# 6.003 (Fall 2009)

# Final Examination *December 17, 2009*

**Name:**

**Kerberos Username:**

**Please circle your section number:**

| *Section* | *Instructor* | *Time* |
|---|---|---|
| 1 | Marc Baldo | 10 am |
| 2 | Marc Baldo | 11 am |
| 3 | Elfar Adalsteinsson | 1 pm |
| 4 | Elfar Adalsteinsson | 2 pm |

**Partial credit will be given for answers that demonstrate some but not all of the important conceptual issues.**

**Explanations are not required and will not affect your grade.**

You have **three hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use four $8.5 \times 11$ sheets of paper (eight sides total).

No calculators, computers, cell phones, music players, or other aids.

| | |
|---|---|
| 1 | /15 |
| 2 | /10 |
| 3 | /14 |
| 4 | /14 |
| 5 | /15 |
| 6 | /16 |
| 7 | /16 |
| Total | /100 |

## 1. CT System with Feedback *[15 points]*

Let $G$ represent a causal system that is described by the following differential equation

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t)$$

where $x(t)$ represents the input signal and $y(t)$ represents the output signal.

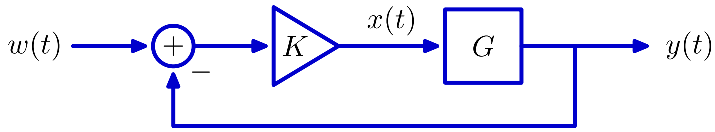
a. Determine the output $y_1(t)$ of $G$ when the input is

$$x_1(t) = \begin{cases} e^{-t}\,; & t \geq 0 \\ 0\,; & \text{otherwise} \end{cases}$$

Enter your result in the box below.

$y_1(t) =$ [ ]

Now consider a feedback loop that contains the $G$ system described on the previous page.[1]




b. Determine a differential equation that relates $w(t)$ to $y(t)$ when $K = 10$. The differential equation should not contain references to $x(t)$.

Enter the differential equation in the box below.

[1] The minus sign near the adder indicates that the output of the adder is $w(t) - y(t)$

c. Determine the values of $K$ for which the feedback system on the previous page is stable. Enter the range (or ranges) in the box below.

## 2. Stepping Up and Down *[10 points]*

Use a small number of delays, gains, and 2-input adders (and no other types of elements) to implement a system whose response (starting at rest) to a **unit-step** signal

$$x[n] = u[n] = \begin{cases} 1 & n \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

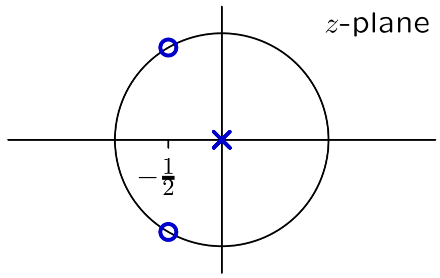
is

$$y[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0, 3, 6, 9, \ldots \\ 2 & n = 1, 4, 7, 10, \ldots \\ 3 & n = 2, 5, 8, 11, \ldots \end{cases}$$

Draw a block diagram of your system below.

