18.06 Professor Strang Quiz 2 April 12, 1999

Your name is: ______________________________

Grading 1 2 3 4

Please circle your recitation:

1)	Mon	2–3	2-131	S. Kleiman
2)	Mon	3–4	2-131	S. Hollander
3)	Tues	11–12	2-132	S. Howson
4)	Tues	12–1	2-132	S. Howson
5)	Tues	12–1	2-131	S. Kleiman
6)	Tues	1–2	2-131	S. Kleiman
7)	Tues	2–3	2-132	S. Howson

1 (30 pts.) (a) Compute the determinant of

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 5 & 0 \\ 1 & 3 & 9 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

(b) Find an orthogonal basis (orthonormal is even better) for the column space of A. Start from a basis and use Gram-Schmidt (and common sense).

(c) If you change the 1 in the upper left corner of A to 2, what is the change in the determinant (I would use cofactors).

2 (24 pts.) An experiment at the nine times $t = -4, -3, -2, -1, 0, 1, 2, 3, 4$ yields the consistent result $b = 0$ except at the last time ($t = 4$) we get $b = 10$. We want the best straight line $b = C + Dt$ to fit these nine data points by least squares.

(a) Write down the equations $Ax = b$ with unknowns C and D that would be solved if a straight line exactly fit the data (it doesn't).

(b) Find the best least squares value of C and D.

(c) This problem is really projecting the vector $b = (0, 0, 0, 0, 0, 0, 0, 0, 10)$ onto a certain subspace. Give a basis for that subspace and give the projection p of b onto the subspace.

3 (22 pts.) Suppose an m by n matrix Q has orthonormal columns.

(a) What is the rank of Q?

(b) Give an expression with no inverses for the projection matrix P onto the column space of Q.

(c) Check that your formula for P satisfies the two requirements for a projection matrix.

4 (24 pts.)

(a) Suppose Q is an orthogonal matrix and $Qx = \lambda x$. Compare the lengths of λx and Qx (using $(Qx)^T(Qx)$) to reach a conclusion about λ.

(b) The Hadamard matrix H has orthogonal columns:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

Project the vector $b = (1, 2, 3, 4)$ onto the line spanned by the *last* column. Then project b onto the subspace spanned by all four columns.

(c) Find the eigenvalues of $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.