pset4

September 7, 2017

1 18.06 Pset 4

Due Wednesday, March 8 at 11am.

1.1 Problem 1

(Similar to Strang, section 3.2, problem 49.)

We showed in class that $C(AB) \subseteq C(A)$. Since the dimension of the column space is the rank, and a subspace always has a dimension $\le$ the dimensionality of the enclosing space, this means that $\text{rank}(AB) \le \text{rank}(A)$.

Using a similar reasoning, show that $\text{rank}(AB) \le \text{rank}(B)$. Hint: consider the transpose $(AB)^T = B^T A^T$.

1.2 Problem 2

(Similar to Strang, section 3.4, problem 26 and 30.)

Find a basis (and the dimension) for each of these subspaces of 3×3 matrices:

- All diagonal matrices
- All symmetric matrices ($A^T = A$).
- All skew-symmetric (anti-symmetric) matrices ($A^T = -A$).
- All matrices whose nullspace contains the vector $(2, 1, -1)$.

2 Problem 3

(Strang, section 3.5, problem 21.)

Suppose $A = uv^T + wz^T$ (it is the sum of two rank-1 matrices).

- Which vectors span the column space of A?
- Which vectors span the row space of A?
- The rank of A is less than 2 if ???????? or if ????????.
- Compute A and its rank if $u = z = (1, 0, 0)$ and $v = w = (0, 0, 1)$. Check your answer with Julia below.

```
In [ ]: u = z = [1,0,0]
        v = w = [0,0,1]
        A = u*v' + w*z'
```

```
In [ ]: rank(A)
```

3 Problem 4

(Based on Strang, section 4.1, problem 9.)

The following is an important property of the very important matrix $A^T A$ (for real matrices) that will come up several times in 18.06:

- If $A^T A x = 0$ then $Ax = 0$. Reason: If $A^T A x = 0$, then Ax is in the nullspace of A^T and also in the ?????? of A, and those spaces are ???????. Conclusion: $N(A^T A) = N(A)$.
- Alternative proof: $A^T A x = 0$, then $x^T A^T A x = 0 = (Ax)^T (Ax)$. Why does this imply that $Ax = 0$? (Hint: if $y^T y = 0$, can we have $y \neq 0$?)
- If A is a random $m \times n$ matrix, what can you conclude about the ranks of $A^T A$ and AA^T? Try it in Julia for a 5×7 random matrix:

In [ ]:
```
A = randn(5,7)
```

In [ ]:
```
rank(A'*A)
```

In [ ]:
```
rank(A*A')
```