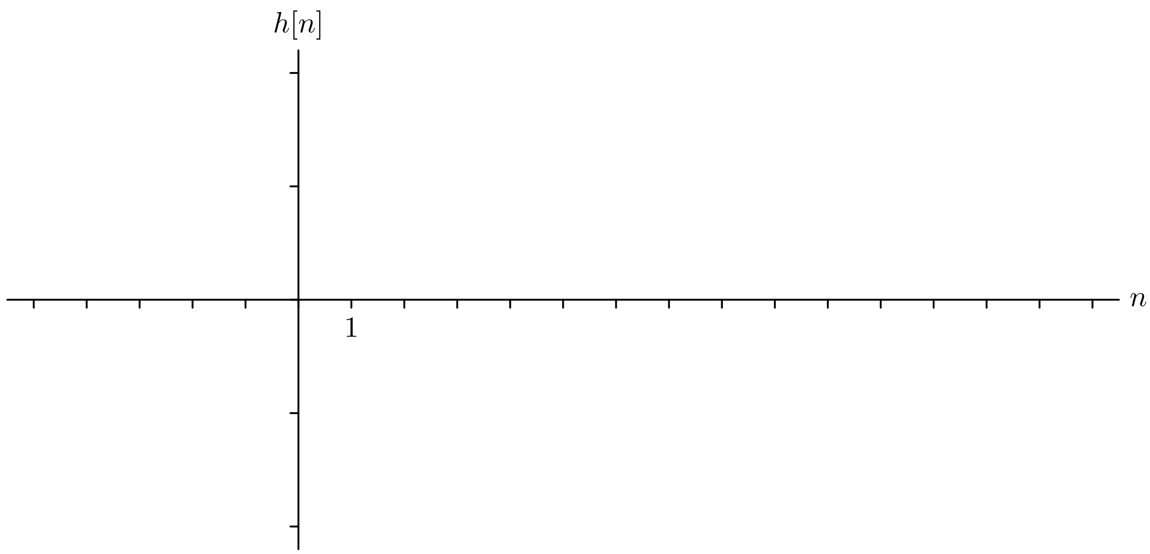
## 3. DT systems *[14 points]*

The pole-zero diagram for a DT system is shown below, where the circle has radius 1.

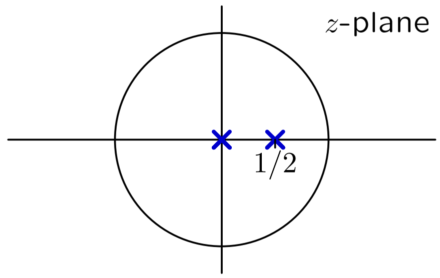



It is known that when the input is 1 for all $n$, the output is also 1 for all $n$.

Sketch the unit-sample response $h[n]$ of the system on the axes below. Label the important features of your sketch.

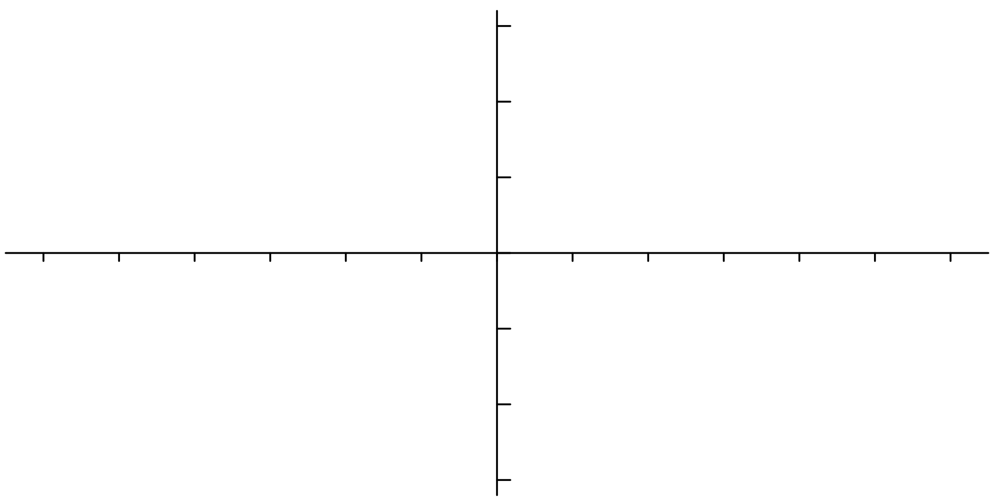


A second DT system has the following pole-zero diagram:

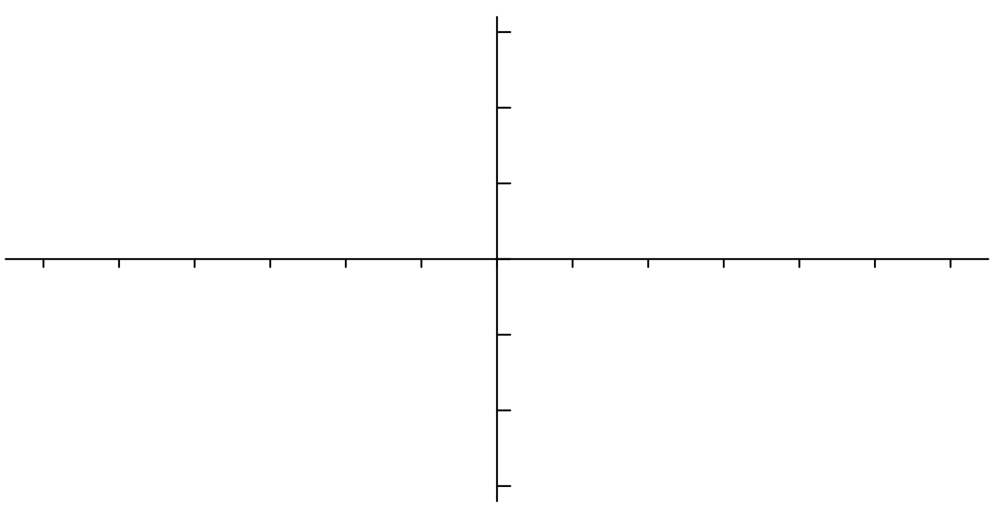



It is known that the system function $H(z)$ is 1 when $z = 1$.

