6.003 (Fall 2009)

Quiz #3 *November 18, 2009*

Name:

Kerberos Username:

Please circle your section number:

Section	*Instructor*	*Time*
1	Marc Baldo	10 am
2	Marc Baldo	11 am
3	Elfar Adalsteinsson	1 pm
4	Elfar Adalsteinsson	2 pm

Partial credit will be given for answers that demonstrate some but not all of the important conceptual issues.

Explanations are not required and will not affect your grade.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use three 8.5×11 sheets of paper (six sides total).

No calculators, computers, cell phones, music players, or other aids.

1	/12
2	/20
3	/18
4	/25
5	/25
Total	/100

1. Impulsive Input *[12 points]*

Let the following periodic signal

$$x(t) = \sum_{m=-\infty}^{\infty} \delta(t-3m) + \delta(t-1-3m) - \delta(t-2-3m)$$

be the input to an LTI system with system function

$$H(s) = e^{s/4} - e^{-s/4}\,.$$

Let b_k represent the Fourier series coefficients of the resulting output signal $y(t)$. Determine b_3.

$b_3 =$

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2. System Design *[20 points]*

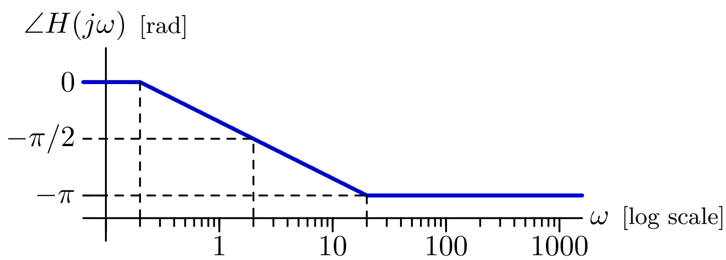
Design a stable CT LTI system H with **all** of the following three properties:

- the impulse response $h(t)$ has the form

$$h(t) = C\delta(t) + De^{-2t}u(t)$$

  where C and D are real-valued constants,

- the angle of $H(j\omega)$ has the following straight-line approximation



- if the input $x(t)$ is 1 for all time, then the output $y(t)$ is 1 for all time.

Determine the system function $H(s)$ that is consistent with these design specifications. If no such a system exists, enter **none**.

$H(s) =$ [ ]

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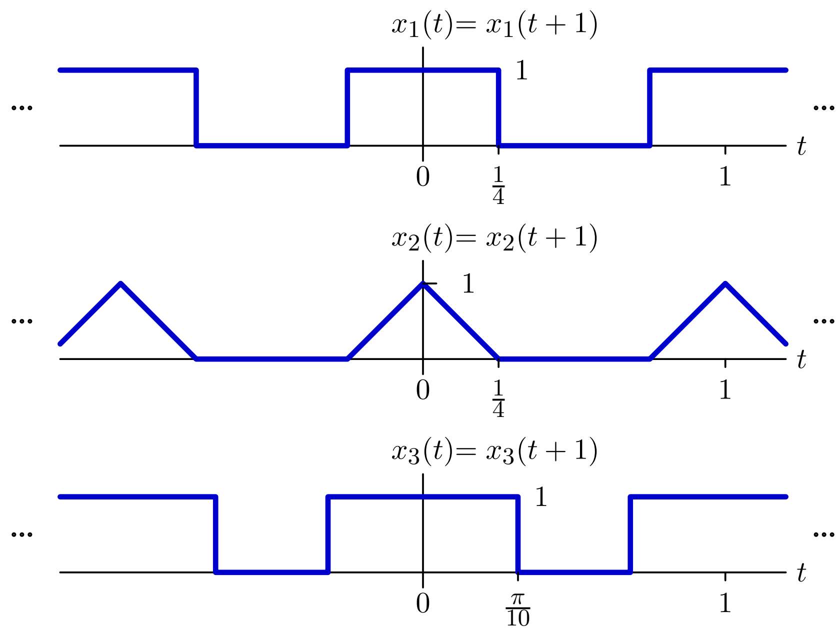
3. Input/Output Pairs *[18 points]*

The following signals are all periodic with period $T = 1$.



