutoff frequency of 16 kHz.

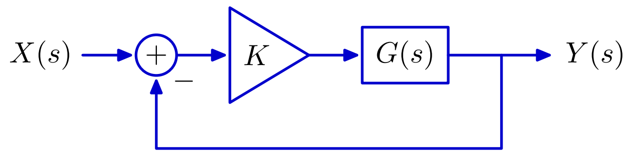



The plot below shows the Fourier transform $Y$ of the output signal, for frequencies between 0 and 20 kHz. Write the number of the harmonic of $x(t)$ that produced each component of $Y$ in the box above that component. If none of 1-7 could have produced this frequency, enter **X**.

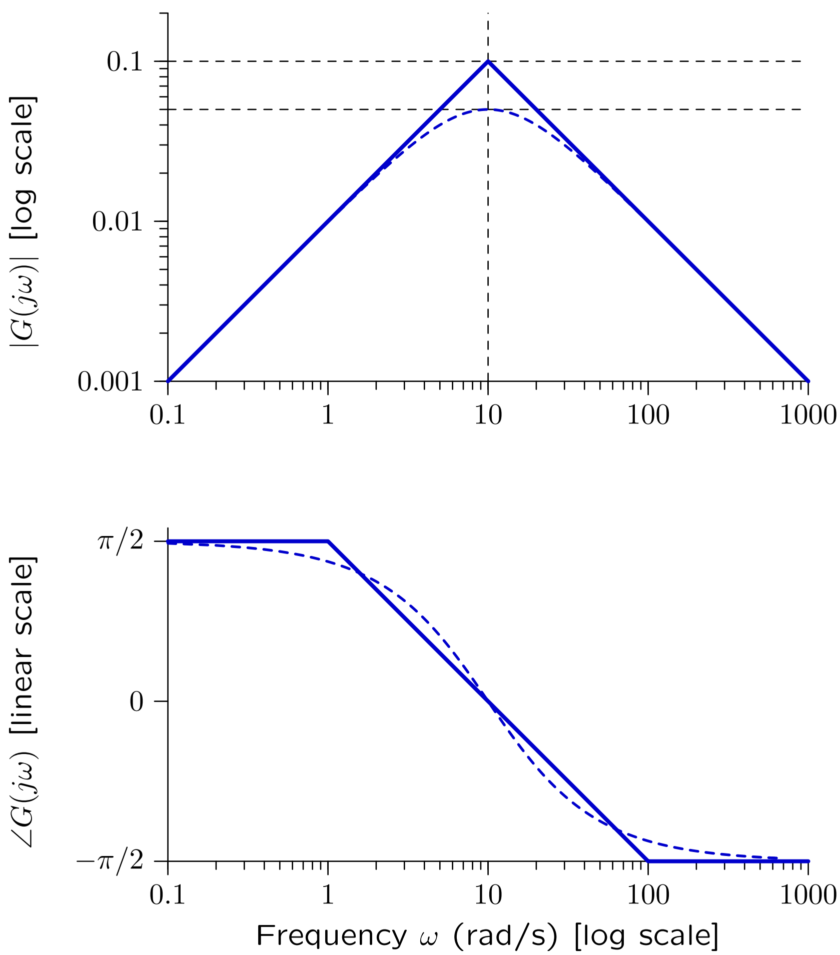


## 4. Feedback *[16 points]*

Let $H(s) = \dfrac{Y(s)}{X(s)}$ represent the system function of the following feedback system