Sketch the magnitude of the frequency response of this system on the axes below. Label the important features of your sketch (including the axes).

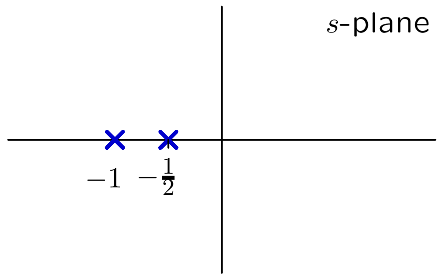


Sketch the angle of the frequency response of this system on the axes below. Label the important features of your sketch (including the axes).



## 4. CT Systems *[14 points]*

A causal CT system has the following pole-zero diagram:



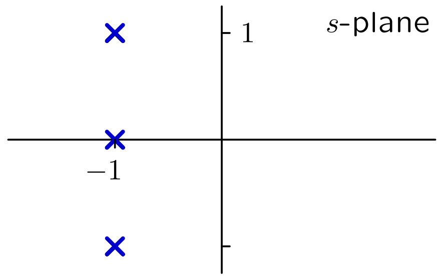

Let $y(t) = s(t)$ represent the response of this system to a unit-step signal

$$x(t) = u(t) = \begin{cases} 1\,; & t \geq 0 \\ 0\,; & \text{otherwise}\,. \end{cases}$$

Assume that the unit-step response $s(t)$ of this system is known to approach 1 as $t \to \infty$. Determine $y(t) = s(t)$ and enter it in the box below.

$y(t) =$ [ ]

A second CT system has the following pole-zero diagram:




Assume that the input signal is

$$x(t) = \begin{cases} 1\,; & \cos t > \frac{1}{\pi} \\ 0\,; & \text{otherwise}\,. \end{cases}$$

Let $a_k$ and $b_k$ represent the Fourier series coefficients of the input and output signals, respectively, where the fundamental (lowest frequency component) of each signal has a period of $2\pi$.

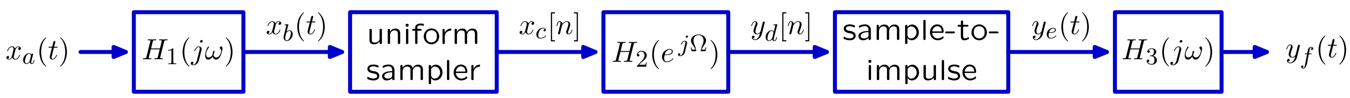
It is known that $\dfrac{b_0}{a_0} = 1$. Determine $\dfrac{b_1}{a_1}$.

$\dfrac{b_1}{a_1} =$ 

| |
|---|
| |

## 5. DT processing of CT signals *[15 points]*

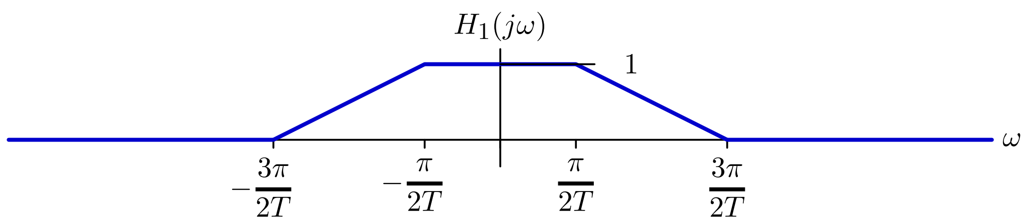
Consider the following system for DT processing of CT signals:

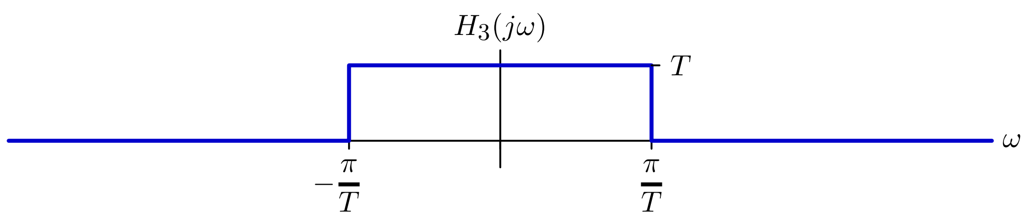



where $x_c[n] = x_b(nT)$ and

$$y_e(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t - nT)\,.$$

The frequency responses $H_1(j\omega)$ and $H_3(j\omega)$ are given below.