Indicate which of the systems on the next page could/could not be linear and time-invariant.

Grading: +3 for each correct answer; −3 for each incorrect answer; 0 for blank or **?**.

$x_1(t)$ → System #1 → $x_2(t)$ System #1 could be LTI? (**yes/no**): ☐

$x_1(t)$ → System #2 → $x_3(t)$ System #2 could be LTI? (**yes/no**): ☐

$x_2(t)$ → System #3 → $x_1(t)$ System #3 could be LTI? (**yes/no**): ☐

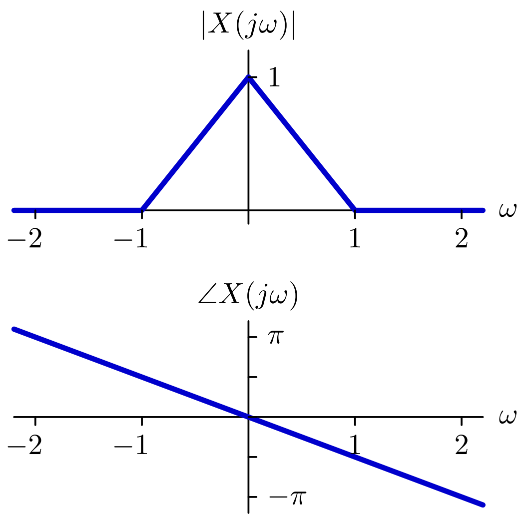
$x_2(t)$ → System #4 → $x_3(t)$ System #4 could be LTI? (**yes/no**): ☐

