pset3

September 7, 2017

1 18.06 pset 3

Due **Wednesday March 1** at 11am.

Note: **Exam 1** is on Friday **March 3** in room **50-340**.

1.1 Problem 1

Suppose that you solve $AX = B$ with

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and find that X is

$$X = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{pmatrix}$$

1.1.1 (a)

What is A^{-1}?

(You should not have to apply brute-force Gaussian elimination to invert any matrices, nor should you use Julia in this part. You should be able to show how to do this quickly *by hand*.)

(This is not because we care about hand calculation *per se*, but rather because it is useful to be able to recognize and exploit special structure in matrices, and to understand the relationship between solving systems of right-hand-sides and finding A^{-1}.)

1.1.2 (b)

Evaluate a simple expression to *check* your answer from (a) by brute-force calculation in Julia.

For example, you can compute $B^{-1}X^{-1}$ by `inv(B) * inv(X)` in Julia. There should be some simple product of matrices or matrix inverses that gives A^{-1}. Figure it out!

```
In [ ]: # here are the matrices B and X in Julia form
        B = [1 1 1 1
             0 2 2 2
             0 0 1 1
             0 0 0 1]
        X = [1 1 0 1
             0 0 1 0
             1 3 1 0
             2 0 0 1]
```

```
In [ ]: inv(B) * inv(X) ## FIX THIS: change to an expression that will give A^-1, and evaluate
```

1.2 Problem 2

Consider the vector space $\mathcal{M}$ of $m \times m$ real-valued matrices for some m, say $m = 4$. True or false (and provide a counter-example if *false*).

1. The symmetric matrices in $\mathcal{M}$ are a subspace (matrices with $A^T = A$).
2. The "skew-symmetric (also called"antisymmetric") matrices (those with $A^T = -A$) in $\mathcal{M}$ are a subspace.
3. The invertible matrices in $\mathcal{M}$ are a subspace.
4. The singular matrices in $\mathcal{M}$ are a subspace.

1.3 Problem 3

(Strang, section 3.2, problem 22.) If $AB = 0$ then the column space of B is contained in the ______ of A. Why?

1.4 Problem 4

(Strang, section 3.2, problem 29.) If A is 4×4 and invertible, what is the nullspace of the 4×8 matrix $B = \begin{pmatrix} A & A \end{pmatrix}$?

1.5 Problem 5

(Strang, section 3.2, problem 23.) The reduced-row echelon form R of a 3×3 matrix with randomly chosen entries is almost sure to be _________. What R is virtually certain if the random matrix is 4×3?

1.6 Problem 6

(Strang, section 3.2, problem 58.) Suppose R is $m \times n$ of rank r, with pivot columns first:

$$R = \begin{pmatrix} I & F \\ 0 & 0 \end{pmatrix}$$

where I is an identity matrix and 0 denotes a block of zeros.

1. What are the shapes of those four blocks of R?
2. Find a *right-inverse* matrix B such that $RB = I$ if $r = m$ (the zero blocks are gone).
3. What is the reduced-row echelon form of R^T?
4. What is the reduced-row echelon form of $R^T R$?

(In the last four parts, indicate both blocks like I or 0 and their shapes.)