a. Assume in this part that $H_2(e^{j\Omega}) = 1$ for all frequencies $\Omega$. Determine $y_f(t)$ when

$$x_a(t) = \cos\left(\frac{\pi}{2T}t\right) + \sin\left(\frac{5\pi}{4T}t\right)\,.$$

$y_f(t) =$

b. For this part, assume that

$$H_2(e^{j\Omega}) = \begin{cases} 1\,; & |\Omega| < \Omega_c \\ 0\,; & \Omega_c < |\Omega| < \pi\,. \end{cases}$$

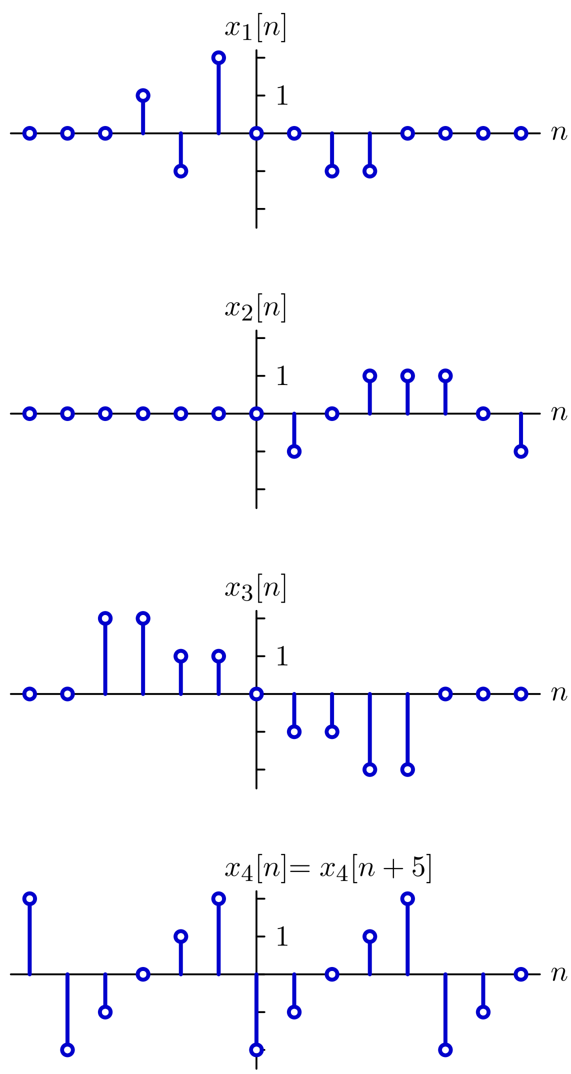
For what values of $\Omega_c$ is the overall system from $x_a(t)$ to $y_f(t)$ linear and time-invariant?

values of $\Omega_c$: [ ]

## 6. Which are True? *[16 points]*

For each of the DT signals $x_1[n]$ through $x_4[n]$ (below), determine whether the conditions listed in the following table are satisfied, and answer **T** for true or **F** for false.

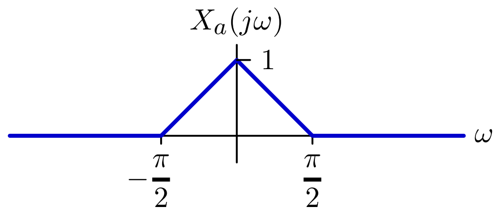
| | $x_1[n]$ | $x_2[n]$ | $x_3[n]$ | $x_4[n]$ |
|---|---|---|---|---|
| $X(e^{j0}) = 0$ | | | | |
| $\int_{-\pi}^{\pi} X(e^{j\Omega})\, d\Omega = 0$ | | | | |
| $X(e^{j\Omega})$ is purely imaginary | | | | |
| $e^{jk\Omega}X(e^{j\Omega})$ is purely real for some integer $k$ | | | | |



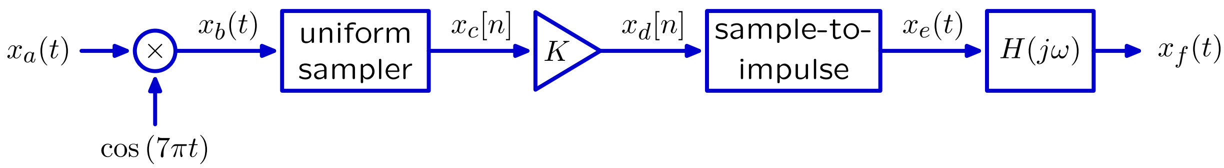
*Worksheet (intentionally blank)*

## 7. Multiplied Sampling *[16 points]*

The Fourier transform of a signal $x_a(t)$ is given below.