$x_3(t)$ → System #5 → $x_1(t)$ System #5 could be LTI? (**yes/no**): ☐

$x_3(t)$ → System #6 → $x_2(t)$ System #6 could be LTI? (**yes/no**): ☐

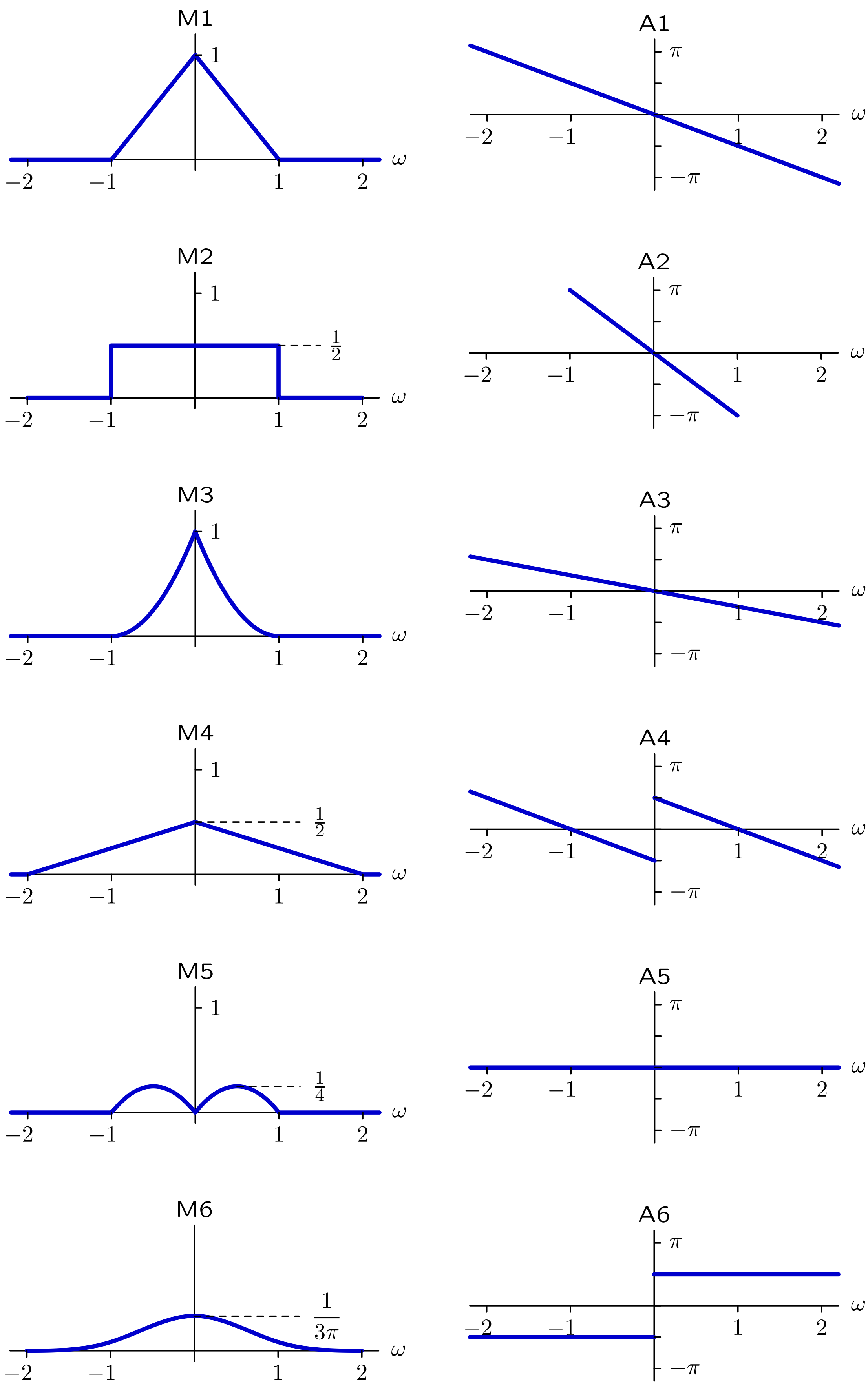
4. Fourier Transforms *[25 points]*

The magnitude and angle of the Fourier transform of a signal $x(t)$ are given in the following plots.



Five signals are derived from $x(t)$ as shown in the left column of the following table. Six magnitude plots (M1-M6) and six angle plots (A1-A6) are shown on the next page. Determine which of these plots is associated with each of the derived signals and place the appropriate label (e.g., M1 or A3) in the following table. Note that more than one derived signal could have the same magnitude or angle.

signal	magnitude	angle
$\frac{dx(t)}{dt}$		
$(x * x)(t)$		
$x\left(t - \frac{\pi}{2}\right)$		
$x(2t)$		
$x^2(t)$		

