MIT 18.06 Exam 2, Spring 2017

Your name: ______________________________

Recitation: ______________________________

problem	**score**
1	/33
2	/33
3	/34
total	/100

Problem 1:

You are given the 6×6 matrix. (Not quite the same matrix as in exam 1: there is a 2 in the lower-right corner rather than a 1.)

$$A = \begin{pmatrix} 1 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & -1 & 2 & -1 & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{pmatrix}.$$

(a) Find the determinant of A. (Hint: elimination.)

(b) What is the projection matrix onto $C(A)$?

(c) If you perform Gram–Schmidt orthogonalization on the columns of A, what is the pattern of nonzero entries in the resulting orthogonal matrix Q? (*Don't* waste your time actually working out the numbers: just put x's where the nonzero entries will be.)

(blank page for your work if you need it)

Problem 2:

The equations of two lines in $\mathbb{R}^n$ are

$$\vec{y}_1(x_1) = \vec{a}_1 x_1 + \vec{b}_1$$

$$\vec{y}_2(x_2) = \vec{a}_2 x_2 + \vec{b}_2$$

where $\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2 \in \mathbb{R}^n$ and x_1 and x_2 are scalars.

Write down a 2×2 system $C\vec{x} = \vec{d}$ of linear equations for $\vec{x} = (x_1, x_2)$ whose solution gives the (x_1, x_2) that **minimizes the distance between the two lines**. That is, find the entries of C and d (in terms of $\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2$) so that $\vec{x} = C^{-1}d$ solves:

$$\min_{x_1, x_2} \left\| \vec{y}_1(x_1) - \vec{y}_2(x_2) \right\| .$$

Hint: write $\vec{y}_1(x_1) - \vec{y}_2(x_2)$ in terms of matrix/vector operations on $\vec{x}$ first.

(blank page for your work if you need it)

Problem 3:

(a) If P projects onto $C(A^T)$, the row space of some $m \times n$ matrix A, then $(I - P)^2 x$ for any $x \in \mathbb{R}^n$ gives a vector in which fundamental subspace?

(b) If A is a symmetric matrix and P is the projection matrix onto $N(A)$, what is PA?

(c) If P is a permutation matrix, what is its QR factorization?

(d) If A and B are two matrices such that $A^T B = 0$, with QR factorizations $A = Q_A R_A$ and $B = Q_B R_B$, write down the QR factorization of the matrix $C = \begin{pmatrix} A & B \end{pmatrix}$ (that is, C is the columns of A followed by the colums of B) in terms of Q_A, Q_B, R_A, R_B. (Hint: what is $Q_A^T Q_B$?)

(blank page for your work if you need it)