This signal passes through the following system



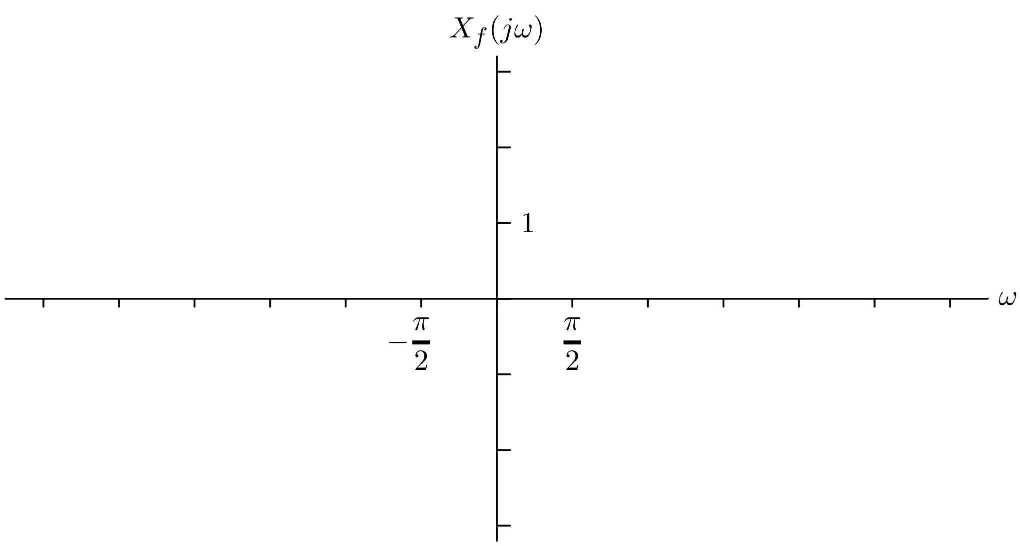

where $x_c[n] = x_b(nT)$ and

$$x_e(t) = \sum_{n=-\infty}^{\infty} x_d[n]\delta(t - nT)$$

and

$$H(j\omega) = \begin{cases} T & \text{if } |\omega| < \frac{\pi}{T} \\ 0 & \text{otherwise}. \end{cases}$$

a. Sketch the Fourier transform of $x_f(t)$ for the case when $K = 1$ and $T = 1$. Label the important features of your plot.



b. Is it possible to adjust $T$ and $K$ so that $x_f(t) = x_a(t)$?

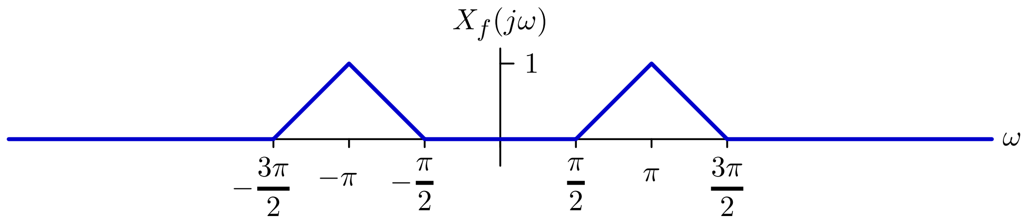
**yes** or **no**:

If yes, specify a value $T$ and the corresponding value of $K$ (there may be multiple solutions, you need only specify one of them).

$T =$ $K =$

If no, briefly explain why not.

c. Is it possible to adjust $T$ and $K$ so that the Fourier transform of $x_f(t)$ is equal to the following, and is zero outside the indicated range?




**yes** or **no**:

If yes, specify all possible pairs of $T$ and $K$ that work in the table below. If there are more rows in the table than are needed, leave the remaining entries blank.

| $T$ | $K$ |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

If no, briefly explain why not.

*Worksheet (intentionally blank)*

*Worksheet (intentionally blank)*

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