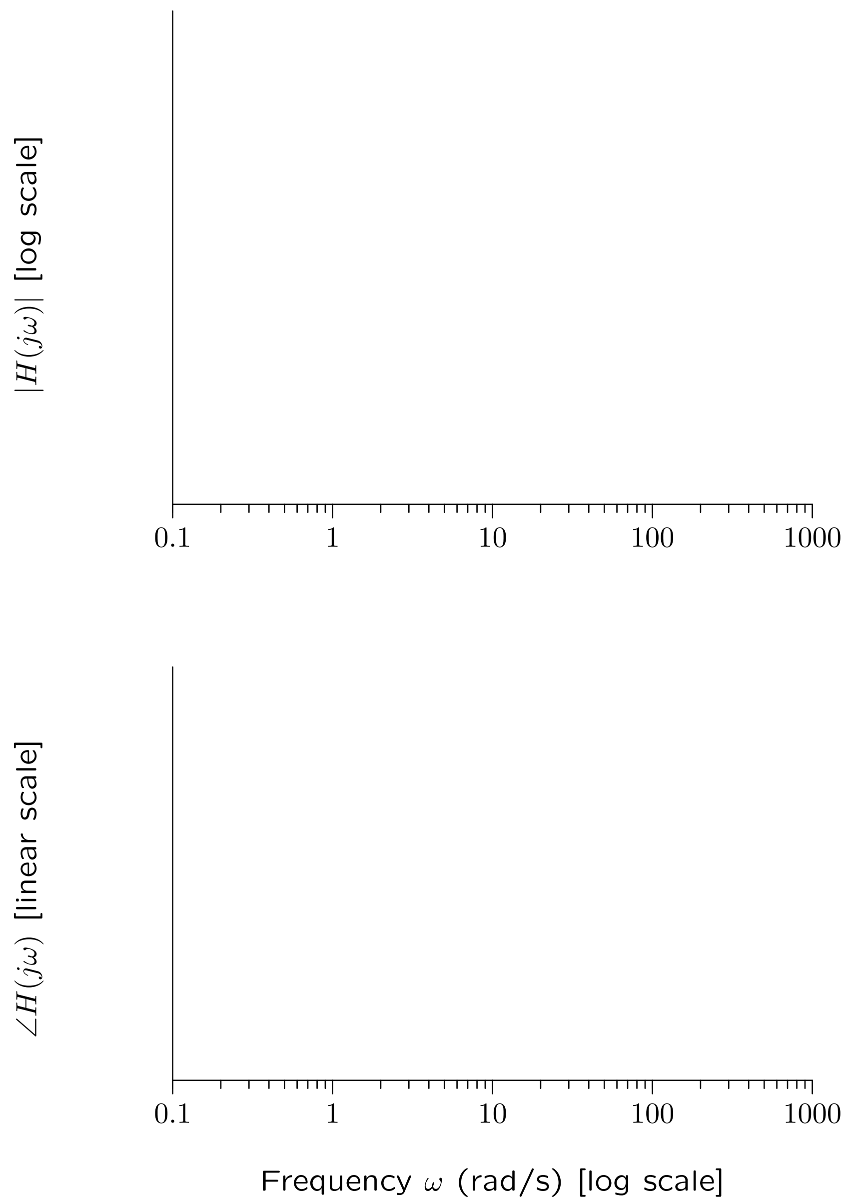

where $G(s)$ represents a linear, time-invariant system. The frequency response of $G(s)$ is given by the following Bode plots (magnitude and frequency plotted on log scales).



**Part a.** Determine a closed-form expression for $g(t)$, the impulse response of $G(s)$.

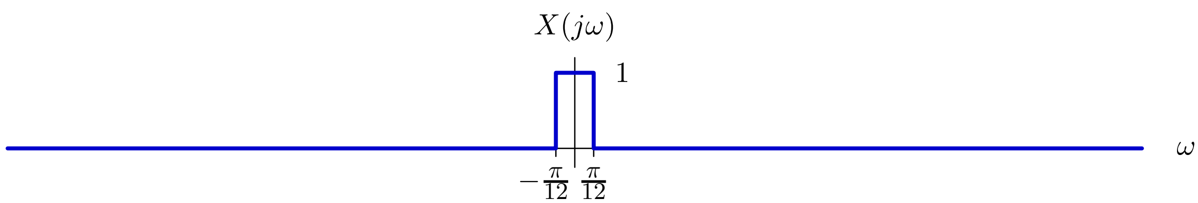
$g(t) =$

**Part b.** Sketch straight-line approximations (Bode plots) for the magnitude (log scale) and angle (linear scale) of $H(j\omega)$ when $K = 81$.
**Clearly label all important magnitudes, angles, and frequencies.**



