Lecture 3: Q&A

Volume of the truncated pyramid

How to integrate to figure out the volume of the pyramid?

Divide it into thin horizontal slices of height dz. Figure out $A(z)$: the area of the slice as a function of its height z (and in terms of a, b, and h). Then the slice has volume $A(z)\,dz$. Now integrate in the range $z = 0 \ldots h$, and you have the volume of the truncated pyramid.

Why are there no fractional powers of a or b in the volume?

I think the following (somewhat) easy case shows that there cannot be fractional powers. Imagine a is just slightly smaller than b. So $a = b - \epsilon$, where ϵ is tiny. The shape is almost a rectangular prism. Most of the deviation is four triangular slabs, one for each side, and you can approximate the volume of each slab, and adjust the volume of a rectangular prism to approximate the volume of the shape with $a = b - \epsilon$.

General

Not convinced about the robustness of easy cases.

True: The method doesn't guarantee correct answers. But it often gives you the correct answer in all cases (even beyond the easy cases). Even if it doesn't, it gives you an answer for some cases, and those cases may be enough for your purposes. Perhaps most important, the method helps you get 'unstuck' and start thinking about a problem, and those thoughts may lead to a full solution.

Where did I learn this if it is not taught in most classes?

From my PhD projects and from a course that I TA'ed as a graduate student. As part of my PhD, I made an approximate model of how the chemical reactions in retinal rod cells compute the logarithm of light intensity. And I made an approximate model for estimating the density of prime numbers. And, as graduate student, I TA'ed a wonderful course, 'Order of magnitude physics', and have thought about those ideas for many years, now teaching my own version (6.055/2.038).

How hard should I try to find a question to put on the sheet?

If something puzzles you, put it down. But no need to force yourself.

I'm not a fan of 'talk to your neighbor'. It just generates a noisy signal, and then I have to unlearn things.

Talking to your neighbor shares a purpose with easy cases: It gets you unstuck and thinking about the problem, formulating ideas, and finding out what parts are confusing. And, being wrong is *good*. It means that you're learning. Imagine being right all the time. You'd die knowing as much as you were born knowing – not a very interesting life.

Can we use these methods for hard problems where it is harder to check the answer?

Yes! Hans Bethe used a variant of discretization for the first solution to a hard problem in quantum electrodynamics (predicting the so-called Lamb shift). Feynman, who worked with Bethe, then figured out a more complete solution. But it was Bethe's discretization approach that opened the door to the later, more accurate approaches.

How do I know when an approximation is good enough?

Hard to say. It depends on the purpose. If no solution exists at all, then a crude approximation may be very useful. Or if it helps you to think about a problem, and get unstuck, an approximation is useful just for that reason. Generally, in the early stages of a project, approximations are more important than exact solutions; and in the later stages, the more accurate solutions become more useful.

Should we cite assistance on the homework?

Yes, just like in a scientific paper – give credit to others (it won't detract from your grade). e.g. If you google something, note that down. If you get a key idea from a colleague, give credit to him or her.