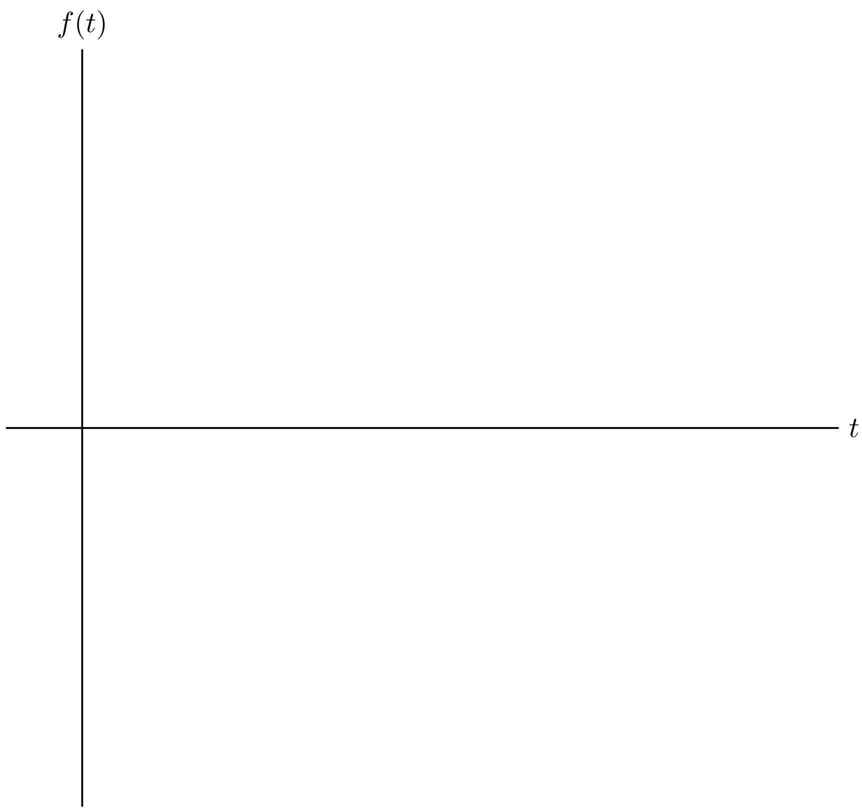

M1
A1
M2
A2
M3
A3
M4
A4
M5
A5
M6
A6
1
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{3\pi}$
π
$-\pi$
ω
-2
-1
1
2

5. Feedback and Control *[25 points]*

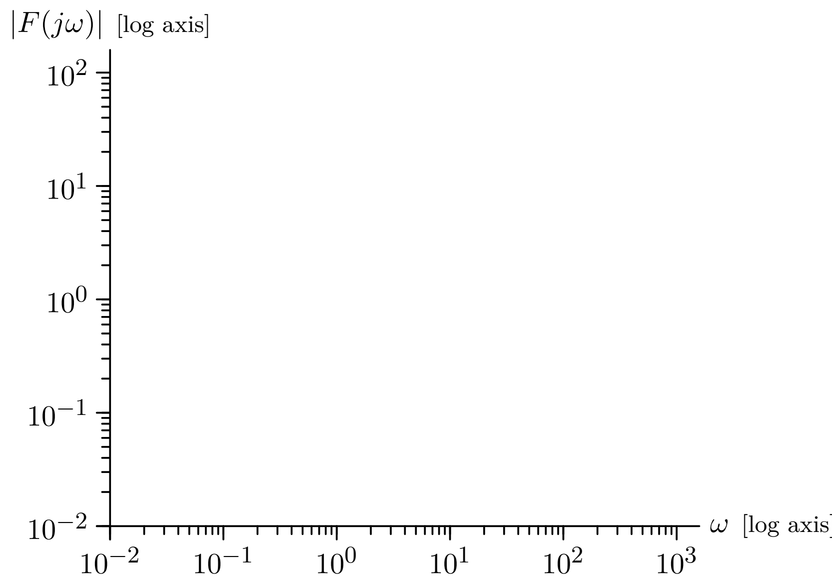
Consider a causal LTI system described by $F(s)$ as follows:

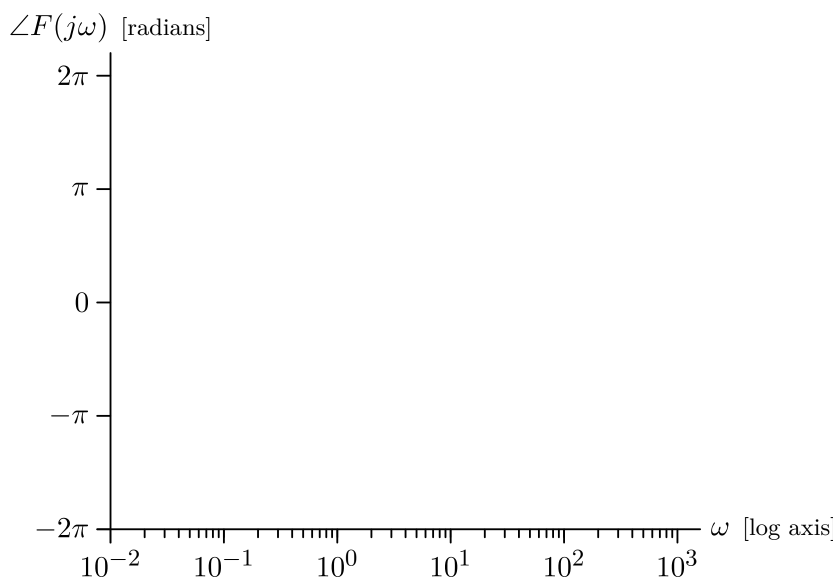
$$F(s) = \frac{s^2 + 2s + 100}{s^2}\,.$$

a. Sketch the impulse response $f(t)$ for this system on the axes below. Label the axes and indicate the important features of your plot.

