18.06 Problem Set 1 Solution

HW1.

a. 1) By definition.

b. 2) $0 = v \cdot v = |v|^2$ so that $v = 0$.

c. 4) By definition.

d. True. Notice that $||w|| = ||-w||$, so this is exactly the triangle inequality.

e. 2) Let the angle between the two vectors be θ. Then $\cos\theta = \frac{|v \cdot w|}{||v|| \cdot ||w||} = \frac{1}{\sqrt{2}}$. So we know $\theta = \pi/4$.

f. 2. This is the geometric interpretation of determinant.

HW2.

a. 3). $x = y = z = 0$ does not satisfy this equation, so it does not pass through 0.

b. (2,3,4).

c. d). This is the sphere centered at 0, with radius 1.

d. (2x,2y,2z).

e. (2,3,4).

f. (2x,2y,2z).

g. This is one side of the space separated by the plane $2x + 3y + 4z = 2020$ where the origin lies in.

HW3.

a. Yes. For any two 3×3 matrices A, B, and any real numbers λ, μ, $\lambda A + \mu B$ is also a 3×3 matrix by definition.

b. No. For any such a matrix A, $0 \cdot A = 0$ is not in this set.

c. Yes. For any two 3×3 matrices $A = (A_{ij}), B = (B_{ij})$ from this set, we have $\sum_{1 \leq i,j \leq 3} A_{ij} = \sum_{1 \leq i,j \leq 3} B_{ij} = 0$. For any real numbers λ, μ, $\lambda A + \mu B = (\lambda A_{ij} + \mu B_{ij})$ is also a 3×3 matrix, and that the sum of all entries is $\sum_{1 \leq i,j \leq 3}(\lambda A_{ij} + \mu B_{ij}) = \lambda \sum_{1 \leq i,j \leq 3} A_{ij} + \mu \sum_{1 \leq i,j \leq 3} B_{ij} = 0$.

d. Yes. There is only one element in this set, which is 0. This is a vector space, as $\lambda \cdot 0 = 0$ for any real number λ.

e. Yes. For any two diagonal matrices matrices A, B, entries are 0 except for the entries on the main diagonal. For any real numbers λ, μ, entries of $\lambda A + \mu B$ are 0 except for the entries on the main diagonal, hence $\lambda A + \mu B$ is also a diagonal matrix.

HW4.

a. Yes. For any two constant functions $f(x) = c_1, g(x) = c_2$, and any real numbers λ, μ, $\lambda f(x) + \mu g(x) = \lambda c_1 + \mu c_2$ is also a constant function.

b. Yes. For any two such functions $f(x), g(x)$, and any real numbers λ, μ, $\lambda f(0) + \mu g(0) = 0$. So it is also a function in this set.

c. No. For any such function $f(x)$, $0 \cdot f(x) = 0$ takes 0 at 0 instead of 17.

d. Yes. For any two such functions $f(x), g(x)$, and any real numbers λ, μ, $\lambda f(17) + \mu g(17) = 0$. So it is also a function in this set.

e. No. For any such function $f(x)$, $0 \cdot f(x) = 0$ takes 0 at 17 instead of 17.

HW5

a. 225

b. 7.416198487095663

31.28897569432403

c. $\cos\theta = \frac{225}{7.416198487095663 \times 31.28897569432403} = 0.9696384473317716.$

$\theta = 0.24704836030593655 = 14.154828622612804°.$

g. $A^2 = \begin{pmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \\ 102 & 126 & 150 \end{pmatrix}$

$\begin{pmatrix} 1 & 4 & 9 \\ 16 & 25 & 36 \\ 49 & 64 & 81 \end{pmatrix}$

h. $x = \begin{pmatrix} 0.0349648997349724 \\ 0.251622660194788 \\ 0.43773815598541144 \end{pmatrix}$