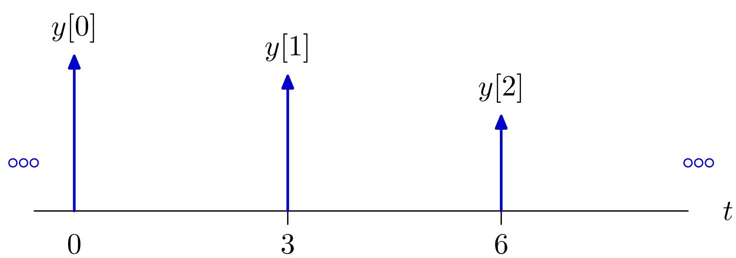
## 5. Triple reconstruction *[16 points]*

A CT signal $x(t)$ is sampled to produce a DT signal $y[n] = x(3n)$. The Fourier transform of $x(t)$ is given below.




We wish to compare two methods of using $y[n]$ to reconstruct approximations to $x(t)$.

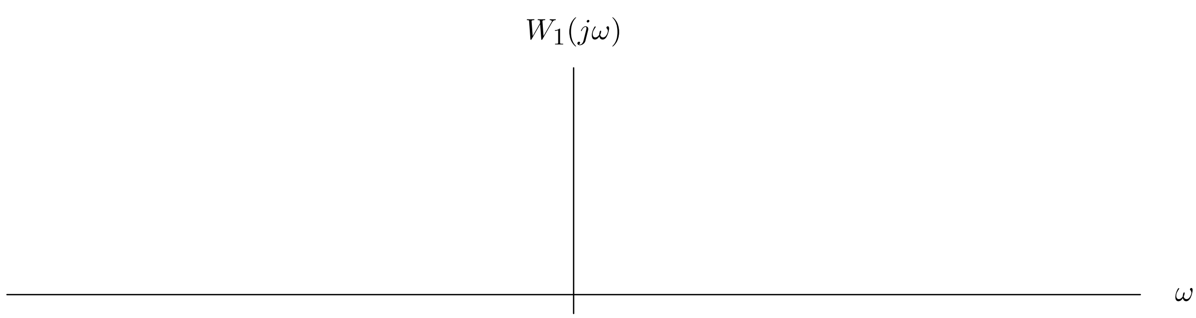
**Part a.** Let $w_1(t)$ represent a signal in which each sample of $y[n]$ is replaced by an impulse of area $y[n]$ located at $t = 3n$. Thus $w_1(t)$ has the following form



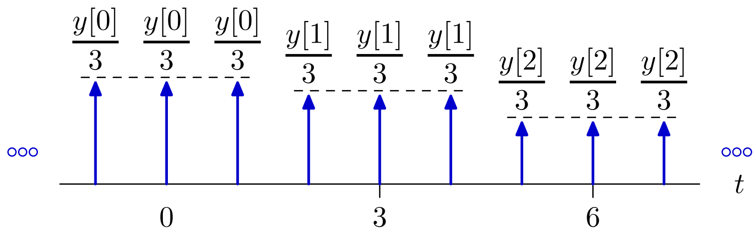

which can be represented mathematically as

$$w_1(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - 3n)\,.$$

Sketch the Fourier transform of $w_1(t)$ on the axes below.
**Label all important features.**



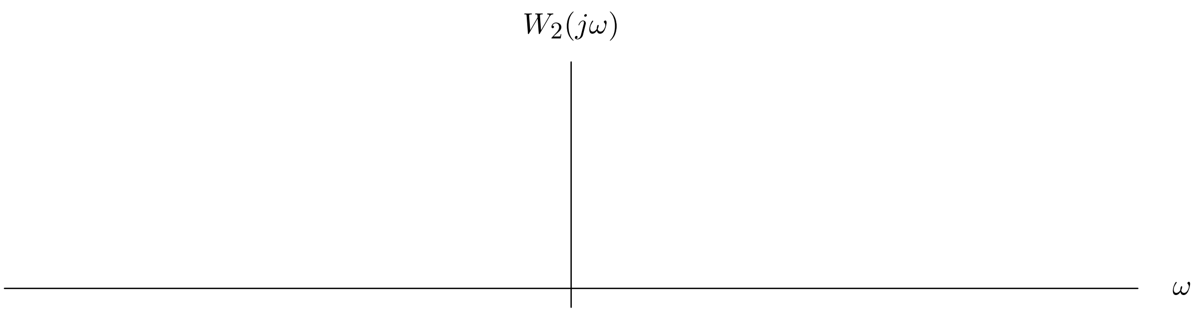
**Part b.** Let $w_2(t)$ represent a signal in which each sample of $y[n]$ is replaced by three impulses (one at $t = 3n-1$, one at $t = 3n$, and one at $t = 3n+1$), each with area $y[n]/3$. Thus $w_2(t)$ has the following form