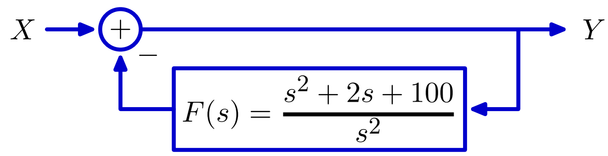


b. Sketch the magnitude and angle of $F(j\omega)$ on the following axes. Notice the log axes for ω and for the magnitude. Indicate the important features of your plots, including extreme values.

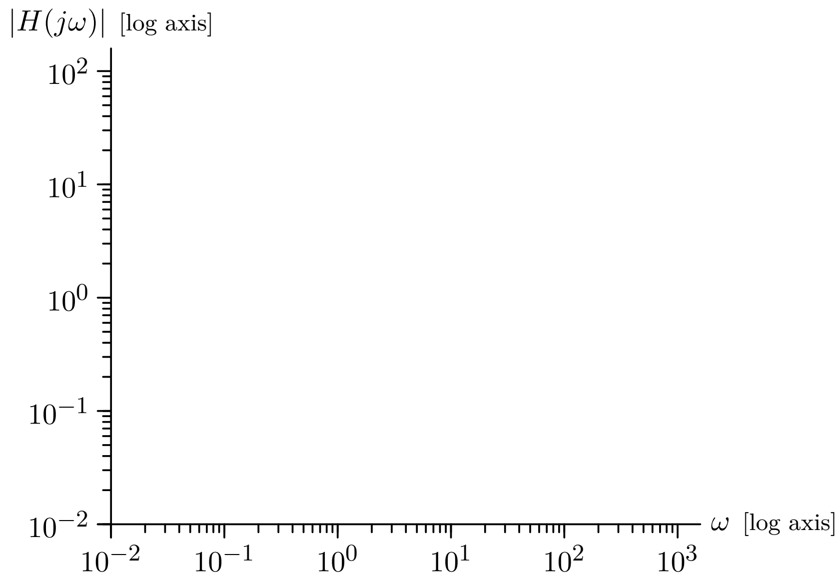


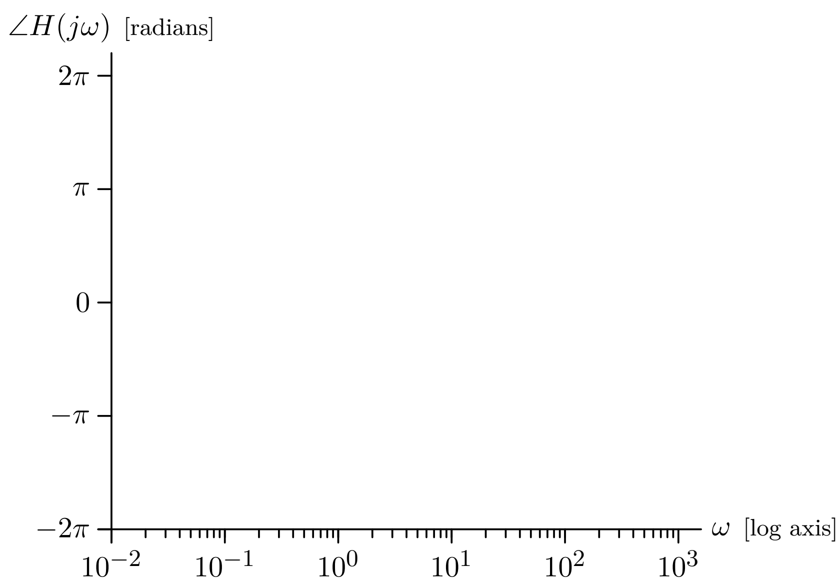


Now consider a feedback system containing $F(s)$ as follows.



c. Let $H(s) = \frac{Y(s)}{X(s)}$ represent the closed-loop system function. Sketch the magnitude and angle of $H(s)$ on the following axes. Notice the log axes for ω and for the magnitude. Indicate the important features of your plots, including extreme values.





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