which can be represented mathematically as

$$w_2(t) = \frac{1}{3} \sum_{n=-\infty}^{\infty} y[n] \Big(\delta(t-3n-1) + \delta(t-3n) + \delta(t-3n+1)\Big) .$$

Sketch the Fourier transform of $w_2(t)$ on the axes below.
**Label all important frequencies as well as the value of $W_2(j\omega)$ at $\omega = 0$.**

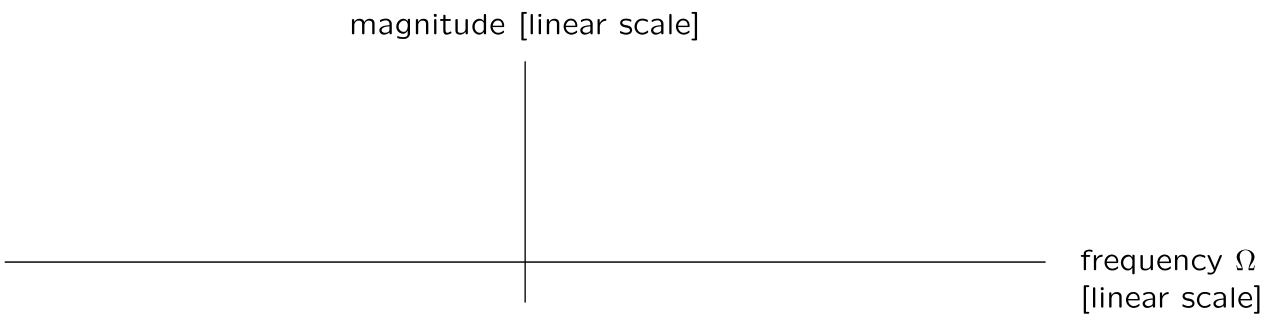


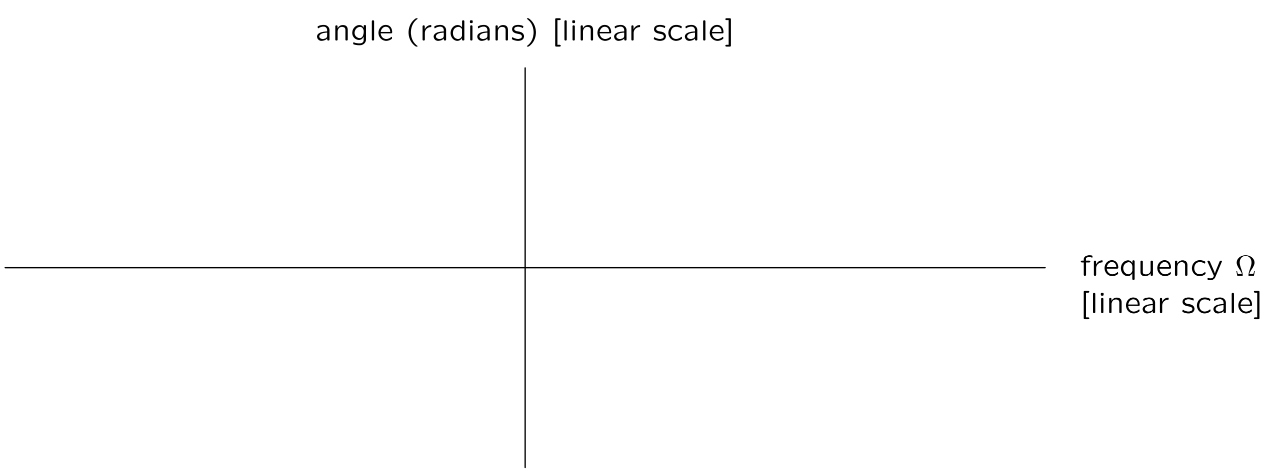
## 6. DT Filtering *[14 points]*

Sketch the magnitude and angle of the frequency response of a linear, time-invariant system with the following unit-sample response:

$$h[n] = \delta[n] - \delta[n-3]\,.$$

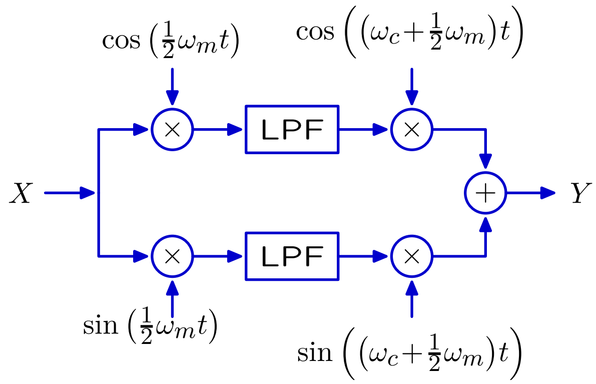
**Label all important magnitudes, angles, and frequencies.** All scales are linear.



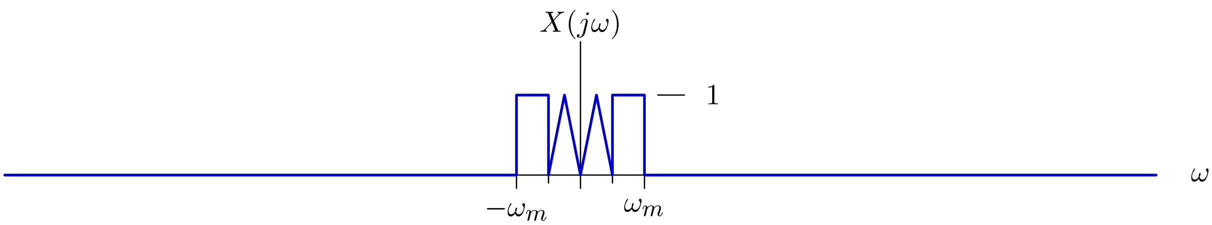



## 7. Bandwidth Conservation *[14 points]*

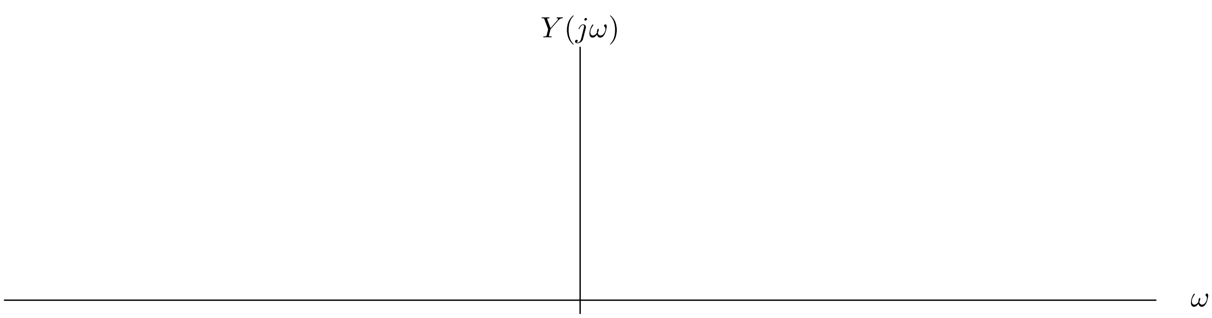
Consider the following modulation scheme, where $\omega_c >> \omega_m$.




Assume that each lowpass filter (LPF) is ideal, with cutoff frequency $\omega_m/2$. Also assume that the input signal has the following Fourier transform.




Sketch $Y(j\omega)$ on the following axes.
**Label all important magnitudes and frequencies.**



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